Trading rule extraction in stock market using the rough set approach

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Abstract

In this paper, we propose the rough set approach to extract trading rules able to discriminate between bullish and bearish markets in stock market. The rough set approach is very valuable to extract trading rules. First, it does not make any assumption about the distribution of the data. Second, it not only handles noise well, but also eliminates irrelevant factors. In addition, the rough set approach appropriate for detecting stock market timing because this approach does not generate the signal for trade when the pattern of market is uncertain. The experimental results are encouraging and prove the usefulness of the rough set approach for stock market analysis with respect to profitability.

Key words: the rough set approach, trading rule, stock market timing

1. Introduction

For a long time, stock market prediction is long-cherished desire of investors, speculators, and industries. Although several studies investigated to predict price movements in stock market, financial time series too complex and noisy to forecast. Many researchers predicted the price movements in stock market using artificial intelligence (AI) techniques during past decades. Kimoto *et al.* (1990) used several learning algorithms and prediction methods for the prediction systems of Tokyo stock exchange prices index (TOPIX). Their system used modular neural network to learn the relationships among various factors. Kamijo and Tanikawa (1990) used recurrent neural network and Ahmadi (1990) employed backpropagation neural network with generalized delta rule to predict the stock market. Yoon and Swales (1991) also performed prediction using qualitative and quantitative data. Some researchers investigated the issue of predicting stock index futures market. Trippi and DeSieno (1992) and Choi *et al.* (1995) predicted daily direction of change in S&P 500 index futures using ANNs. Duke and Long (1993) executed daily prediction of German government bond futures using feedforward backpropagation neural network.

Recent research tends to include novel factors and to hybridize several AI techniques. Hiemstra (1995) proposed fuzzy expert systems to predict stock market returns. He suggested that ANNs and fuzzy logic could capture the complexities of the functional mapping because they do not require the functional specification of the function to approximate. A more recent study of Kohara *et al.* (1997) incorporated prior knowledge to improve the performance of stock market prediction. Tsaih *et al.* (1998) integrated the rule-based technique and the ANNs to predict the direction of the S&P 500 stock index

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futures on a daily basis.

Previous research using AI techniques almost predicted the price of every trading day, week, and month. It is more important, however, to determine stock market timing, when to buy and sell stocks, than to predict the price movement for everyday because investors in stock market generally do not trade everyday. If investors trade their stocks everyday, they are charged to tremendous amount of fee for trade. Market timing is an investment strategy which is used for the purpose of obtaining the excess return. Traditionally excess return is achieved by switching between asset classes in anticipation of major turning points in stock market (Waksman *et al.*, 1997).

In this paper, we propose a new rule-based technique, so called rough set approach, to determine market timing for stock market. Rough set approach is very valuable to extract trading rules. First, it does not make any assumption about the distribution of the data. Second, it can generate profitable market timing because it not only handles noise well, but also eliminates irrelevant factors (Ruggiero, 1997). In addition, the rough set approach appropriate for detecting stock market timing because this approach does not generate the signal for trade when the pattern of market is uncertain.

The rest of the paper is organized into five sections. The next section reviews the basic concept of the rough set approach and their applications in business. In the third section, we explain the trading rule extraction and review the concept of market timing in stock market. In the fourth section, we describe the research data and execute experiments. In the fifth section, empirical results are summarized and discussed. In the following section, conclusions and future research issues are presented.

2. The rough set approach and their applications in business

The rough set concept proposed by Pawlak (1982) assume that there is some information which can be associated with every object of the universe. The same information can characterize the objects, and these are indiscernible in the view of available information about them. The so-called indiscernible relation from this view is the mathematical basis of the rough set theory.

