Air-Sea Heat Flux Estimation by Ocean Data Assimilation Using Satellite and TOGA/TAO Buoy Data

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ABSTRACT

A data assimilation system for a 1-dimensional mixed layer model has been constructed using the adjoint method. The classical adjoint method does not work well for the mixed layer variabilities due to the occurrence of spikes in the gradient of the cost function. To solve this problem, the two techniques of scaling the cost function and optimization in the frequency space are used. As a result, the heat flux can be reliably estimated with an accuracy of $8Wm^{-2}$ rms error in the identical twin experiments. We then applied this system to the tropical Pacific TOGA-TAO buoy data. The air-sea heat flux as well as the mixed layer variability were estimated in close approximation to the buoy data, particularly on time scales longer than the seasonal one.

1. Introduction

Construction of a 4-dimensional data assimilation system capable of providing an integrated dataset of the mixed layer variability and the air-sea heat flux is quite important for studies of climate system dynamics and the mechanisms driving the ocean circulation . For model variables in the mixed layer, however, different dynamics operate between the deepening and the shoaling phase, so that data assimilation systems show strong nonlinearity, and optimal solutions are difficult to obtain.

In this study, prior to constructing a data assimilation system with a 3-dimensional mixed layer model, a simpler assimilation system using a 1-dimensional bulk mixed layer model is constructed with the adjoint method, which enables us to identify the nature of the above problem and to gain useful information concerning possible improvements to the assimilation system. To highlight how the optimal solution can be obtained for the mixed layer problem, we first conducted an idealized assimilation experiment using the simulated data. This data assimilation system is then applied to the tropical Pacific TOGA-TAO region, incorporating observation data taken from satellites and the TOGA-TAO buoy into the model. The estimated air-sea heat flux as well as the mixed layer variability were in close approximation to the buoy data, particularly on time scales longer than the seasonal one.

2. Data assimilation model

The mixed layer model used in this study is based on the 1-dimensional bulk mixed layer model (e.g., Kraus and Turner, 1967). This model can represent the variability of the depth-integrated mixed layer and has only two variables, that is, the mixed layer temperature T_m, and the mixed layer depth h_m.

The variability of the mixed layer temperature is expressed as follows:

$$h_m \frac{\partial T_m}{\partial t} + (adv.) = \frac{Q_{net}}{rho_0 C} - \Delta T w_e, \tag{1}$$

where $(adv.)$ represents the effect of the advection and is set as a constant in this study for simplicity, and Q_{net},.ρ_0, C, and ΔT are the net air-sea heat flux, the reference density, the specific heat of the sea water, and the temperature difference between water in and below the mixed layer, respectively.

The governing equations of h_m and w_e vary with the sign of the turbulence kinetic energy budget (TKE), depending on whether the mixed layer is deepening or shoaling. The TKE is based on the parameterization by Davis et al. (1981).

$$E_m = m_0 u_*^3 + m_s S - \frac{\alpha g h_m}{2\rho_0 C} Q_{net} - \frac{m_c \; alphagh_m}{4\rho_0 C} (|Q_{net}| - Q_{net}), \tag{2}$$

where α is the thermal expansion coefficient ($= 2.5 \times 10^{-4 \circ} C^{-1}$). In this equation, the first two terms on the rhs denotes the energy production of wind stirring (where $u_* = (|\tau|/\rho_0)^{1/2}$ is frictional velocity) and current shear. The last two terms denote the energy sources of heat flux. In particular, the last term acts only in the cooling condition to represent the convective effect. Because the shear production rate S is not available from observations, we are unable to estimate the shear production term accurately and set as $m_s = 0$. The values of the other coefficients are taken as $m_0 = 0.5$ and $m_c = 0.83$ following Qiu and Kelly (1993).

