Fuzzy Mean Method with Bispectral Features for Robust 2D Shape Classification

Youngwoon Woo*, Soowhan Han**

Abstract

In this paper, a translation, rotation and scale invariant system for the classification of closed 2D images using the bispectrum of a contour sequence and the weighted fuzzy mean method is derived and compared with the classification process using one of the competitive neural algorithm, called a LVQ(Learning Vector Quantization). The bispectrum based on third order cumulants is applied to the contour sequences of the images to extract fifteen feature vectors for each planar image. These bispectral feature vectors, which are invariant to shape translation, rotation and scale transformation, can be used to represent two-dimensional planar images and are fed into an classifier using weighted fuzzy mean method. The experimental processes with eight different shapes of aircraft images are presented to illustrate the high performance of the proposed classifier.

Key words : Bispectrum, LVQ, Weighted Fuzzy Mean

Ⅰ. Introduction

The studies on two-dimensional object recognition problem have broad applications such as satellite image identification, the characterization of biomedical images, and the recognition of industrial parts by robots for product assembly. Most of these shape recognition systems require an object to be classified in situations where the position, orientation and distance of the object are time-varying. Additionally, the systems are required to be tolerant to noisy shapes results from the segmentation of objects in varying backgrounds as well as non-ideal imaging conditions. There have been over a dozen prior research efforts to improve the performance of system including Fourier descriptors[1], autoregressive modeling method[2]-[4], dynamic alignment process of contour sequences[5], and neural network approach[6]-[10].

The accuracy on pattern recognition problems, while keeping simplicity of the overall system, depends on two important factors. One is to extract feature vectors representing a 2D object image. The feature vectors should have the small dimensionality for real-time process, the similarity between intraclass. In this study, the boundary of a closed planar shape is characterized by an ordered sequence that represents the Euclidean distance between the centroid and all boundary pixels since the overall shape information is contained in the boundary of the shape. Then, the contour sequence is normalized with respect to the size of image. This normalization includes the amplitude and the duration of the contour sequence. Next the bispectrum based on third order cumulants is applied to this normalized contour sequence as a means of feature selection. Higher order spectra (bispectrum, trispectrum) play an important role in digital signal processing due to their ability of preserving nonminimum phase information, as well as information due to deviations from Gaussianity and degrees of nonlinearities in time series[11]. In the previous works for recognition systems[8][9], the spectrum feature vectors were extracted from power spectrum density of contour sequence. However, the power spectrum of contour sequence is corrupted by white gaussian noise power $E[n^2]=\delta_n^2$ in the all frequency components where the bispectrum is not. The reason for that will be shown in next section and Han's work presents that the bispectral feature vector has a better noisy tolerant characteristic[10]. Therefore, in this investigation of 2D object classification, the bispectral components of the normalized contour sequence of an object image are utilized as feature vectors. These bispectral feature vectors have enough shape information to represent each 2D object, a property to be invariant in size, shift, and rotation, and are used as the input of fuzzy classifier.

Another factor is to select an appropriate classifier architecture for this particular recognition task. In a recent year, the neural network algorithms[8]-[10] and the fuzzy memberships functions[12]-[14] are widely used. However, the hybrid neural structure with back-propagation and counter-propagation in [8] and with two fuzzy ART modules in [9] is relatively

* Dept. of Computer Eng., Dongeui Univ.
** Dept. of Multimedia Eng., Dongeui Univ.

complicated, and the fuzzy ARTMAP in [9] had used the *five*-voting strategy (repeat five simulations with different ordering of training patterns) to avoid the ordering influence of training patterns. Moreover it is hard to select an optimal matching of specific neural network architecture for this kind of recognition system among many different neural models. Thus, a triangular fuzzy membership function and an weighted fuzzy mean method are utilized as a classifier. This fuzzy classifier has a simple structure and it can easily improve the classification results by an weighted fuzzy mean extracted from analyzing the bispectral feature vectors. In the experimental procedure, the proposed fuzzy classifier is tested with eight different shapes of aircraft images and compared with the results of the previous work[10] using a LVQ, one of the competitive neural classifier.