Any set of indiscernible objects is called elementary set and forms a basic atom of knowledge about the universe (Dimitras *et al.*, 1999). Any subset of the universe can either be expressed precisely in the view of the atoms or roughly only. In the latter case, a certain subset can be characterized by two ordinary sets which are called lower and upper approximations. For each concept X the greatest definable set contained in X and the least definable set containing X can be computed. The former set is called a *lower approximation* of X and the letter is called an *upper approximation* of X (Pawlak *et al.*, 1995). The lower approximation consists of all of he objects that certainly belong to the concept while the upper approximation consists of any objects that possibly belongs to the concept (Ruggiero, 1997). The concept X is called rough if the upper approximation is not equal to the lower approximation of X (Jagielska *et al.*, 1999).

Generally, the knowledge about objects can be represented in the shape of an information table. The rows of the table are objects, the columns are attributes and the intersections of rows and columns are filled with attribute values.

The important point in the rough set approach is the discovery of the dependencies between attributes. It can be assumed that the set of attributes depends on the set of attributes. Another important concept is the reduction of attributes. This process is performed for finding the minimal subset assuring the same quality of classification. The attributes in information table are composed of two kinds of attributes, the condition attributes and the decision attributes. As for condition attributes and decision attribute, there are another measure of dependency, which can represent full dependency and partial dependency.

The *reducts* in the rough set approach is sets that contain the same elements as other sets but possess the fewest attributes (Ruggiero, 1997). Elimination of superfluous attributes can help to extract strong and non-redundant classification rules (Jagielska *et al.*, 1999). Intersection of all the reducts is called *Core*. The core is a collection of the most relevant and strong attributes (Dimitras *et al.*, 1999).

As the rough set approach to data analysis can handle imperfect data full of uncertainty and vagueness, it atones for other approaches dealing with data uncertainty such as probability theory, evidence theory, and fuzzy set theory, etc. Using this approach one analyses only facts hidden in data, it needs no additional information about data and doesn't correct inconsistencies manifested in data, instead rules extracted are categorized into certain and possible forms.

The applications of this approach on business area are mainly on the evaluation and prediction of bankruptcy and market research. These works were made mainly by the Greek information about bankruptcy. One of the earliest applications was done in the work of Slowinski and Zopounidis (1995). They used the rough set approach to evaluate bankruptcy. Slowniski *et al.* (1997) compared rough set approach with discriminant analysis for prediction of company acquisition in Greece and Dimitras *et al.* (1999) compared it with logit analysis to predict business failure.

The rough set approach, however, may not be proper in the evaluation of bankruptcy because of the excessive production of rules for the predicted cases. In the work of Slowinski *et al.* (1997), 24 rules were given to the 30 cases. In addition, the case will occurs that the rules extracted cannot be applicable to the holdout samples solely by the rough set approach. This problem is crucial in the field of bankruptcy prediction because all the judgment is to be given to all new cases to come.

3. Trading rule extraction and market timing

Extracting trading rules from the stock market is alternative way of detecting market timing. Market timing is an investment strategy which is used for the purpose of obtaining the excess return. Traditionally excess return is achieved by switching between asset classes in anticipation of major turning points in the stock market (Waksman *et al.*, 1997). Detecting market timing means determining to buy and sell to get an excess return from trading.

The market timing systems usually employs profitable rule base to capture the major turning points. It is very important to determine stock market timing than to predict the price movement for everyday because investors in stock market generally do not trade everyday. If investors trade their stocks in everyday, they are charged to tremendous amount of fee for trade. In addition, although there are an infinite number of possible trading rules by which we could trade, it seems that only a few of them would have made a profit.

There has been much debate regarding the development of trading systems using historical data. We agree that the future is never exactly like the past, however, a common investment approach is to employ systems that would probably have worked well in the past and that seem to have a reasonable chance of doing well in the future. So, we define a goal of system as finding trading rules for market timing which would have yielded the highest return over a certain period.