When the value of TKE is positive ($E_m \geq 0$), the mixed layer becomes deeper and the equations of w_e and h_m are as follows:

$$w_e = \frac{2E_m}{\alpha g h_m \Delta T}, \tag{3}$$

$$\frac{\partial h_m}{\partial t} = w_e \tag{4}$$

In the shoaling phase ($E_m < 0$), there is no entrainment velocity, and the mixed layer depth change due to turbulent mixing is such that h_m detrains instantly to the Monin-Obukhov depth:

$$h_m = \frac{2\rho_0 C m_0 u_*^3}{\alpha g Q_{net}}, \tag{5}$$

$$w_e = 0 \tag{6}$$

(cf. Kraus and Turner, 1967). This depth is obtained from Eq. (2) , which expresses the TKE balance between the effects of wind stirring and stabilization due to surface warming.

In the variational adjoint method for the above mixed layer system, the difference between observational data and the corresponding model values (mixed layer temperature in our case) is formulated in terms of a cost function. In addition, the temporal smoothness of the heat flux, which is the control variable, is included in the costfunction.

$$J = \frac{1}{2}\left(T_m - T_{obs}\right)^2 + \frac{w_s}{2}\left(\frac{partialQ_{net}}{\partial t}\right), \tag{7}$$

where the w_s is the weight parameter for the smoothness.

The problem is to search for a minimum in the above cost function while satisfying the model equations (Eq. (1) - Eq. (6)) is the problem of a constrained minimization and needs to be transformed into an unconstrained problem. This can be achieved by formulating the augmented Lagrange function using the Lagrange multiplier $(\hat{T}, \hat{h}, \hat{w}, \hat{E})$. Based on Sasaki (1970)'s "strong constraint" formalism, the Lagrange function can be expressed as:

$$L = J + \hat{T}(eq.T_m) + \hat{h}(eq.h_m) + \hat{w}(eq.w_e) + \hat{E}(eq.E_m). \tag{8}$$

The change in the model equations for the entrainment velocity and mixed layer depth between the deepening phase (Eqs. (3), (4)) and the shoaling phase (Eqs. (5), (6)) can be formally expressed using the step function:

$$\phi(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ 0 & \text{for } x < 0. \end{cases} \tag{9}$$

The adjoint equations can be obtained by the first-order variation of the Lagrange function (Eq. 8).

$$-\frac{\partial \hat{T}}{\partial t} = (T_m - T_{obs}) \tag{10}$$

$$\begin{aligned} -\phi(E_m)\frac{\partial \hat{h}}{\partial t} + \phi(-E_m) &= -\hat{T}\frac{\partial T_m}{\partial t} - \phi(E_m)\hat{w}\frac{2E_m}{\alpha g h_m^2 \Delta T} \\ &\quad -(1+\phi(-Q_{net})m_c)\hat{E}\frac{\alpha g Q_{net}}{2\rho_0 C} \end{aligned} \tag{11}$$

$$\phi(E_m)\hat{w} = \hat{h} - \Delta T\hat{T} \tag{12}$$

$$\hat{E} = \phi(E_m)\hat{w}\frac{2}{\alpha g h_m \Delta T} \tag{13}$$

The gradient of the cost function for the control variable, Q_{net}, can be also obtained by the variation of the Lagrange function as follows:

$$\begin{aligned} \frac{\partial L}{\partial Q_{net}} = &-\frac{\hat{T}}{\rho_0 C} + \phi(-E_m)\frac{2\rho_0 C m_0 u_*^3 \hat{h}}{\alpha g Q_{net}^2} \\ &+(1+\phi(-Q_{net})m_c)\frac{\alpha g h_m \hat{E}}{2\rho_0 C} + w_s\frac{\partial^2 Q_{net}}{\partial t^2}. \end{aligned} \tag{14}$$

3. Assimilation of TOGA-TAO buoy data

An identical twin experiment following the adjoint equations (Eqs. (9)- (14)), in which both approaches of a scaling of the cost function and the optimization in the frequency space are used, revealed that the estimated heat flux and mixed layer variability were very close temporal evolution to that of the control run: the RMS error of the estimated air-sea heat flux in our approach is 8 Wm^{-2}, while the RMS error in the first guess is 46 Wm^{-2}. Thus, our assimilation system has the potential to estimate the air-sea heat flux as well as the mixed layer variability very well and is therefore applied to estimation of the air-sea heat flux in the equatorial Pacific ocean using the mixed layer temperature measured by the TOGA-TAO buoy.