II. Shape Information and Bispectral Feature Extraction

In this portion of the study, the boundary of a closed planar shape is characterized by an ordered sequence that represents the Euclidean distance between the centroid and all contour pixels of the digitized shape. Clearly, this ordered sequence carries the essential shape information of a closed planar image. The bispectral feature extraction from a closed planar image is done as follows. First, the boundary pixels are extracted by using contour following algorithm and the centroid is derived [15][16]. The second step is to obtain an ordered sequence in a clockwise direction, b(i), that represents the Euclidean distance between the centroid and all boundary pixels. Since only closed contours are considered, the resulting sequential representation is circular as equation (1).

$$b(i)=\sqrt{(x_i-x_c)^2+(y_i-y_c)^2}$$
$$b(N+i)=b(i) \quad i=1,2,3,\ldots\ldots,N \tag{1}$$

where (x_c,y_c) : the centroid of an image, (x_i,y_i) : the contour pixel, N : the total number of boundary pixels.

This Euclidean distance remains unchanged to a shift in the position of original image. Thus the sequence $b(i)$ is invariant to translation. The next step is to normalize the contour sequence with respect to the size of image. Scaling a shape results in the scaling of the samples and duration of the contour sequence. Thus scale normalization involves both amplitude and duration normalization. The normalized duration of the sequence, 256 points fixed, is obtained by resampling operation and function approximation. This is shown in equation (2).

$$c(k)=b(k*N/256) \quad k=1,2,3\ldots\ldots,256 \tag{2}$$

where N is the total number of boundary pixels.

After duration normalization, amplitude is divided by sum of contour sequence and removed the mean. It is shown in equation (3) and (4).

$$d(k)=c(k)/s \quad k=1,2,3,\ldots\ldots,256 \tag{3}$$

$$d(k)=d(k)-mean(d(k)) \tag{4}$$
$$\text{where } s=c(1)+c(2)+c(3)+\ldots\ldots+c(256)$$

This sequence $d(k)$ is invariant to translation and scaling. In a forth, bispectral feature measurement is taken into the contour sequence. The spectral density of the sequence $d(k)$ utilized in this paper is a third-order spectrum, called a bispectrum. In general, the higher-order spectra can address noise suppression, and preserve nonminimum phase information as well as the information due to degrees of nonlinearities[17]. In this study to 2D shape classification, the bispectrum of contour sequence $d(k)$, instead of power spectrum, is investigated for feature vectors because of its better noisy-tolerant characteristic [10][17]. The *n*th order cumulants spectrum of contour sequence d(k) is defined as

$$\begin{aligned} &H_n(\omega_1,\cdots,\omega_{n-1}) \\ &=\frac{1}{(2\pi)^{n-1}}\sum_{\tau_1=-\infty}^{+\infty}\cdots\sum_{\tau_{n-1}=-\infty}^{+\infty}C_d(\tau_1,\cdots,\tau_{n-1}) \\ &=F(\omega_1)\cdots F(\omega_{n-1})F^*(\omega_1+\cdots+\omega_{n-1}) \end{aligned} \tag{5}$$

where C_d, F are cumulants and Fourier transform of the sequence $d(k)$, respectively. For the special cases where n=2(power spectrum) and n=3 (bispectrum) :

$$\begin{aligned} H_2(\omega) &=\frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}C_d(\tau)e^{-j\omega\tau} \\ &=F(\omega)F^*(\omega) \end{aligned} \tag{6}$$

where $C_d(\tau)=E[d(k)d(k+\tau)]$: Expectation of $d(k)d(k+\tau)$.

$$\begin{aligned} &H_3(\omega_1,\omega_2) \\ &=\frac{1}{(2\pi)^2}\sum_{\tau_1=-\infty}^{+\infty}\sum_{\tau_2=-\infty}^{+\infty}C_d(\tau_1,\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)} \\ &=F(\omega_1)F(\omega_2)F^*(\omega_1+\omega_2) \end{aligned} \tag{7}$$

where $C_d(\tau_1,\tau_2)=E[d(k)d(k+\tau_1)d(k+\tau_2)]$ and $|\omega_1|\le\pi,\ |\omega_2|\le\pi,\ |\omega_1+\omega_2|\le\pi$.