4. Research data

The research data used in this study is KOSPI 200 from May 1996 through October 1998. KOSPI 200 is the underlying index of KOSPI 200 future which is the first derivative instrument in Korean stock market. Futures are the standard forms that decide the quantity and price in the certified market at a certain future point of time. The general functions of futures market are supplying information about future price of commodities, speculation and hedging (Kolb and Hamada, 1988). We collected samples of 660 trading days. Many previous stock market analyses have used technical or fundamental indicator. In general, fundamental indicators are mostly used for long-term trend analysis while technical indicators are used for short-term pattern analysis. In this research, we use the technical indicators as input variables. Nine technical indicators are selected by the review of domain expert and past research. Table 1 gives selected input variables and their formulas.

Features	Names of feature	Formulas
A1	Stochastic %K	$\frac{C_t - L_n}{H_n - L_n} \times 100$
A2	Stochastic %D	$\frac{\sum_{i=0}^{n-1} \%K_{t-i}}{n}$
A3	RSI (relative strength index)	$100-\frac{100}{1+\frac{\sum_{i=0}^{n-1} Up_{t-i}/n}{\sum_{i=0}^{n-1} Dw_{t-i}/n}}$
A4	Momentum	$C_t - C_{t-4}$
A5	ROC (rate of change)	$\frac{C_t}{C_{t-n}} \times 100$
A6	A/D Oscillator	$\frac{H_t - C_{t-1}}{H_t - L_t}$
A7	CCI (commodity channel index)	$\frac{(M_t - SM_t)}{(0.015 \times D_t)}$
A8	OSCP (price oscillator)	$\frac{MA_5 - MA_{10}}{MA_5}$
A9	Disparity 5 days	$\frac{C_t}{MA_5} \times 100$

<Table 1> Technical indicators (Achelis, 1995; Gifford, 1995; Chang *et al.*, 1996; Edwards and Magee, 1997; Choi, 1995)

Note) C: Closing price, L: Low price, H: High price, MA: Moving average of price, M_t : $\frac{(H_t + L_t + C_t)}{3}$,

SM_t : $\frac{\sum_{i=1}^{n} M_{t-i+1}}{n}$, D_t : $\frac{\sum_{i=1}^{n} |M_{t-i+1} - SM_t|}{n}$,

Up: Upward price change, Dw: Downward price change

The rough set approach needs the data which are discretized in advance because the rule extraction process does not has the discretization algorithm. The thresholds of discretization are presented in Table 2. These thresholds are selected by the review of the textbook about technical analysis in stock market. Table 3 presents summary statistics for each variable.

Features	Category 1	2	3
A1 (Stochastic %K)	[0, 25]	(25, 75]	(75, ∞)
A2 (Stochastic %D)	[0, 25]	(25, 75]	(75, ∞)
A3 (RSI)	[0, 30]	(30, 70]	(70, 100]
A4 (Momentum)	(-∞, 0]	(0, ∞)	
A5 (ROC)	(-∞, 100]	(100, ∞)	
A6 (A/D Oscillator)	(-∞, 0.5]	(0.5, ∞)	
A7 (CCI)	(-∞, 0]	(0, ∞)	
A8 (OSCP)	(-∞, 0]	(0, ∞)	
A9 (Disparity 5 days)	(-∞, 100]	(100, ∞)	

<Table 2> Discretizing threshold (Murphy, 1986; Achelis, 1995; Gifford, 1995; Chang *et al.*, 1996; Edwards and Magee, 1997; Choi, 1995)

Features	Max	Min	Mean	Standard Deviation
A1 (Stochastic %K)	151.02	0.00	43.69	33.77
A2 (Stochastic %D)	118.57	0.00	43.65	28.70
A3 (RSI)	100.00	0.00	43.58	29.21
A4 (Momentum)	10.90	-11.35	-0.44	3.24
A5 (ROC)	129.14	81.51	99.55	5.86
A6 (A/D Oscillator)	1.71	-0.10	0.48	0.32
A7 (CCI)	229.14	-212.44	-12.89	81.59
A8 (OSCP)	8.14	-9.09	-0.39	99.72
A9 (Disparity 5 days)	114.72	87.25	99.72	3.16

<Table 3> Summary statistics

5. Experimental Results

As mentioned earlier, in this study, the rough set approach is used to find the profitable rules. It produces following results. First, the core of attributes is empty. This implies that no single attribute perfectly explain the characteristics of decision classes. This may be owing to complex relationship among attributes in stock market.

