주파수 오프셋을 갖는 이중 경로 상에서의 One-Tap 등화기 뱅크를 갖는 OFDM 시스템의 BER 성능

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BER Performance of the OFDM System with One-Tap Equalizer Bank under the Two-ray Multipath Channel with Frequency Offset

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Abstract

It is well known that the OFDM transmission is weak against the frequency offset. We evaluate the BER performance of the OFDM system with guard interval and simple one-tap equalizer bank. For the small frequency offset, the loss in E_b/N_o is about *1dB* at required BER = 10^{-5}, when the mean value of the second-ray's attenuation coefficient is *0.25* and the normalized frequency offset, which is normalized about OFDM symbol time, is *5%*.

I. Introduction

Recently, the Orthogonal Frequency Division Multiplexing (OFDM) transmission scheme has been given much attention to support from several to hundreds *Mbps* in wireless communications, because the OFDM is strong against multipath channel and has large bandwidth efficiency.

However, the OFDM transmission has also some disadvantages. One of them is that the OFDM transmission is weak against the frequency offset. For the single carrier system, the frequency offset results in the rotation and attenuation of the useful signal component. In the multi-carrier system, the frequency offset makes not only the rotation and attenuation of the useful signal component but also Inter-Channel Interference (ICI) due to breaking down the orthogonality among the subcarriers. The ICI due to the frequency offset is the main reason for the degradation of the BER performance in the multi-carrier systems [1].

To compensate the phase rotation and attenuation of the useful signal and ICI in OFDM system without Guard Interval (GI), Multiple Input Multiple Output (MIMO) equalizers, which require a complex structure, are necessary [2]. Further, by employing GI, we can improve the performance and reduce the complexity of equalizer structure which is simple one-tap equalizer bank structure.

In this paper, we analyze the BER performance of the OFDM systems with simple one-tap equalizer bank. We consider only the OFDM system with GI and assume that the maximum delay spread is smaller than GI. In addition, we verify the analysis by using simulation.

This paper is organized as follows. Section II shows the signal representations in OFDM system and explains the channel characteristics of the OFDM system with frequency offset. In Section III, we analyze the BER performance of the OFDM system with one-tap equalizer bank. In Section IV, we show some results of the analysis and verify that by using simulation. Finally, we conclude this paper in Section V.

II. Signal Representation

The transmitting signal in OFDM system is represented as follows [3]:

$$T(t) = \sum_{n=-\infty}^{+\infty} \sum_{k=0}^{N-1} \frac{P}{\sqrt{T_s}} a_{n,k} g(t-nT_s) e^{j2\pi f_k t} e^{j2\pi f_c t} \quad (1)$$

where n is the time index, N is the number of the subcarrier, k is the index of subcarrier, T_s is the effective OFDM symbol duration, P is the transmitted power, $a_{n,k}$ is transmitted data symbol of the k-th subcarrier at time n, f_k is the frequency of the k-th subcarrier, f_c is the carrier frequency and $g(t)$ is the pulse shape function. We assume that the pulse shape function is rectangular function.

This signal is transmitted through the multipath channel. In this paper, we use two-ray multipath channel model. One is the mean component of the received signal. The other is the fluctuating component. Its impulse response is given as follows:

$$h(t) = \delta(t) + \alpha\delta(t-\tau) \quad (2)$$

where α is the attenuation coefficient and τ is the delay spread. We assume that α is a random variable with Rayleigh distribution and τ is a random variable with

Uniform distribution. The received signal is given as the summation of the response of channel and AWGN as follows:

$$R'(t)=T'(t)*h(t)+n(t) \quad (3)$$

where "*" represents the convolution operation and $n(t)$ is the AWGN, which has the two-side power spectral density, $N_0/2$. We rewrite the received signal as the eq. (4) by inserting eq. (1) and eq. (2) into eq.(3).

$$R'(t)=\sum_{n=-\infty}^{+\infty}\sum_{k=0}^{N-1}[\frac{P}{\sqrt{T_s}}a_{n,k}g(t-nT_s)e^{j2\pi(f_c+f_k)t} + a\frac{P}{\sqrt{T_s}}a_{n,k}g(t-\tau-nT_s)e^{j2\pi(f_c+f_k)(t-\tau)}+n(t) \quad (4)$$