If the observed contour sequence $d(k)=s(k)+n(k)$ where $s(k)$: the zero mean contour sequence without noise, $n(k)$: the zero mean white gaussian noise sequence and they are independent, equations (6) and (7) becomes

$$\begin{aligned} &H_2(\omega) \\ &=\frac{1}{2\pi}\sum_{\tau=-\infty}^{+\infty}C_s(\tau)e^{-j\omega\tau}+\frac{1}{2\pi}\sum_{\tau=-\infty}^{+\infty}C_n(\tau)e^{-j\omega\tau} \quad (8) \\ &=H_s(\omega)+H_n(\omega) \\ &=H_s(\omega)+\frac{1}{2\pi}\left(\frac{N_0}{2}\right) \quad (9) \end{aligned}$$

where $C_s(\tau)=E[s(k)s(k+\tau)]$ and

$C_n(\tau)=E[n(k)n(k+\tau)]=\frac{N_0}{2}\delta(\tau)$.

$$H_3(\omega_1,\omega_2)=\frac{1}{(2\pi)^2}\sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty}C_s(\tau_1,\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)}+\frac{1}{(2\pi)^2}\sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty}C_n(\tau_1,\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)} \quad (10)$$

$$=H_s(\omega_1,\omega_2)+H_n(\omega_1,\omega_2)$$

$$=H_s(\omega_1,\omega_2)+\gamma_n$$

$$=H_s(\omega_1,\omega_2)+E[n^3(k)] \quad (11)$$

where $C_s(\tau_1,\tau_2)=E[s(k)s(k+\tau_1)s(k+\tau_2)]$ and $C_n(\tau_1,\tau_2)=E[n(k)n(k+\tau_1)n(k+\tau_2)]=\gamma_n\delta(\tau_1,\tau_2)$.

In equation (11), the noisy bispectrum $H_n=E[n(k)^3]=\gamma_n$ becomes zero because of skewness of noisy density function, which means the bispectrum suppress the white noisy portion and the extracted feature vectors have better noisy tolerance than the feature vectors from the power spectrum. The performance comparisons with noise for robust 2D shape classification between the power spectral and the bispectral features are shown in [10]. And trispectrum with cumulants order n=4 contains the noisy spectrum because of kurtosis of noisy density and the higher spectra with cumulant order more than n=4 have not widely used yet because of their computational complexity and the difficulty of feature extraction from n-dimensional spectrum space. Therefore the bispectrum is utilized for feature selection of 2D shape images in this paper.

The magnitude of bispectrum derived in a forth step, $|H_3(\omega_1,\omega_2)|$, is unchanged even after the sequence d(k) is circular shifted because the magnitude of Fourier transform, $|F(\omega)|$, is not changed[18]. Thus $|H_3(\omega_1,\omega_2)|$ is invariant to the rotation of an image. Finally, the two dimensional bispectral magnitude (256 by 256) is projected to vertical axis (ω_1) by taking mean value of each column for feature extraction. It is shown in equation (12).

h[k]=[mean(kth column of $|H_3(\omega_1,\omega_2)|$)] (12)
where k=1,2,....256.

The first column and the row in the magnitude of bispectrum contain all zero value because the normalized contour sequence has a zero mean. It means h(1) is always zero. And the projected bispectral components exceed to the sixteenth have very small values (near zero). Thus, for fast classification process with reliable accuracy, the projected bispectral components from the second to the sixteenth (h(2), h(3),.....h(16)) are chosen to be used as feature vectors to represent each image shape, which are fed into a proposed fuzzy classifier for classification process. These feature vectors have the desired format for planar image classification system, which means they are invariant to translation, rotation and scaling of the shape and highly tolerant to the noise. Figure 1, 2 and 3 show the fifteen projected bispectral feature vectors of rotated image, two different shapes and 10dB noisy image.