Second, 280 reducts are obtained from experiment. Derived reducts and their strength are presented in Appendix. The strength means the number of objects satisfying the rule. The rough set approach produces several number of reducts. To extract rules from the several sets of reducts, Dimitras *et al.* (1999) used following two criteria. First, the reduct should include as small a number of attributes possible. Second, the reduct should not miss the attributes judged by the domain expert as the most significant for evaluation of the firms. These criteria, however, are arbitrary and subjective ones.

In this study, we use following criteria to extract rules. First, we select reduct which has the largest strength. The largest strength means many objects of historical data support the rule. The reducts selected are twelve reducts which are the reduct #1 ~ #12 in Appendix. These reducts are supported by 60 strength respectively. Then we extract rules from each selected reduct. In the process of extracting rules, we permit 51% of the minimum level of discrimination and 21 of the minimum strength for each rule. Then we apply the results of extracted rules to our trading strategy in Table 4. In the process of applying the rules, the reducts which do not produce 'Buy' signal are excluded because these reducts do not produce profit or loss any more. Finally, the reduct #1, #2, #3, #4, #6, #9, #10, #11, and #12 are selected and they are applied to out-of-sample data to validate the generalization power.

Trading profit earned from simulated trading results are presented in Table 5. While the underlying index decreased more than 58% during the modeling period, we can find rules that would yield the high level of profit were it used to trade stocks on a given set of historical data.

```
If today's signal is UP.
        Then
                If the previous day's decision was BUY.
                Then SELL stocks;
        Else BUY stocks;
If today's signal is NONE (If any rules do not matched to today's market condition) or DOWN.
        Then
                If the previous day's decision was BUY.
                Then SELL stocks;
        Else do not trade.
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<Table 4> Trading strategy

Trading profit earned from simulated trading results are presented in Table 5. While the underlying index decreased more than 58% during the modeling period, we can find rules that would yield the high level of profit were it used to trade stocks on a given set of historical data. The rules derived by modeling data are applied to validation period to verify the effectiveness of the proposed approach. In the derived trading rules sets, the reduct #3 produces the best performance for validation