ANALYSIS OF THREE DIEMNSIONAL STRUCTURES BY FOLDED PLATE THEORY

-Analysis of Rectangular Plates with Elastic Support-

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1. INTRODUCTION

Composite materials are, generally, strong in tension. When an element is designed based on tension load, it will have a thin section which is weak against any loading type other than in-plane on-axis tension load. This requires the section modulus to be increased by means of joining thin-walled plates or sandwich panels. Even the one-dimensional element, after the frame is analyzed, requires additional study by the methods explained for thin-walled sections. The thin panels of such sections are weak against the loads normal to these panels. The longitudinal stringers are added between transverse diaphragms to take care of such loads. The diaphragms transmit the loads from stringers to the walls of the beam by means of in-plane shear.

Any curved surface can be considered as continuations of certain types of triangular plates. Therefore, the theory of nonprismatic folded plates can be applied to any type of shell structures[1].

If the finite difference method is used to integrate the differential equations, some elaborate work is necessary. A method of solving such a problem was reported by Kim, D. H.[1, 2, 3, 4]. The process of calculation is straightforward. A very high degree of accuracy can be obtained by this method[1, 2].

With the method of analysis as metioned above available, the problem is reduced to solving a plate, either prismatic or non-prismaic, with arbitrary elastic boundary conditions.

In this paper, the result of analysis of a rectangular plate, with arbitrary elastic boundary conditions is presented. Finite difference method is used in analysis.

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2. FINITE DIFFERENCE METHOD (F.D.M.)

The method used in this paper requires the deflection influence surfaces. Since no reliable analytical method is available for the subject problem, F.D.M. is applied to the governing equation of the special orthotropic plates[4, p292],

$$D_1\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 w}{\partial y^4} = -q(x,y) + kw + Nx\frac{\partial^2 w}{\partial x^2} + Ny\frac{\partial^2 w}{\partial y^2} + 2Nxy\frac{\partial^2 w}{\partial x \partial y} \tag{1}$$

where $D_1=D_{11}$, $D_2=D_{22}$, $D_3=(D_{12}+2D_{66})$.

The number of the pivotal points required, in the case of the order of error Δ^4, where Δ is the mesh size, is five for the central differences. This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, ω, M_x, M_y, are used instead of equation(4),[1, 4].

$$\frac{\partial^2 M_x}{\partial x^2} + 4D_{66}\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial M_y}{\partial y^2} = -q(x,y) + kw(x,y) + Nx\frac{\partial^2 w}{\partial x^2} + Ny\frac{\partial^2 w}{\partial y^2} + 2Nxy\frac{\partial^2 w}{\partial x \partial y} \tag{2}$$

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \tag{3}$$

$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \tag{4}$$

If F.D.M. is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim, D. H.[1, 2, 3] is very efficient to solve such equations.

3. NUMERICAL EXAMINATION

$[A/B/A]_r$ type laminate is considered. The material properties are ;

E_1 = 38.6 GPa,	E_2 = 8.27 GPa,	
ν_{12} = 0.26,	ν_{21} = 0.0557,	a=b=1m.
G_{12} = 4.14 GPa,	H = 0.005×3×3=0.045 m,	

The thickness of a ply is 0.005m. As the r increases, B_{16}, B_{26}, D_{16} and D_{26} stiffnesses decrease and the equations for the special orthotropic plates can be used. For simplicity, it is assumed that A=0°, B=90°, and r=3. Then D(1,1)=227219 Nm., D(1,2)=1668.02 Nm., D(2,2)=133930.2 Nm. and

D(6,6)=31438.12 Nm. The plate geometry and applied load is as given in Table 1 and Figure 1.

Table 1. Cases used for numerical example.

CASE / Forces	CASE A	CASE B	CASE C	CASE D
K (N/m^2)	10^6	10^9	10^{12}	∞
N_x (N)	1000	1000	1000	1000
N_y (N)	100	100	100	100
N_{xy} (N)	50	50	50	50

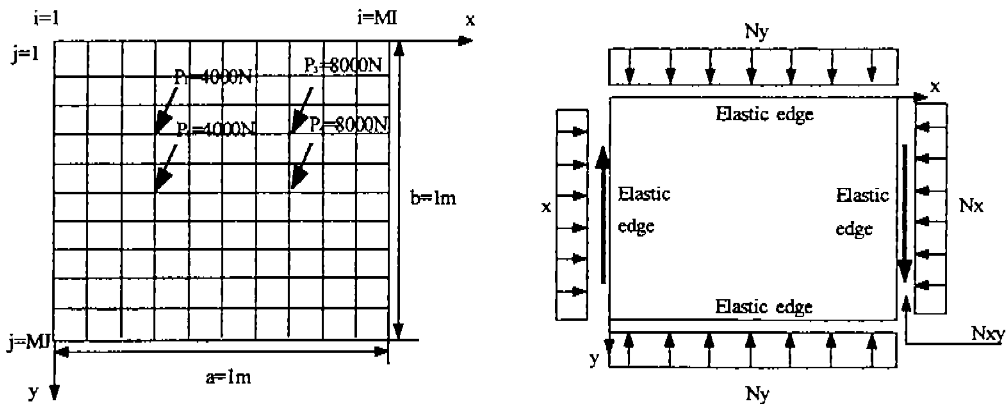

Figure 1. The plate geometry and applied load.