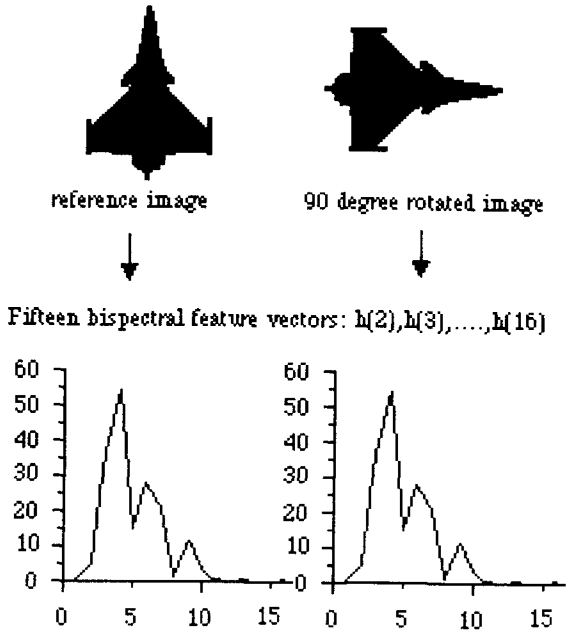

Figure 1. Bispectral feature vectors extracted from reference and rotated images

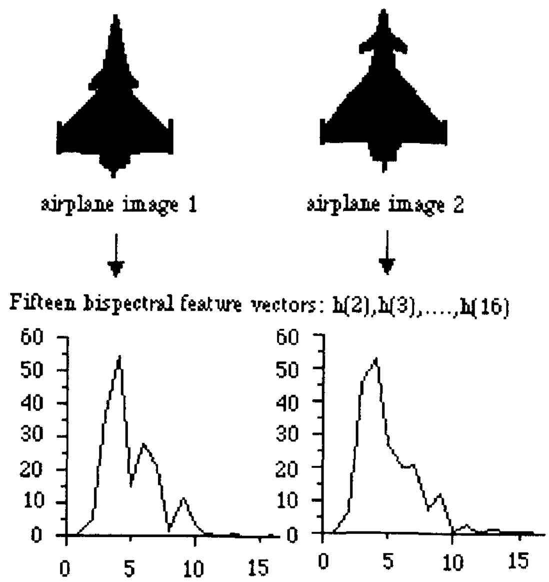

Figure 2. Bispectral feature vectors extracted from two different images

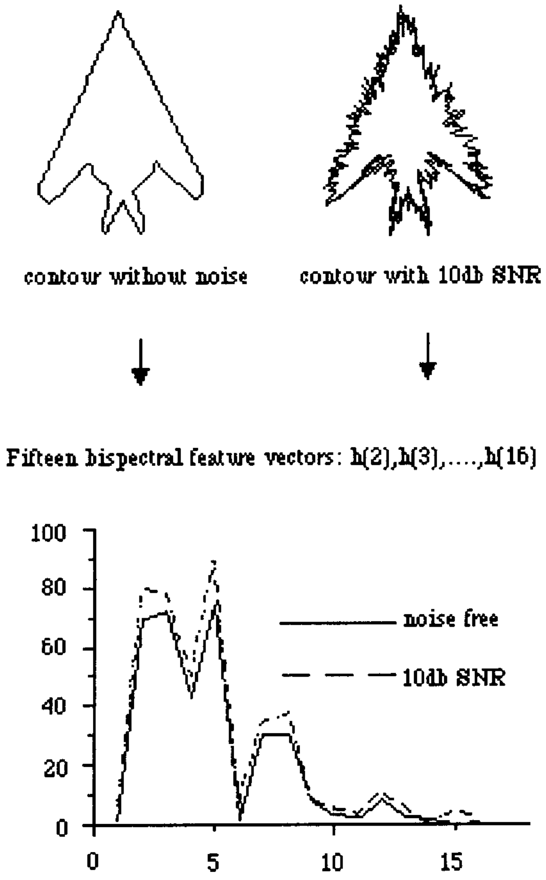

Figure 3. Bispectral feature vectors extracted from the contour images without noise and with 10dB SNR

III. The Proposed Classifier Using Weighted Fuzzy Mean Mehtod

There are various methods to construct the fuzzy classifier depending on the type of fuzzy membership function and the calculation method of mean value for membership grades[14]. The two of most popular types for fuzzy membership function are triangle and trapezoid[19]. And the arithmetic mean written in equation (13), the harmonic mean in equation (14) and the weighted mean in equation (15) are widely used for the calculation of mean value[20].

$$h_1(\mu_1(x), \mu_2(x), \ldots, \mu_n(x)) = \frac{1}{n} \sum_{i=1}^{n} \mu_i(x) \qquad (13)$$

$$h_{-1}(\mu_1(x), \mu_2(x), \ldots, \mu_n(x)) = \frac{n}{\sum_{i=1}^{n} \frac{1}{\mu_i(x)}} \qquad (14)$$

$$h_w(\mu_1(x), \mu_2(x), \ldots, \mu_n(x); w_1, w_2, \ldots, w_n) = \sum_{i=1}^{n} \mu_i(x) \cdot w_i \ , \ \left(\sum_{i=1}^{n} w_i = 1 \right) \qquad (15)$$

where μ_i is an ith membership grade, w_i is an ith weight and n is the number of fuzzy membership functions.