period. While the underlying index decreased about 17% during the validation period, the trading strategies followed by the derived rules get 31.5% for the reduct #3 (the derived rules set #3 in Table 5) of trading profit. This means the investor who follows the rules from the reduct #3 can earn 48.3% of profit during the validation period than the investor who follows "buy and hold" strategy. The description of rules of the reduct #3 are illustrated in Table 6. The other rules in Table 5 are alternatively good trading rules although there is minor difference in simulated performance. These experimental results demonstrate that the rough set approach is promising method for extracting profitable trading rules.

	Accumulated profit (assume initial investment of 10,000 won) (the accumulated rate of return / the excess return than following "buy and hold" strategy)	
	Modeling period (May 1996 – November 1997)	Validation period (January 1998 – October 1998)
Following Buy & Hold	4,167 won (-58.3%/ 0%)	8,317 won (-16.8%/ 0%)
Following the rules set 1 (Reduct #1: {A1, A2, A6, A8})	10,872 won (8.7%/ 67.0%)	12,480 won (24.8%/ 41.6%)
Following the rules set 2 (Reduct #2: {A1, A2, A8})	9,539 won (-4.6%/ 53.7%)	12,660 won (26.6%/ 43.4%)
Following the rules set 3 (Reduct #3: {A1, A3, A6, A8})	10,830 won (8.3%/ 66.6%)	13,153 won (31.5%/ 48.3%)
Following the rules set 4 (Reduct #4: {A1, A6, A8})	10,177 won (1.8%/ 60.1%)	11,816 won (18.2%/ 35.0%)
Following the rules set 5 (Reduct #6: {A3, A6, A8})	9,881 won (-1.2%/ 57.1%)	10,744 won (7.4%/ 24.2%)
Following the rules set 6 (Reduct #9: {A4, A7, A8})	9,689 won (-3.1%/ 55.2%)	12,140 won (21.4%/ 38.2%)
Following the rules set 7 (Reduct #10: {A4, A8, A9})	9,430 won (-5.7%/ 52.6%)	12,688 won (26.9%/ 43.7%)
Following the rules set 8 (Reduct #11: {A5, A7, A8})	9,689 won (-3.1%/ 55.2%)	12,140 won (21.4%/ 38.2%)
Following the rules set 9 (Reduct #12: {A5, A8, A9})	9,430 won (-5.7%/ 52.6%)	12,688 won (26.9%/ 43.7%)

<Table 5> The profit of buying and selling simulations

	Elementary conditions				Decision class	
No.	A1	A3	A6	A8	A10	Strength
#1	2	2	2	1	Up	29
#2	2	2	1	1	Down	21
#3	3	3	2	2	Up	47
#4	2	2	1	2	Down	21
#5	1	1	2	1	Down	21
#6	2	1	2	1	Down	33
#7	1	1	1	1	Down	104

<Table 6> Description of derived rules from the reduct #3

7. Concluding remarks

This study intends to mine profitable trading rules using the rough set approach for Korea Stock Price Index 200 (KOSPI 200) futures. The rough set approach is very valuable to extract trading rules because it can be used to discover dependencies in data while eliminating the superfluous factors in noisy stock market data. Additional advantage of the proposed approach is that trading rules generated by the rough set approach produce the trading