DEM interpolation using spectral information

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Abstract

Generation of a Digital Elevation Model (DEM) in remote sensing is an important application. The process of DEM generation often requires interpolation. This paper is aimed to introduce a class of interpolation algorithms using spectral information, which is widely used in geophysical applications, and to examine the applicability of the method to DEM interpolation. The interpolation process can be explained in two steps. The first step is for finding spectral information from the known data and the second step is finding missing data so as to follow the spectral trend found in the previous step. The interpolation algorithm has been tested for a real DEM data and problems in the DEM interpolation are discussed.

Introduction

Many of human's physical observations are performed in discrete and finite form due to various reasons. And such limit often requires to find missing data by interpolating the known data. In remote sensing, Digital Elevation Model (DEM), which is important for cartographic applications, is a typical case that is required to be interpolated. Recently, Kim (1998) showed comparisons of various DEM interpolation techniques such as averaging, nearest neighborhood interpolation, and kriging. Yet, another interpolation method has been introduced by Claerbout (1992) and been widely used in geophysical applications (Claerbout, 1991, Ji, 1996(a)(b), Zhang et. al, 1992). His recent work (Claerbout, 1999) also showed many possibilities of various applications. Basically his interpolation method uses spectral information of the known data or of the data after interpolation to interpolate. This paper is aimed to introduce the interpolation techniques using the spectrum information of the known data and to examine the applicability of the method in DEM interpolation.

Theory

Claerbout (1992) has given us a direction for interpolation in his book as follows:

> ``A method for restoring missing data is to ensure that the restored data, after specified filtering, has minimum energy."

He has also suggested choosing the filter to have an amplitude spectrum that is an inverse to the spectrum we want for the interpolated data. Such filters can be found in many ways (Ji, 1991, Zhang et. al, 1992), but a prediction-error filter (PEF) would be the one of the best choices since a PEF absorbing all spectrum of a input data with only few terms (i.e. short FIR filter). Therefore, the method can be summarized in two steps procedure as follows:

Step 1 -- Finding a filter whose spectrum is inverse to the one of the known data or a PEF from the known data

Step 2 -- Finding missing data so as the filtered output to have minimum energy.

The profound explanation and detailed implementation techniques of the interpolation method can be found in his book (Claerbout, 1992). However for the completeness of this paper, I describe the only its essence below.

In algebraic description, the first step is finding a PEF **p** by solving the following linear equation described in convolution

$$\mathbf{p} * \mathbf{d}_k \sim 0 \qquad (1)$$

where $\mathbf{d}_k$ represent the known data. In the case of 2-D, the PEF with a size of 3 by 5 has the following form

001xx
xxxxx
xxxxx

where 001 describes the coefficient values that is constrained as it is but the coefficient of the other locations should be found by solving equation (1) in a least-squares sense.

The second step, finding missing data, can be accomplished by solving the following linear equation

$$\mathbf{p}*(\mathbf{d}_k + \mathbf{d}_m) \sim 0$$

Where $\mathbf{d}_m$ represent the missing part of data. Again the finding missing data $\mathbf{d}_m$ is a least-squares problem.

DEM interpolation examples

For the analysis, a real DEM were used as a reference DEM. The size of this DEM was 464 by 749 pixels. Figure 1 shows the reference DEM and its spectrum. To compare with Kim's (1998) results, I extract the reference DEM sample points randomly and uses these random points to interpolate the rest of DEM.

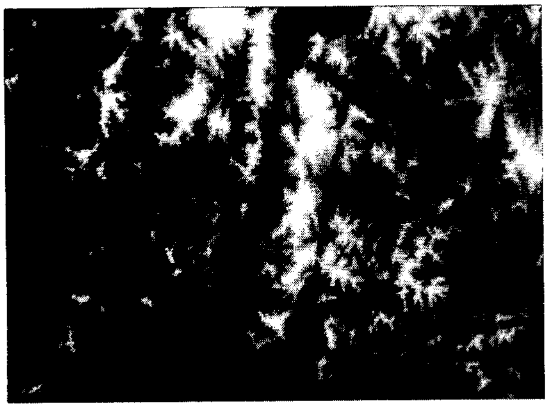
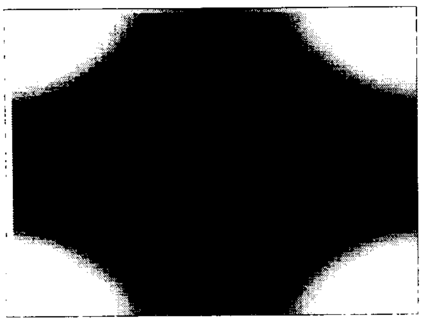