The proposed classification method in this work uses the triangular type of fuzzy membership function and the weighted fuzzy mean method whose variance is utilized for weights. The triangular type of fuzzy membership functions is useful where the only one reference feature set is available as in this study. The one of advantage of the proposed method is not required the training stage unlike the neual network structure. Only 120 fuzzy membership functions (15 × 8: the one membership function for each of fifteen dimensional feature values × the eight reference aircraft images) and 15 variances for each dimensional value of feature vector are established for classification process. This preprocessing step is much faster and easier than that of the conventional training stage of neural classifier. Another advantage of this fuzzy classifier is the use of a variance as an weight. In general, it is hard for the neural classifiers to improve the performance results because they are highly depend on the architectures, learning algorithm and training order[9][21]. However, the improvement of classification results for the proposed method is easily achieved with the new information extracted from analyzing the characteristics of the bispectral feature vectors. That is shown in next section. Therefore the triangular fuzzy membership function and the weighted fuzzy mean method using variance are utilized as a proposed classification method.

IV. Experimental Results and Performance Assessment

The methodology presented in this paper, for the classification of closed planar shape, was evaluated with eight different shapes of aircraft. They are shown in figure 4.

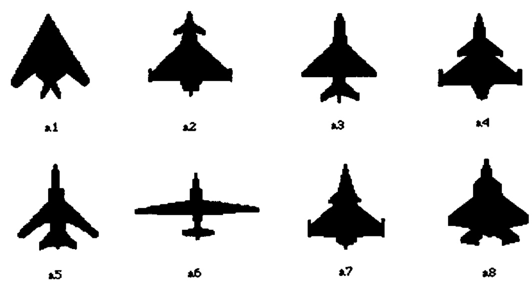

Figure 4. Eight different shapes of reference aircraft images

From each reference shape of aircraft, 36 noisy-free patterns were generated by rotating the original image with 30 degree increment and scaling with three factor (1, 0.8 and 0,6). And forty noisy corrupted patterns were made by adding four different level of random gaussian noise (25dB, 20dB, 15dB, 10dB SNR : ten noisy patterns for each SNR) to 36 noisy-free patterns. Thus the data set for each reference aircraft image has 36 noisy-free patterns and 1440 (40×36) noisy corrupted patterns. The number of total test patterns becomes 11808 (1476×8 reference image). The sample contour images for a4 and a7 with noise-free and with 10dB SNR are shown in figure 5.

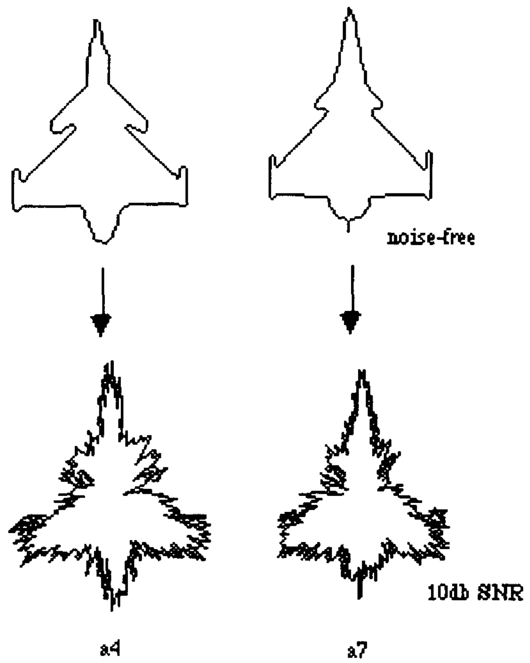

Figure 5. The sample contour images for a4 and a7 with noise-free and with 10dB SNR

The construction of a proposed method and the classification process are done by as follows. First, the fifteen fuzzy membership functions for each reference aircraft image are established by using each of the fifteen dimensional bispectral feature values. The fuzzy membership functions are defined by equation (16).

$$y = 0.01 \cdot (x - f_i) + 1 \quad (-100 + f_i < x < f_i)$$

$$y = -0.01 \cdot (x - f_i) + 1 \quad (f_i < x < 100 + f_i) \qquad (16)$$

otherwise $y = 0$

where 0.01 is the slope of a fuzzy membership functions, x is an input feature value, f is an reference feature value and y is an membership grade for x.