The received signal is down-converted by multiplying the carrier frequency. Practically, the frequency offset is present due to the difference between transmitter and receiver oscillator. If we consider the frequency offset, the down-converted signal is given as follows:

$$R(t)=\sum_{n=-\infty}^{+\infty}\sum_{k=0}^{N-1}[\frac{P}{\sqrt{T_s}}[1+ae^{-j2\pi(f_c+f_k)\tau}] \times a_{n,k}g(t-nT_s)e^{j2\pi(f_k+\Delta f)t}+n(t)e^{-j2\pi(f_c-\Delta f)t} \quad (5)$$

where Δf is the frequency offset. After the down-conversion, eq. (5) pass through the Matched Filter (MF). In this processing, We assume that the maximum delay spread is smaller than GI. We can generally divide the signal into two part. One is in-phase part, and the other is quadrature part. For the in-phase and quadrature part output signal of the MF are given as follows [4]:

$$\begin{aligned} z^{Re} &= \int_0^{T_s} r(t)\sqrt{2P}\cos(2\pi f_r t)dt+n \\ &= \frac{P}{2\pi(f_r-f_t)}\sin(2\pi(f_r-f_t)T_s)+n \\ z^{Im} &= \int_0^{T_s} r(t)\sqrt{2P}\sin(2\pi f_r t)dt+n \\ &= \frac{P}{2\pi(f_r-f_t)}[\cos(2\pi(f_r-f_t)T_s)-1]+n \end{aligned} \quad (6)$$

where f_r is carrier frequency of the receiver and f_t is carrier frequency of the transmitter. By eq. (6), the MF output signal of the l-th subcarrier is given as follows:

$$\begin{aligned} u_{n,l} = \sum_{k=0}^{N-1} & P(1+ae^{-j2\pi(f_c+f_k)\tau}) \\ & \times[\frac{\sin(2\pi(f_l-f_k+\Delta f)T_s)}{2\pi(f_l-f_k+\Delta f)T_s} \\ & + i\frac{\cos(2\pi(f_l-f_k+\Delta f)T_s)-1}{2\pi(f_l-f_k+\Delta f)T_s}]a_{n,k}+N_{n,k} \end{aligned} \quad (7)$$

where $N_{n,k}$ is the output noise of the matched filter due to the AWGN at the l-th subcarrier, and has Gaussian distribution [5]. We assume that the $N_{n,k}$ is iid process. By that assumption, $N_{n,k}$ is replaced with N. In addition, $f_c\tau$ is larger than 1, because f_c is much greater than τ. It makes the phase term of the second signal component into uniform random variable, θ, over $[0,2\pi]$. Finally, we use normalized frequency offset, ΔfT_s, then the MF output, eq. (7) is changed into eq. (8).

$$\begin{aligned} u_{n,l} = \sum_{k=0}^{N-1} & P(1+ae^{i\theta})[\frac{\sin(2\pi(l-k+\Delta fT_s))}{2\pi(l-k+\Delta fT_s)} \\ & + i\frac{\cos(2\pi(l-k+\Delta fT_s))-1}{2\pi(l-k+\Delta fT_s)}]a_{n,k}+N \end{aligned} \quad (8)$$

In eq. (8), the summation for the subcarrier index, k, is the ICI due to the frequency offset and the *sine* and *cosine* term is the attenuation and phase rotation of the useful signal. While $u_{n,l}$ is used for the decision in OFDM

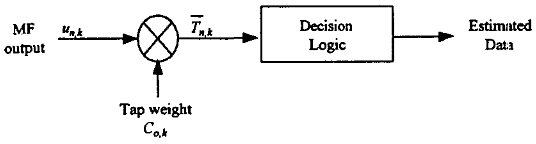

Figure 1. Equalizer structure

system without equalizer bank, $u_{n,l}$ is used for the input of the equalizer bank in OFDM system with equalizer bank.