signals only when the rules are fired. Each of the rules extracted produces the signals less than 20% of time. The rough set approach do not produce the trading signals when the pattern of market is uncertain because the selection of reduct and the extraction of rules are controlled by the strength of each reduct and rule. This is very important to detect market timing because market timing is detected by capturing the major turning points in data. In addition, investors in stock market generally do not trade everyday owing to tremendous amount of fee for trade.

The derived rules form the experiments are alternatively good trading rules although there is minor difference in simulated performance. The experimental results are very encouraging and prove the usefulness of the rough set approach for stock market analysis with respect to profitability.

We need a more extensive validation process because the future is never exactly like the past. In addition, the rough set approach, however, does not has the general guidance for the discretization of continuous data, we need a comparative study about the effects on results of different methods for discretization including domain knowledge, equal frequency approach, and entropy minimization method.

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Appendix

No.	Reduct	Strength
1	{A1, A2, A6, A8}	60
2	{A1, A2, A8}	60
3	{A1, A3, A6, A8}	60
4	{A1, A6, A8}	60
5	{A2, A9}	60
6	{A3, A6, A8}	60
7	{A3, A7}	60
8	{A3, A9}	60
9	{A4, A7, A8}	60
10	{A4, A8, A9}	60
11	{A5, A7, A8}	60
12	{A5, A8, A9}	60
13	{A2, A3, A8}	59
14	{A1, A2, A3, A6}	58
15	{A2, A3, A6, A8}	58
16	{A2, A6, A8}	58
17	{A1, A2, A6}	56
18	{A3, A7, A9}	56
19	{A7, A8, A9}	56
20	{A1, A2, A3}	55
21	{A2, A3, A6, A9}	55
22	{A1, A3, A4, A6, A8}	54
23	{A1, A3, A5, A6, A8}	54
24	{A2, A3, A6}	54
25	{A3, A4, A6, A7, A8}	54
26	{A3, A5, A6, A7, A8}	54
27	{A1, A2, A3, A8}	53
28	{A3, A4}	53
29	{A3, A5}	53
30	{A1, A4, A6, A8}	51
31	{A1, A5, A6, A8}	51
32	{A3, A4, A7, A9}	51
33	{A3, A5, A7, A9}	51
34	{A4, A6, A7, A8}	51
35	{A5, A6, A7, A8}	51
36	{A1, A4, A8}	50
37	{A1, A5, A8}	50
38	{A3, A6, A7}	49
39	{A3, A4, A6, A8, A9}	48
40	{A3, A5, A6, A8, A9}	48
41	{A1, A4, A6, A7, A8}	47
42	{A1, A5, A6, A7, A8}	47
43	{A1, A3, A4, A8}	45
44	{A1, A3, A5, A8}	45
45	{A1, A3, A6, A7, A8}	45
46	{A4, A6, A8, A9}	45
47	{A5, A6, A8, A9}	45
48	{A1, A2, A3, A9}	43
49	{A2, A3, A6, A7}	43
50	{A3, A4, A7, A8, A9}	43
51	{A3, A5, A7, A8, A9}	43
52	{A1, A3, A6, A7}	42
53	{A4, A7, A9}	42
54	{A5, A7, A9}	42
55	{A6, A7, A9}	42
56	{A1, A3, A8}	40
57	{A2, A4, A6, A8, A9}	40
58	{A2, A5, A6, A8, A9}	40
59	{A1, A2, A9}	39
60	{A1, A2, A3, A7}	38
61	{A1, A2, A4, A6}	38
62	{A1, A2, A5, A6}	38
63	{A1, A3, A4, A6, A9}	38
64	{A1, A3, A5, A6, A9}	38
65	{A2, A6, A7, A8}	38