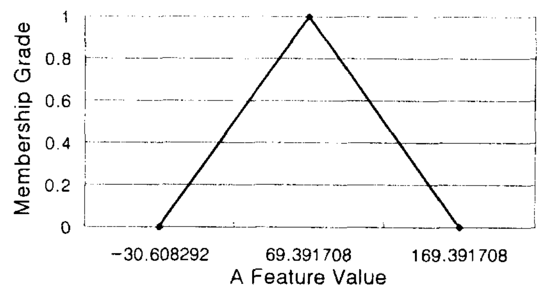

Figure 6. A fuzzy membership function extracted from a first feature value for an aircraft image, a1

All of membership functions are configured as a triangular type shown in figure 6. The number of total fuzzy membership functions becomes 120 for the eight different type of aircraft images. In a second step, the

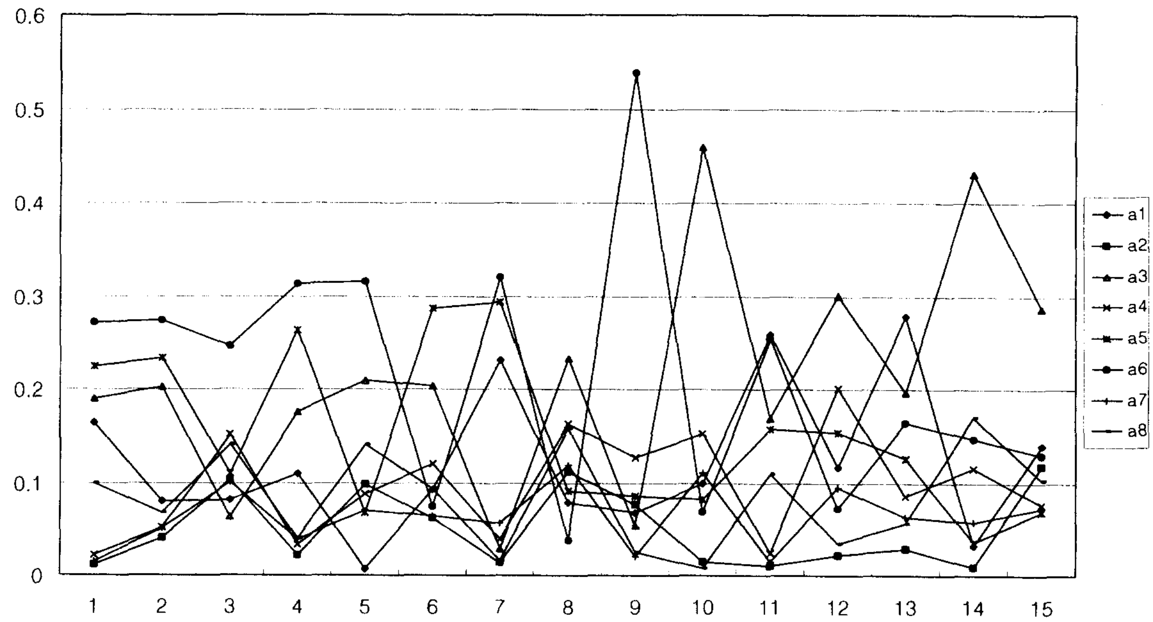

Figure 7. Normalized amplitudes of bispectral features

variances for each of the normalized fifteen dimensional feature values with eight reference aircraft images are derived by equation (17) and (18). The normalized feature values from reference aircraft a1 to a8 are shown in figure 7. Those variances are utilized as the weights for classification process.

$$m_j = \frac{1}{8} \sum_{i=1}^{8} x_{ij} \quad (j=1..15) \tag{17}$$

where m_j is a mean of jth feature values for the eight different aircraft images and x_{ij} is a jth feature value for aircraft image ai

$$vr_j = \frac{1}{8} \sum_{i=1}^{8} (x_{ij} - m_j)^2 \quad (j=1..15) \tag{18}$$

where vr_j is a variance of *j*th feature values for the eight different aircraft images(a1,a2,...,a8).

In a third, the fifteen bispectral feature values of the incoming test aircraft image are applied to the corresponding fuzzy membership function for each of eight reference shapes and the membership grades are computed by equation (16). The fifteen membership grades for any one of eight reference shapes present the degree of similarity with that reference shape.