Ⅲ. BER Performance Analysis

The structure of equalizer and decision logic part is shown in Figure 1. The output of the equalizer, $\overline{T}_{n,l}$, is used for detection. So, we can evaluate the BER performance by using equalizer output signal. To find output signal of equalizer, we find the tap-weight value, which is generally calculated by Minimum Mean Square Error (MMSE) criterion. The l-th subcarrier tap weight which satisfies the MMSE criterion is founded as follows:

$$\begin{aligned} & minimize, MSE = \min E[\varepsilon_{n,l}\,\varepsilon^*_{n,l}] \\ & \Leftrightarrow E[\varepsilon_{n,l}\,u^*_{n,l}]=0 \end{aligned} \quad (9)$$

where $\varepsilon_{n,l}$ is the error between the desired value, $Pa_{n,l}$ and the input signal of the equalizer, $u_{n,l}$. To find tap-weight value, we consider the specific modulation, coherent QPSK. Then, the optimal tap weight value of the l-th subcarrier, $C_{o,l}$, is given as follows:

$$C_{o,l}=\frac{(1+ae^{i\theta})\Theta_{l,l}}{\sum_{k=0}^{N-1}|(1+ae^{i\theta})\Theta_{l,k}|^2+N_0/4E_b} \quad (10)$$

where E_b is the average energy per bit and $\Theta_{l,k}$ is attenuation and phase rotation due to the frequency offset and given as follows:

$$\Theta_{l,k}=\frac{\sin(2\pi(l-k+\Delta fT_s))}{2\pi(l-k+\Delta fT_s)}+i\frac{\cos(2\pi(l-k+\Delta fT_s))-1}{2\pi(l-k+\Delta fT_s)} \quad (11)$$

Then, the l-th subcarrier output of equalizer is given as follows:

$$\overline{T}_{n,l}=C_{o,l}\times u_{n,l} \quad (12)$$

The decision is performed by using $\overline{T}_{n,l}$. The eq. (12) contains three random variables. One is attenuation coefficient, a. The second is the transmitted data symbol sets, $\overrightarrow{a}_n$. The other is the phase term, θ due to the delay spread. To simplify the problem, we use the conditional bit error probability about the three random variables. For QPSK, the signal is divided into two parts. One is in-phase part and the other is quadrature part. Each part is the same as the BPSK. The BER of in-phase and quadrature part is given as follows [5]:

$$\begin{aligned} P^{Re}_{B,k}(\overline{T}_n|\overrightarrow{a}_n,a,\theta) &= Q(\frac{Th-m_{Re}}{\sigma}) \\ P^{Im}_{B,k}(\overline{T}_n|\overrightarrow{a}_n,a,\theta) &= Q(\frac{Th-m_{Im}}{\sigma}) \end{aligned} \quad (13)$$

where "Th" is the threshold value of the in-phase detection part, m is the mean of the MF output, σ is the standard derivation of the noise at the MF output, "$_{Re}$" means the in-phase part, "$_{Im}$" means the quadrature part, and the $Q(x)$ is defined as follows:

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-\frac{u^2}{2})du \tag{14}$$

From eq. (13), the conditional mean of the in-phase part and noise variance is given as follows:

$$m_{Re}=real\{\sum_{k=0}^{N-1}C_{o,l}\times P(1+ae^{j\theta})a_{n,k}\times \Theta_{l,k}\}$$
$$\sigma^2=|C_{o,l}|^2\times N_0/2 \tag{15}$$

where $real\{x\}$ means the real part of x. Using eq. (10) and (15), the conditional BER of the in-phase part is given as follows:

$$P^{Re}_{B,l}(\overline{T}_n|\overrightarrow{a_n},a,\theta)$$
$$=Q(\sqrt{\frac{2E_b}{N_0}}real\{\sum_{k=0}^{N-1}\frac{C_{o,l}}{|C_{o,l}|}(1+ae^{-j\theta})a_{n,k}\Theta_{l,k}\}) \tag{16}$$

Similarly with the in-phase case, the conditional BER of the quadrature part is given as follows:

$$P^{Im}_{B,l}(\overline{T}_n|\overrightarrow{a_n},a,\theta)$$
$$=Q(\sqrt{\frac{2E_b}{N_0}}imag\{\sum_{k=0}^{N-1}\frac{C_{o,l}}{|C_{o,l}|}(1+ae^{j\theta})a_{n,k}\Theta_{l,k}\}) \tag{17}$$

where $imag(x)$ means the imaginary part of x. The symbol error occurs when in-phase or quadrature error occurs. So, we can find conditional symbol error probability as follows:

$$P_{E,l}(\overline{T}_n|\overrightarrow{a_n},a,\theta)=1-(1-P^{Re}_{B,l}(\overline{T}_n|\overrightarrow{a_n},a,\theta))\times(1-P^{Im}_{B,l}(\overline{T}_n|\overrightarrow{a_n},a,\theta)) \tag{18}$$