Figure 1 The reference DEM data

At first, for interpolation, I used 3 by 3 size Laplacian filter as an approximation to the PEF. This implies that the DEM is somewhat smoothly changing. Figure 2 shows the spectrum of the Laplacian.



Spectrum of Laplacian

Figure 2 The Spectrum of Laplacian.

Figure 3 shows some representative results of the DEM interpolation obtained from 20%, 10%, 5%, and 1% of the reference DEM, respectively. The performance of the interpolation can be summarized using the root mean square error (RMSE) of the interpolated DEM in Figure 4. The accuracy of the interpolated DEM was quite satisfactory and comparable to the result of kriging method (Kim, 1998).

However, the interpolation results tends to be smooth as the number of known DEM data samples decreases. This is due to the Laplacian filter whose spectral property is too simple, monotonically increase from low to high frequencies. To restore the high frequency components of the DEM data, we can use a PEF obtained from another part of data whose spectral property is similar to the missing data region. Figure 6 shows such example. The upper-left picture of Figure 6 is original DEM data and upper-right picture is the data to be interplated where all missing samples are located right half region (the known samples are only 10% of right half). Lower-left of Figure 6 shows the interpolation result using the Laplacian and lower-right using PEF (size of 5 by 5). RMSEs for both results were 5.85m for the Laplacian and 3.77m for the PEF, respectively. We can observe that the one using PEF contains more high frequency components. This can be deduced by examining the spectral property (Figure 6) of the PEF used for the interpolation.

Conclusion

In this paper, I applied the new interpolation method that uses spectral information to DEM interpolation. Using an Laplacian filter as an inverse of the data spectrum was quite successful for the purpose DEM interpolation and the accuracy was comparable to the kriging method. The interpolation result tend to be smooth as the number of known data samples decrease. In such case, using the PEF obtained from a region of similar spectral property helped to restore the high frequency component of the DEM data reliably.

Acknowledgements

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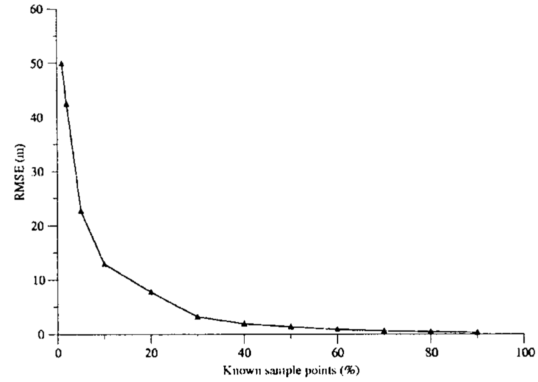
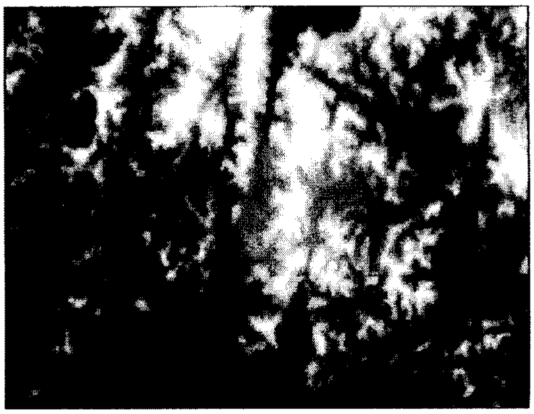