Finally the weighted mean values of the membership grades for each of eight reference aircraft shape are computed by equation (19) and (20), and a reference shape of aircraft image having the largest weighted mean value (the largest h_i) is chosen as a classification result.

$$w_j = vr_j \quad (j=1..15) \tag{19}$$

where w_j is an weight for the *j*th feature value.

$$\begin{aligned} h_i(&\mu_{i1}(x), \mu_{i2}(x), \ldots, \mu_{i15}(x); \\ &w_1, w_2, \ldots, w_{15}) \\ &= \sum_{j=1}^{15} \mu_{ij}(x) \cdot w_j\,, \quad (i=1..8) \end{aligned} \tag{20}$$

where μ_{ij} is an membership grade for the *j*th feature value of aircraft image *ai* computed by equation (16), and h_i is an weighted fuzzy mean value for each of eight reference aircraft images.

The experimental process was performed under six different experimental environments. The experiments of 2, 4, and 6 are performed by the proposed method and the others are performed by the neural classifier, a LVQ, used in previous work[10]. These are as follows.

Experiment 1.
Neural classifier algorithm: LVQ1 with 8 output clusters (one cluster for each reference shape).
Training data set: only the 8 reference aircraft images.

Experiment 2.
Classifier algorithm: the weighted fuzzy mean using variance.
Reference data set for membership function: same as training data set of experiment 1.

Experiment 3.
Classifier algorithm: LVQ1 with 8 output clusters (one cluster for each reference shape).
Training data set: 8 reference patterns + 32 noisy patterns (4 noisy patterns with 25dB SNR generated from each of 8 reference images).

Experiment 4.
Classifier algorithm: same as 2.
Reference data set for membership function: average of training data set of experiment 3.

Experiment 5.
Neural classifier algorithm: an improved LVQ algorithm called LVQ3 with 16 output clusters (two clusters for each reference shape).
Training data set: 8 reference patterns + 32 noisy patterns (4 noisy patterns with each of 25dB, 20dB, 15dB and 10dB SNR generated from each of 8 reference images).

Experiment 6.
Classifier algorithm: same as 2.
Reference data set for membership function: average of training data set of experiment 5.

Under each of six different experimental environments, 11808 of total test patterns (1476 patterns for each reference image) were evaluated. The overall classification results of experiments 1-6 are summarized in table 1. In table 1, the average results of a proposed fuzzy classifier are compared with the results of a LVQ neural classifier. In experiments of 4 and 6 for a fuzzy classifier, the membership functions for each of eight reference shapes are constructed with a noise-free pattern and four of randomly selected noisy patterns. It means the classification results slightly depend on the selection of noisy patterns. By the same case, in experiments of 3 and 5 for a LVQ classifier, the classification results depend on the selection of noisy patterns for training. Therefore the five independent experiments with different styles of noisy patterns keeping the same SNR were evaluated and the results were averaged.

The classification results with both of a LVQ and a fuzzy classifier can be increased by adding some noisy patterns to training process and to construction of membership function, respectively. These are shown in the results of experiment 1, 3 and 5 with the LVQ neural classifier and in the results of experiment 2, 4 and 6 with the proposed method. And table 1 shows that both classification methods perform well to recognize the eight different shapes of images where the signal power is relatively larger than the noise power. However, the experimental results of 1, 2, 3, and 4 with 10dB SNR show that a LVQ neural classifier is more easily affected by noise. It means that the proposed classification method measuring the

Table 1. Comparison of the classification performance using average results

	Experiment 1 (LVQ1)	Experiment 2 (Weighted Fuzzy Mean)	Experiment 3 (LVQ1)	Experiment 4 (Weighted Fuzzy Mean)	Experiment 5 (LVQ3)	Experiment 6 (Weighted Fuzzy Mean)
Noise-free	288/288 (100%)	288/288 (100%)	288/288 (100%)	288/288 (100%)	288/288 (100%)	288/288 (100%)
25dB	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)
20dB	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)
15dB	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)	2880/2880 (100%)
10dB	2765/2880 (96.01%)	2806/2880 (97.43%)	2767/2880 (96.08%)	2814/2880 (97.71%)	2852/2880 (99.03%)	2859/2880 (99.27%)
Total # of correctly classified patterns(%)	11693/11808 (99.03%)	11732/11808 (99.37%)	11695/11808 (99.04%)	11742/11808 (99.44%)	11780/11808 (99.76%)	11787/11808 (99.82%)

weighted fuzzy mean is more effective than a LVQ measuring the Euclidean distance where the images are highly corrupted by noise.