The applied load is the uniform load q=10000 N/m^2 plus the wheel loads as shown by Figure 1. The influence of the modulus of foundation, k, is studied by changing k values from 10×10^6 N/m^2 to ∞ N/m^2. Table 2 shows the deflections at the wheel load points for each case, under changing values of k. From Table 3 to Table 5 show the moment Mx, My, and Mxy at the wheel load points for each case.

Table 2. Deflection for each case at the load point of the plate.

(unit : m)

CASE / Load Point	CASE A	CASE B	CASE C	CASE D
(4,4)	0.282991E-06	0.291389E-06	0.281376E-06	0.281368E-06
(4,6)	0.142907E-06	0.145815E-06	0.144086E-06	0.144080E-06
(8,4)	-0.295377E-07	-0.308847E-07	-0.272046E-07	-0.272102E-07
(8,6)	-0.313308E-07	-0.271252E-07	-0.290294E-07	-0.290340E-07

Table 3. M_x for each case at the load points of the plate.

(unit : m)

CASE / Load Point	CASE A	CASE B	CASE C	CASE D
(4,4)	0.105098E+02	0.107567E+02	0.104113E+02	0.104112E+02
(4,6)	0.458656E+01	0.464212E+01	0.458462E+01	0.458455E+01
(8,4)	-0.957057E+00	-0.104449E+01	-0.789470E+00	-0.790260E+00
(8,6)	-0.104542E+01	-0.862662E+00	-0.848574E+00	-0.849395E+00

Table 4. My for each case at the load points of the plate.

(unit : m)

CASE / Load Point	CASE A	CASE B	CASE C	CASE D
(4,4)	0.131479E+02	0.135117E+02	0.130757E+02	0.130753E+02
(4,6)	0.616322E+01	0.628839E+01	0.621359E+01	0.621333E+01
(8,4)	-0.131469E+01	-0.138042E+01	-0.120631E+01	-0.120660E+00
(8,6)	-0.139430E+01	-0.120552E+01	-0.128537E+01	-0.128562E+01

Table 5. M_{xy} for each case at the load points of the plate.

(unit : m)

CASE / Load Point	CASE A	CASE B	CASE C	CASE D
(4,4)	0.417182E-01	0.104814E-01	0.800236E-01	0.800340E-01
(4,6)	0.678170E-01	0.545268E-01	0.728692E-01	0.728750E-01
(8,4)	-0.255833E-01	-0.148311E-01	-0.965409E-02	-0.967337E-02
(8,6)	-0.474360E-02	-0.361662E-02	-0.210712E-02	-0.210921E-02

4. CONCLUSION

The finite difference method (F.D.M.) is used to obtain the deflection influence surfaces in this paper. In order to reduce the required number of pivotal points, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, Mx, and My, are used instead of the fourth order partial differential equation for the special orthotropic plate. If F.D.M. is applied to these equations, the resulting matrix equation is huge in sizes, but the tri-diagonal matrix calculation scheme used by Kim, D. H. is very efficient to solve such problems.

The effect of the modulus of foundation, the in-plane Nx, Ny, Nxy forces of the special orthotropic plate, and the concentrated attached mass on the plate, on the deflection and moment is thoroughly studied and the result is given in tables to provide a guideline to the design engineers.

REFERENCES

[1] Goldberg, J. E. and Kim, D. H.(1996) "Analysis of triangularly folded plate roofs of umbrella type", Proc. of 16th General Congress of Applied Mechanics, Tokyo, Japan, Oct., p. 280

[2] Kim, D. H.,(1967) "Theory of non-prismatic folded plate structures", Trans. Korea Military Academy (ed. Lee, S. H.), 5, Aug., pp. 182-268.

[3] Kim, D. H.,(1966) Matrix analysis of multiple shells, Proc. Korean Soc. Civ. Engrs., 13(4), 9.

[4] Kim, D. H., *Composite Structures for Civil and Architectural Engineering*, E & FN SPON, Chapman & Hall, London, 1995.