66	{A1, A3, A4, A7, A8}	37
67	{A1, A3, A5, A7, A8}	37
68	{A2, A3, A6, A8, A9}	37
69	{A2, A8, A9}	37
70	{A3, A4, A6, A9}	37
71	{A3, A5, A6, A9}	37
72	{A2, A3, A7}	36
73	{A1, A3, A4, A6, A7}	35
74	{A1, A3, A5, A6, A7}	35
75	{A1, A4, A6, A8, A9}	35
76	{A1, A5, A6, A8, A9}	35
77	{A2, A3, A9}	35

No.	Reduct	Strength
78	{A1, A3, A4}	34
79	{A1, A3, A5}	34
80	{A2, A3, A6, A7, A8}	34
81	{A2, A4, A6, A7, A8}	33
82	{A2, A5, A6, A7, A8}	33
83	{A1, A2, A4, A7, A8}	32
84	{A1, A2, A5, A7, A8}	32
85	{A1, A2, A6, A7, A8}	32
86	{A1, A4, A6, A7}	32
87	{A1, A5, A6, A7}	32
88	{A2, A7, A8}	32
89	{A1, A3, A6, A8, A9}	31
90	{A1, A3, A6, A9}	31
91	{A1, A4}	31
92	{A1, A5}	31
93	{A2, A3, A8, A9}	31
94	{A1, A3, A4, A6}	30
95	{A1, A3, A5, A6}	30
96	{A1, A3, A6}	30
97	{A2, A3, A4, A6, A9}	29
98	{A2, A3, A5, A6, A9}	29
99	{A2, A6, A7}	29
100	{A3, A4, A6, A7, A9}	29
101	{A3, A5, A6, A7, A9}	29
102	{A1, A2, A4, A8}	28
103	{A1, A2, A5, A8}	28
104	{A1, A2, A6, A9}	28
105	{A2, A7, A9}	28
106	{A4, A7, A8, A9}	28
107	{A5, A7, A8, A9}	28
108	{A1, A3, A4, A6, A7, A8}	27
109	{A1, A3, A5, A6, A7, A8}	27
110	{A2, A3, A4, A6, A7}	27
111	{A2, A3, A5, A6, A7}	27
112	{A3, A6, A7, A8}	26
113	{A1, A2, A4, A6, A8, A9}	25
114	{A1, A2, A5, A6, A8, A9}	25
115	{A3, A6, A8, A9}	25
116	{A3, A6, A9}	25
117	{A1, A2, A3, A6, A7}	24
118	{A2, A3, A4, A8, A9}	24
119	{A2, A3, A5, A8, A9}	24
120	{A2, A3, A4, A7, A8}	23
121	{A2, A3, A5, A7, A8}	23
122	{A2, A6, A8, A9}	23
123	{A3, A7, A8, A9}	22
124	{A1, A2, A6, A7}	21
125	{A1, A3, A4, A8, A9}	21
126	{A1, A3, A5, A8, A9}	21
127	{A2, A3, A4, A6, A8}	21
128	{A2, A3, A5, A6, A8}	21

No.	Reduct	Strength
129	{A2, A8}	21
130	{A2, A7}	20
131	{A3, A4, A6, A7}	20
132	{A3, A5, A6, A7}	20
133	{A4, A6, A9}	20
134	{A5, A6, A9}	20
135	{A1, A2, A3, A6, A9}	19
136	{A1, A2, A4, A6, A7, A8}	19
137	{A1, A2, A5, A6, A7, A8}	19
138	{A1, A2, A6, A8, A9}	19
139	{A1, A3, A4, A6, A8, A9}	19
140	{A1, A3, A5, A6, A8, A9}	19
141	{A1, A3, A6, A7, A8, A9}	18
142	{A1, A3, A7, A8, A9}	18
143	{A1, A2, A4, A6, A7}	17
144	{A1, A2, A4, A6, A8}	17
145	{A1, A2, A5, A6, A7}	17
146	{A1, A2, A5, A6, A8}	17
147	{A1, A3, A4, A7, A8, A9}	17
148	{A1, A3, A5, A7, A8, A9}	17
149	{A2, A3, A6, A7, A8, A9}	17
150	{A2, A4, A6, A7}	17
151	{A2, A5, A6, A7}	17
152	{A1, A3, A4, A7, A9}	16
153	{A1, A3, A5, A7, A9}	16
154	{A2, A3, A4, A6, A7, A8}	16
155	{A2, A3, A4, A7, A9}	16
156	{A2, A3, A5, A6, A7, A8}	16
157	{A2, A3, A5, A7, A9}	16
158	{A1, A2, A3, A7, A9}	15
159	{A3, A4, A6, A8}	15
160	{A3, A5, A6, A8}	15
161	{A1, A2, A3, A4, A6, A8}	14
162	{A1, A2, A3, A5, A6, A8}	14
163	{A1, A4, A7, A8}	13
164	{A1, A5, A7, A8}	13
165	{A2, A3, A4, A7, A8, A9}	12
166	{A2, A3, A5, A7, A8, A9}	12
167	{A3, A4, A7, A8}	12
168	{A3, A5, A7, A8}	12
169	{A1, A2, A3, A4}	11
170	{A1, A2, A3, A5}	11
