

A PROPER PROBABILITY FUNCTION  
FOR N-AXIAL EASY AXIS DISTRIBUTION

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### 1. INTRODUCTION

Information about magnetic easy axis distribution is very important to understand magnetic properties of permanent magnet [1] as well as thin-film recording media [2]. Many authors have been reported magnetic analysis techniques based on the probability function of Gaussian (G) and Lorentzian (L) to describe the easy axis distribution of partially aligned system. However, G and L are not proper probability function because they do not show the uniaxial symmetry of  $f(\phi + \pi) = f(\phi)$ . In addition, they are not applicable to a sample having widely distributed easy axes because they don't satisfy the condition of  $\int_{-\pi/2}^{\pi/2} f(\phi) d\phi = 1$ . In this paper, we report a new probability function for the magnetic easy axis distribution and its advantages over G and L.

### 2. THEORY AND DISCUSSIONS

Let's consider conditions that a probability function of the easy axis orientation should obey. The function should satisfy a n-axial symmetry of  $f(\phi + \pi/n - \phi_c, d) = f(\phi - \phi_c, d)$ , a sum rule of  $\int_{\phi_c - \pi/2n}^{\phi_c + \pi/2n} f(\phi - \phi_c, d) d\phi = 1$ , and a relation of  $f(\phi - \phi_c, d) \geq 0$ . Here,  $\phi_c$  and  $\phi$  are angles of the maximum probable orientation and the easy axis orientation, respectively.  $d$  is the half width at half maximum (HWHM) of the probability function. One can find a function expressed by Eq. (1) satisfies all conditions mentioned above.

$$f(\phi - \phi_c, d) = \frac{n}{\pi} \frac{\tan(nd)(1 + \tan(nd))}{\tan^2(n(\phi - \phi_c)) + \tan^2(nd)} \quad (1)$$

Eq. (1) satisfies  $f(\phi - \phi_c = \pi/2, d \neq \pi/2n) = 0$  and  $f(\phi - \phi_c, d = \pi/2n) = n/\pi$ . It should be noted that the HWHM ( $d$ ) of  $\pi/2n$  means uniform distribution of easy axis for the n-axial symmetry case. Fig. 1 shows typical examples of evaluating  $f(\phi - \phi_c, d)$ . Direct comparison was carried out by fitting G and L to the data obtained by evaluating present function, as seen in Fig. 2. Present function with a small HWHM shows L behavior. Definite integral,  $I(d)$  defined

by  $I(d, n) = \int_{\phi_c - \pi/2n}^{\phi_c + \pi/2n} f(\phi - \phi_c, d) d\phi$ , is evaluated with  $n = 1$  to test the sum rule. The definite integral of our function gives exact 1 as seen in Fig. 3. Rigorously speaking, if the definite integral of a function deviates from 1, the function can not be used as a probability function. Fortunately, the definite integral of G shows negligibly small deviation from 1 when HWHM is smaller than 30 deg. Hence, previous works based on G with a smaller HWHM than 30 deg. are valid. However, the constraints in using G and L as a probability function will be severe for the biaxial case.

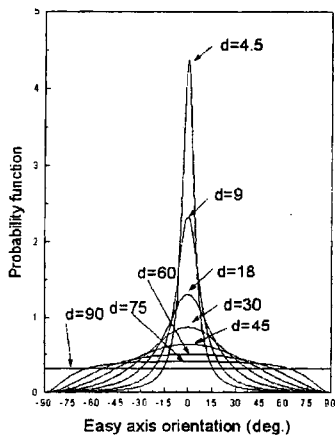


Fig.1 New probability function.

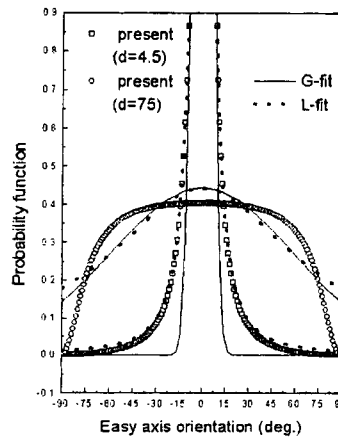


Fig.2 Comparisons by fitting.

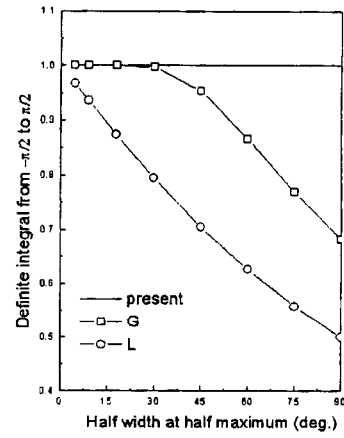


Fig.3 Sum rule test.

### 3. CONCLUSIONS

We present a new probability function for n-axis easy axis distribution and discuss its properties. Our probability function is mathematically correct and practically accurate. Furthermore, it can be used to accurately describe high-n-axis angular symmetry. This is a unique and remarkable advantage over G and L.

### REFERENCES

- [1] Y. B. Kim and Han-min Jin, *J. Magn. Magn. Mat.* **173**, 93 (1997).
- [2] J. W. Harrell *et al.* *J. Appl. Phys.* **81**, 3800 (1997).