Then, we use total probability about $\overrightarrow{a_n}$.

$$P_{E,l}(\overline{T}_n|a,\theta)=\sum_{a_n}P_{E,l}(\overline{T}_n|\overrightarrow{a_n},a,\theta)\Pr(\overrightarrow{a_n}) \tag{19}$$

where $Pr(\overrightarrow{a_n})$ is a priori probability of $\overrightarrow{a_n}$. We assume that $\overrightarrow{a_n}$'s are given equally likely signals and symmetrical likelihoods. However, there are too many possible data set. So, we consider only two adjust data set. That makes analysis simple and analysis error.

$$P_{E,l}(\overline{T}_n|a,\theta)$$
$$\approx\sum_{k=-1}^{k=1}P_{E,l}(\overline{T}_n|a_{n,l+k},a,\theta)\Pr(a_{n,l\pm1},a_{n,l}) \tag{20}$$

Then, we use total probability about a and θ.

$$P_{E,l}(\overline{T}_n)=\int_0^{2\pi}\int_0^{\infty}P_{E,l}(\overline{T}_n|a,\theta)f(a)f(\theta)dad\theta \tag{21}$$

where $f(a)$ is the probability density function of a and $f(\theta)$ is the probability density function of θ. $f(a)$ is Rayleigh distribution and $f(\theta)$ is Uniform distribution over $[0,2\pi]$. Next, we find symbol error probability by averaging about total N subcarriers.

$$P_E=\frac{1}{N}\sum_{l=1}^{N-1}P_{E,l}(\overline{T}_n)$$
$$=P_{E,l}(\overline{T}_n) \tag{22}$$

In this processing, we use that the symbol error probability of every subcarrier is symmetric about the phase term, θ and the transmitted data, $a_{n,k}$. So, Total average symbol error probability is same as that of the l-th subcarrier. For Gray-coded M-PSK modulation, the relation between the symbol error probability, P_E, and the bit error probability, P_B, is given as follows [5]:

$$P_B\approx\frac{1}{\log_2 M}P_E \tag{23}$$

where M is number of modulated symbols.

Parameter	Value
Data rate	155Mbps
Carrier frequency, f_c	30GHz
Number of subcarriers	128
Modulation type	Coherent QPSK
Symbol duration, T_s	1.65 μs
Gurd Interval, T_g	103ns
Delay, τ	Uniform R.V. [50,100ns]
Attenuation Coeff. a	Rayleigh R.V.
Equalizer	LMS (100 training sequence, Δ=0.05)
No. of OFDM symbol	1000(256kbit)

Table 1. System parameter values used in the analysis and the simulation

For the OFDM system without equalizer bank, the BER performance can be found by same procedure of that with equalizer bank, if we use tap-weight value as one.

IV. Analysis Results

The OFDM system is aimed to support the data rate from a few *Mbps* to hundreds *Mbps*. We use system parameter values given in Table 1. The target data rate is *155Mbps*. We consider the data service and set the required BER as 10^{-5}, which is reasonable for the data transmission in wireless communications. Simulation is done to verify the analysis. The number of the OFDM symbol block is 1000. We assume that the channel characteristic does not change during one OFDM block duration. That means that simulation generate the random variables according to the OFDM block duration. To increase the accuracy of the simulation, we repeat simulation several times. We employee least-mean-square (LMS) equalizer. Because it is very popular in commercial communication system. The analysis and simulation are done according to the variation of the normalized frequency offset and the channel condition which is determined by the mean value of attenuation coefficient a.

Figure 2 shows the BER performance of two OFDM systems with and without simple one-tap equalizer bank according to the variation of the normalized frequency offset, Δf = *0, 5* and *10%* for the mean value of the second-ray's attenuation coefficient, $E[a]$ = *0.2*. As the normalized frequency offset increases, the BER performance degradation also increases. For the OFDM system without equalizer bank, the required BER = 10^{-5} can be attained at the E_b/N_0 = *19.5dB* in case of no frequency offset. However, it can not be attained if there is frequency offset. The BER performance degradation is very large according to the increasing normalized frequency offset. For the OFDM system with simple one-tap equalizer bank, the BER performance degradation occurs according to increasing frequency offset. However, if the normalized frequency offset is smaller than *5%*, the performance degradation is smaller than *0.5dB* at the required BER = 10^{-5}. For the large normalized frequency offset, the performance degradation is very large. For example, when the normalized frequency offset is *10%*, the loss in E_b/N_0 is about *4dB* comparing with the case of no frequency offset at the required BER = 10^{-5}.