171	{A2, A3, A7, A8}	11
172	{A2, A4, A6, A8}	11
173	{A2, A4, A7, A9}	11
174	{A2, A5, A6, A8}	11
175	{A2, A5, A7, A9}	11
176	{A2, A7, A8, A9}	11
177	{A3, A4, A8, A9}	11
178	{A3, A4, A9}	11
179	{A3, A5, A8, A9}	11

No.	Reduct	Strength
180	{A3, A5, A9}	11
181	{A1, A2, A3, A4, A8, A9}	10
182	{A1, A2, A3, A5, A8, A9}	10
183	{A1, A2, A7, A8}	10
184	{A2, A3, A4, A7}	10
185	{A2, A3, A5, A7}	10
186	{A1, A2, A3, A4, A6}	9
187	{A1, A2, A3, A5, A6}	9
188	{A1, A2, A8, A9}	9
189	{A1, A6, A8, A9}	9
190	{A2, A4, A7, A8, A9}	9
191	{A2, A5, A7, A8, A9}	9
192	{A1, A2, A4, A8, A9}	8
193	{A1, A2, A5, A8, A9}	8
194	{A1, A4, A8, A9}	8
195	{A1, A5, A8, A9}	8
196	{A1, A6, A7, A8}	8
197	{A2, A3}	8
198	{A3, A6, A7, A8, A9}	8
199	{A1, A2, A3, A4, A7, A8}	7
200	{A1, A2, A3, A5, A7, A8}	7
201	{A1, A2, A3, A6, A8, A9}	7
202	{A1, A2, A3, A6, A8}	7
203	{A1, A2, A4, A6, A9}	7
204	{A1, A2, A5, A6, A9}	7
205	{A2, A3, A7, A8, A9}	7
206	{A2, A3, A4, A6, A8, A9}	6
207	{A2, A3, A5, A6, A8, A9}	6
208	{A2, A6, A9}	6
209	{A1, A2, A3, A4, A6, A7}	5
210	{A1, A2, A3, A4, A8}	5
211	{A1, A2, A3, A5, A6, A7}	5
212	{A1, A2, A3, A5, A8}	5
213	{A1, A2, A4, A6, A7, A9}	5
214	{A1, A2, A5, A6, A7, A9}	5
215	{A1, A3, A8, A9}	5
216	{A1, A4, A7, A8, A9}	5
217	{A1, A5, A7, A8, A9}	5
218	{A1, A8}	5
219	{A2, A4, A6, A7, A9}	5
220	{A2, A5, A6, A7, A9}	5
221	{A2, A6, A7, A8, A9}	5
222	{A1, A2, A4}	4
223	{A1, A2, A5}	4
224	{A1, A3, A6, A7, A9}	4
225	{A1, A3, A7, A8}	4
226	{A1, A4, A6, A7, A9}	4
227	{A1, A5, A6, A7, A9}	4
228	{A4, A6, A7}	4
229	{A5, A6, A7}	4
230	{A2, A3, A4, A9}	3

No.	Reduct	Strength
231	{A2, A3, A5, A9}	3
232	{A2, A4, A7, A8}	3
233	{A2, A5, A7, A8}	3
234	{A1, A2, A3, A4, A6, A9}	2
235	{A1, A2, A3, A4, A7}	2
236	{A1, A2, A3, A5, A6, A9}	2
237	{A1, A2, A3, A5, A7}	2
238	{A1, A2, A3, A8, A9}	2
239	{A1, A2, A4, A7}	2
240	{A1, A2, A5, A7}	2
241	{A1, A2, A6, A7, A9}	2
242	{A1, A2}	2
243	{A1, A3, A4, A6, A7, A9}	2
244	{A1, A3, A4, A7}	2
245	{A1, A3, A5, A6, A7, A9}	2
246	{A1, A3, A5, A7}	2
247	{A1, A4, A6, A9}	2
248	{A1, A5, A6, A9}	2
249	{A1, A6, A7, A8, A9}	2
250	{A2, A3, A4, A8}	2
251	{A2, A3, A5, A8}	2
252	{A3, A4, A7}	2
253	{A3, A5, A7}	2
254	{A1, A2, A3, A4, A7, A8, A9}	1
255	{A1, A2, A3, A5, A7, A8, A9}	1
256	{A1, A2, A3, A7, A8, A9}	1
257	{A1, A2, A4, A7, A8, A9}	1
258	{A1, A2, A4, A7, A9}	1
259	{A1, A2, A5, A7, A8, A9}	1
260	{A1, A2, A5, A7, A9}	1
261	{A1, A2, A6, A7, A8, A9}	1
262	{A1, A2, A7}	1
263	{A1, A3, A4, A9}	1
264	{A1, A3, A5, A9}	1
265	{A1, A3, A7, A9}	1
266	{A1, A3, A9}	1
267	{A1, A3}	1
268	{A1, A4, A6}	1
269	{A1, A5, A6}	1
270	{A1, A6}	1
271	{A2, A3, A6, A7, A9}	1
272	{A2, A3, A7, A9}	1
273	{A2, A4, A6, A9}	1
274	{A2, A5, A6, A9}	1
275	{A3, A6, A7, A9}	1
276	{A3, A8}	1
277	{A4, A6, A7, A9}	1
278	{A5, A6, A7, A9}	1
279	{A6, A8, A9}	1
280	{A7, A9}	1