The TOGA-TAO buoy used here is located at $8°N$, $165°E$ and the data duration is 1172 days (from Feb. 5, 1992 to Apr. 24, 1995). The TOGA-TAO buoy provides good sea-truth data for the mixed layer. The wind stress is calculated from the buoy observations following the equations of Hellerman and Rosenstein (1983). The latent and sensible heats are calculated using the equations of Smith (1980) and Large and Pond (1982), respectively, and the NCEP reanalysis data is used for the radiation flux when calculating the net heat flux. The mixed layer is defined from the sea surface to the depth where the temperature is $2°C$ below SST.

When the heat flux derived from the buoy data is used to force the mixed layer model, some parameterization is necessary to match the observed mixed layer variability because of the model deficiency. In our experiment, the effect of horizontal advection ($(adv.)$ terms in Eq. (1)) in the heat balance is set to give a constant value, $-16Wm^{-2}$, based on the preliminary analysis. In addition, the effect of shear is also parameterized by taking the wind stress to increase as $\tau' = \tau \times 1.5 + 0.06$. In the shoaling phase, the minimum mixed layer depth is set to 50 m and maximum shoaling speed to -75 m/day. The results of the model forced by the buoy flux with these parameterizations show a similar mixed layer variability to the in-situ observations (see Table 1).

For the first guess of the assimilation experiment, the heat flux from the NCEP reanalysis data is used. The model result shown in Fig. 1 is rather different from the buoy observations due to the lowering of the heat flux.

In contrast, the assimilation result (Fig. 2) shows better agreement with the buoy observations especially on seasonal and longer time scales. The RMS difference between both values is $34.6Wm^{-2}$ and its low-pass filtered value over 90 days is $28Wm^{-2}$ (Table 2). Also, the accuracy of the mixed-layer depth is improved greatly by the assimilation despite the fact that mixed layer temperature alone are assimilated. These results imply that our data assimilation system with the variational adjoint method can greatly contribute to the heat budget analysis in the upper ocean.

4. Summary and future work

A data assimilation system for a 1-dimensional mixed layer model has been constructed using the adjoint method. The classical adjoint method does not work well for the mixed layer variabilities due to the occurrence of spikes in the gradient of the cost function. These spikes are caused by the different responses of the mixed layer model to flux perturbations associated with the change of the dynamical regime. To solve this problem, the two techniques of scaling the cost function and optimization in the frequency space are used. As a result, the heat flux can be reliably estimated with an accuracy of $8Wm^{-2}$ rms error in the identical twin experiments.

We then applied this system to the tropical Pacific TOGA-TAO buoy data. The air-sea heat flux as well as the mixed layer variability were estimated in close approximation to the buoy data, particularly on time scales longer than the seasonal one.

For further study, the mixed layer model used here must be extended to 3-dimensions in order to represent the advective effect explicitly. A data assimilation system with a 3-dimensional mixed layer model would be very useful for the study of the heat budget in the upper ocean. For more accurate representations of the mixed layer variability, good parameterization of the shear effect is also necessary. We could then expect data assimilation systems to be effective in determining the basic parameters of the shear production from observational data.