Figure 3 shows the BER performance of two OFDM systems with and without simple one-tap equalizer bank

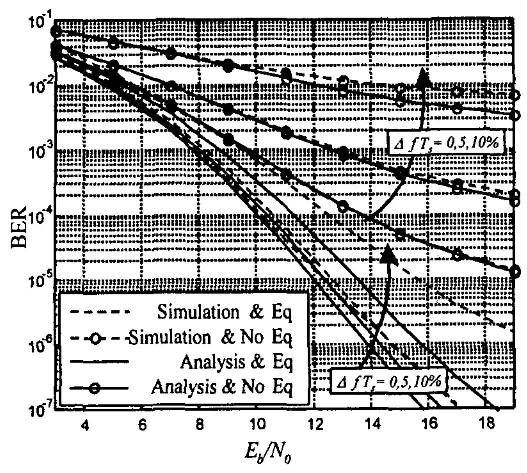

Figure 2. BER performance of the OFDM system with and without the equalizer bank according to the variation of the normalized frequency offset under the two-ray multipath channel with $E[\alpha]$=0.2

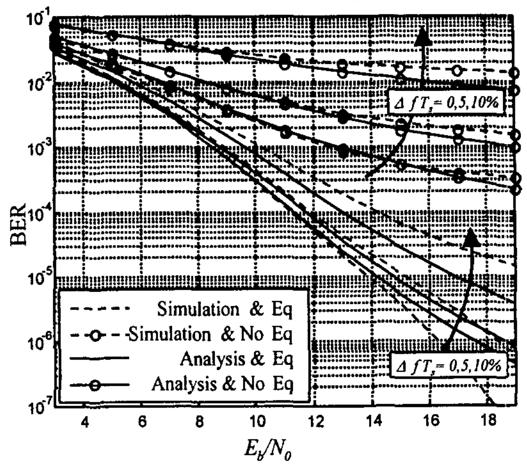

Figure 3. BER performance of the OFDM system with and without the equalizer bank according to the variation of the normalized frequency offset under the two-ray multipath channel with $E[\alpha]$=0.3

according to the variation of the normalized frequency offset for $E[\alpha]$ = *0.25*. Comparing with figure 2, we know that the performance degradation is larger as the channel condition is worse. For OFDM system with simple one-tap equalizer bank, the loss in E_b/N_0 is about *1dB* when ΔfT_s is *5%* comparing with no frequency offset case. As the normalized frequency offset becomes larger, the loss in E_b/N_0 becomes larger. When the large normalized frequency offset is *10%*, the loss in E_b/N_0 is about *6dB*.

In figure 2 with figure 3, we know that the simple one-tap equalizer bank can successfully compensate the attenuation and phase rotation of the useful signal for the small normalized frequency offset, $\Delta f \leq 5\%$. The required BER can be supported by increasing transmission power slightly without employing any other techniques. However, the loss in E_b/N_0 is very large, for the large frequency offset. Because, the ICI increases more significantly, as the frequency offset becomes larger. To overcome ICI, other techniques are considered [6,7].

V. Conclusions

In this paper, we analyzed the BER performance of two OFDM systems with and without a simple one-tap equalizer bank under the two-ray multipath channel with frequency offset. While the OFDM system without equalizer bank could not support the required BER = 10^{-5}, the system with simple one-tap equalizer bank could support the required BER.

For the OFDM system with simple one-tap equalizer bank, we could obtain the required BER by increasing the *1dB* transmission power without applying any other techniques, when the small normalized frequency offset, ΔfT_s, was *5%*, and the mean value of the second-ray's attenuation coefficient, $E[\alpha]$, was *0.25*. That reason was that the simple one-tap equalizer bank could compensate the attenuation and phase rotation of the useful signal due to the frequency offset.

As the frequency offset became larger, the ICI was more critical to the BER performance than the attenuation and phase rotation of the useful signal. The loss in E_b/N_0 was about *6dB*, when the normalized frequency offset was *10%* and the $E[\alpha]$ was *0.25*. The simple one-tap equalizer bank could not compensate ICI due to the frequency offset. So, other techniques were considered to compensate ICI.

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