V. Conclusion

The high classification results including the performance comparison with a LVQ show that the weighted fuzzy classifier with the fifteen bispectral feature vectors extracted from the normalized contour sequences of planar shapes, performs well to classify the different shapes of aircraft even the aircraft images are rotated, scaled and significantly corrupted by noise. Additionally, the fuzzy classifier can easily improve the classification results by analyzing the incoming feature vectors and also it does not require the training stage as the neural network classifier does.

In a near future, more realistic data such as the satellite images or the biomedical images will be tested and investigated for the practical applications. At the same time, the way to segment the shape of image from noisy background should be considered.

References

[1] C. C. Lin and R. Chellappa, "Classification of partial 2-D shapes using Fourier descriptors, "*IEEE Trans. on PAMI*, Vol. 9, No. 5, pp. 686-690, Sep., 1987.

[2] S. R. Dubois and F. H. Glanz, "Autoregressive Model Approach to Two-Dimensional Shape Classification," *IEEE Trans. on PAMI*, Vol. 8, No. 1, Jan., 1986

[3] I. Sekita, T. Kurita, and N. Otsu, "Complex Autoregressive Model for Shape Recognition," *IEEE Trans. on PAMI*, Vol. 14, No. 4, April 1992.

[4] M. Das, M. J. Paulik, and N. K. Loh, "A Bivariate Autoregressive Modeling Technique for Analysis and Classification of Planar Shapes," *IEEE Trans. on PAMI*, Vol. 12, No. 1, Jan., 1990.

[5] L. Gupta and M.D. Srinath, "Invariant Planar Shape Recognition Using Dynamic Alignment," *Pattern Recognition*, Vol.21, No. 3, pp. 235-239, 1988.

[6] L. Gupta and M.R.Sayeh and R. Tammana, "A neural network approach to robust shape classification," *Pattern Recognition*, Vol. 23, No. 6, pp. 563-568, 1990.

[7] Lilly and M. B. Reid, "Robust position, scale, and rotation invariant object recognition using higher-order neural networks," *Pattern Recognition*, Vol. 25, No. 9, pp. 975-985,1992.

[8] B. H. Cho, "Rotation, translation adn scale invariant 2-D object recognition using spectral analysis and a hybrid neural network," *Florida Institute of Technology*, Melbourne, Florida, U.S.A., Ph.D. Thesis 1993.

[9] S. W. Han, "Robust Planar Shape Recognition Using Spectrum Analyzer and Fuzzy ARTMAP," *Journal of Fuzzy Logic and Intelligent Systems*, Vol 5, No. 4, pp. 33-40, June, 1997.

[10] S. W. Han, "A Study on 2-D Shape Recognition Using Higher-Order Spectral and LVQ," *Journal of Fuzzy Logic and Intelligent Systems*, Vol 9 No. 3 pp. 285-293, 1999.

[11] C. L. Nikias and M.R. Raghuveer, "Bispectrum estimation : A digital signal processing framework," *Proc. IEEE*, Vol. 75, No.7, pp.869-891, July, 1987.

[12] A. Kandel, *Fuzzy Techniques in Pattern Recognition*, John Wiley & Sons, Inc., 1982.

[13] H. -J, Zimmermann, *Fuzzy Set Theory - and Its Applications*, Kluwer Academic Publishers, 1996.

[14] F. Hoppner, F. Klawonn, R. Kruse and F. Klowan, *Fuzzy Cluster Analysis : Methods for*

Classification, Data Analysis and Image Recognition, John Wiley & Son, June, 1999.

[15] R. C. Gonzalez adn R.E. Woods, *Digital Image Processing*, Addison-Wesley Publishing Company, Inc., 1992.

[16] W. K. Patt, *Digital Image Processing, 2nd ed.*, Wiley Interscience, 1991.

[17] C. L. Nikias and J. M. Mendel, *Signal Processing with Higher-Order Spectra*, United Signals & Systems, Inc., 1990.

[18] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing*, Prentice-Hall, N.J., 1975.

[19] D. Dubois and H. Prade, *Fuzzy Sets and Systems*, Academic Press, New York, 1980.

[20] G. J. Klir and T. A. Folger, *Fuzzy Sets, Uncertainty, and Information*, Prentice Hall, New Jersey, 1988.

[21] L. Fausett, *Fundamentals of Neural Networks : Architectures, Alogrithm, and Applications*, Prentice Hall, 1994