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Table 1: Difference of the mixed layer variability from buoy data

	Temperature	Depth
Buoy flux case	1.12°C	15.0 m
NCEP flux case	2.54°C	204 m
Assimilation result	0.69°C	17.0 m

Table 2: Difference of the heat flux from buoy data

unit in Wm^{-2}	Total	90 day <	< 90 day
NCEP flux	40.0	28.5	27.7
Assimilation result	34.6	19.4	28.2

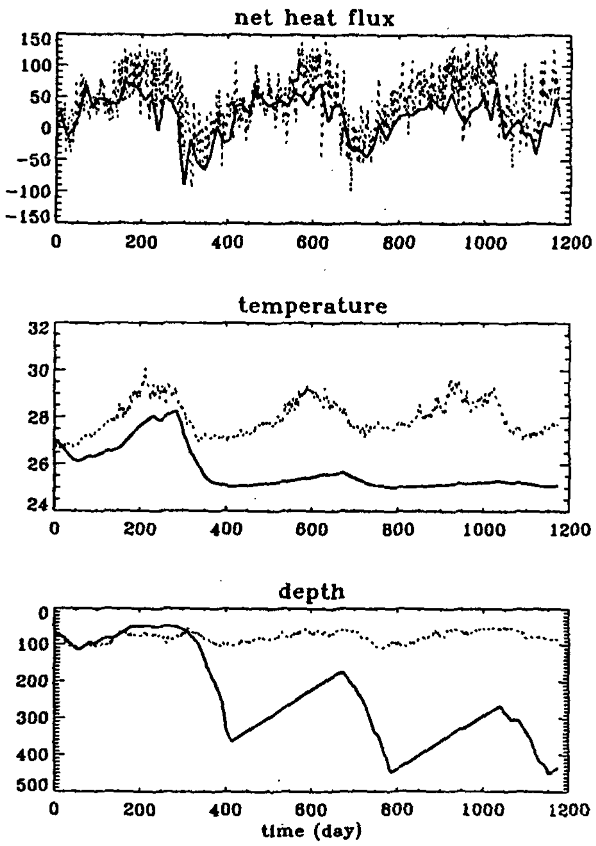

Figure 1: Time series of the net heat flux (upper), mixed layer temperature(middle), mixed layer depth (lower) . Dotted and solid lines denote the buoy observation and first guess, respectively.

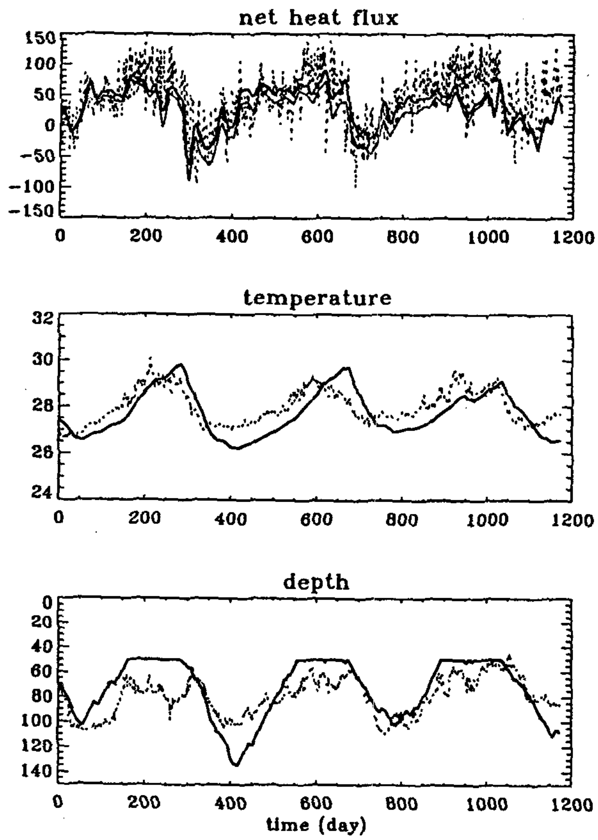

Figure 2: Time series of the net heat flux (upper), mixed layer temperature(middle), mixed layer depth (lower). Dotted and solid lines denote the control run and assimilation result, respectively. NCEP heat flux is also indicated in the upper panel as a thin line.