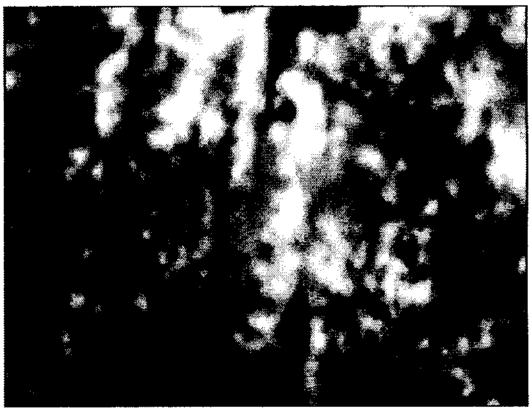
Figure 4. A line graph of interpolated DEM accuracy (RMSE vs. % of reference DEM data used).

interpolated from 20% of data



interpolated from 10% of data

interpolated from 5% of data



interpolated from 1% of data

Figure 3. The results of DEM interpolation using different number of known data samples (20%-upper-left, 10%-upper-right, 5%-lower-left, 1%-lower-right).

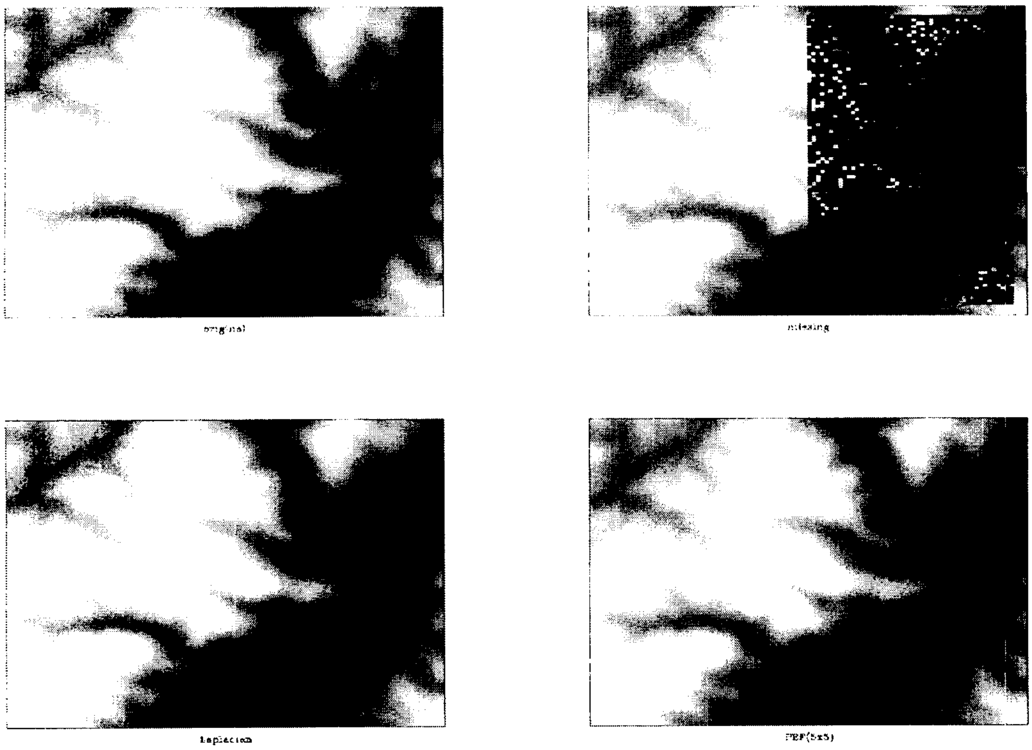


Figure 5. **(upper left) original DEM , (upper right) DEM before interpolation (lower left) The interpolated DEM using 3 by 3 size Laplacian as filter. (lower right) The interpolated DEM using 5 by 5 size PEF as filter.**

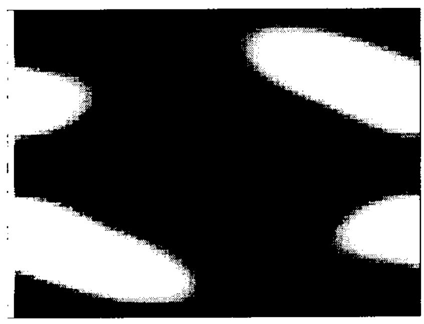


Figure 6. The spectrum of PEF (5 by 5) used in Figure 5.