고정이득 궤환 안정화를 위한 충분조건

강 환일
명지대학교 전기,정보제어공학부
전화 : (0335) 330-6476 / 팩스 : (0335) 321-0271

A Sufficient condition for constant gain feedback stabilization

Hwan Il Kang
Division of Electrical & Information control Eng. Myong Ji University Yongin, Gyunggi-do, 449-728
Tel: 82-335-330-6476 Fax: 82-335-321-0271, email: hwa

Abstract

We consider a negative unity feedback control system in which a controller and a given minimum phase transfer function are in cascade. This paper present a sufficient condition for the existence of a constant gain controller under which the overall closed loop characteristic polynomial is stable. This sufficient condition is based on Lehnigk's lemmas.

I. 서론

Consider a unity feedback closed-loop control system such that a constant gain controller and a given plant having a transfer function are in cascade. The main question is: Does there exist a positive or negative constant gain for which the characteristic polynomial of the closed loop system is stable? We can obtain the existence of constant gain by the classical approach such as the root locus method[1], the Nyquist stability criterion or the Routh-Hurwitz criterion. By using the root locus method or the Nyquist stability criterion, we can determine the existence of a constant gain controller graphically. By the Routh-Hurwitz criterion, we can determine the existence of a constant gain controller analytically. However, if the order of the denominator and the numerator of the transfer function is higher, determining the existence of a constant gain controller involves a lots of inequalities. In this paper, we present a sufficient condition for the existence of a constant controller analytically. The main results are based on Lehnigk's lemmas. We present the lemmas in section 2 and then the main result in section 3.

II. Notations & Lemmas

Let three polynomials be $f_0(s), f_1(s)$ and $f(s) = f_0(s) + f_1(s)$which we write as

$$f_0(s) = \sum_{\nu=0}^{n} b_\nu s^\nu \qquad (b_\nu > 0)$$

$$f_1(s) = \sum_{\nu=0}^{n_1} c_\nu s^\nu \qquad (c_{n_1} \neq 0, n_1 < n)$$

$$f(s) = \sum_{\nu=0}^{n} a_\nu s^\nu \qquad (a_n = b_n > 0)$$

with

$$a_\nu = b_\nu + c_\nu \qquad (0 \le \nu \le n_1)$$
$$a_\nu = b_\nu \qquad (n_1 + 1 \le \nu \le n)$$

The number π_0 is a number of multiplicity of the zeros at the origin of $f_0(s)$. τ_0 is the number of factors

$A_\nu s^2 + B_\nu s - C_\nu$ and $A_\nu s^2 - C_\nu$ and $B_\nu s - C_\nu$ for $A_\nu, B_\nu, C_\nu > 0$

Let ℓ_0 and ℓ_1 be the number of zeros with negative real parts of $f_0(s)$ and $f_1(s)$, respectively. The number r_0 denotes the number of zeros with nonnegative real parts of $f_0(s)$.

A polynomial called a *normal* polynomial that satisfies the condition that none of the factors of $f_0(s)$ is of the form

$A_\nu s^2 - B_\nu s - C_\nu$ or $A_\nu s^2 - C_\nu$ for any $A_\nu, B_\nu, C_\nu > 0$

Now we describe two lemmas by Lehnigk[2].

Lemma 1[2, p. 221]: Let $f_0(s)$ be a normal polynomial and of degree n, and let $f_1(s) = \sum_{\nu=0} c_\nu s^\nu$ with $c_{n_1} > 0$ be stable and of degree $n_1 < n$. For $f(s) = f_0(s) + f_1(s)$ to be stable, it is necessary and sufficient that the inequality

1) $\pi_0 + \tau_0 \le n_1 + 1$

holds

and that one of the following inequalities hold which corresponds to the given numbers n and n_1:

2) n even, n_1 even

$$r_0 \le x_0 + x_1 - \begin{pmatrix} 1 \quad \text{if } n = 0(\text{mod}4) > 0 \text{ and } n_1 = 0 \\ 0 \ \textit{otherwise}(c_{n_{1-1}} > 0 \text{ if } n - n_1 = 0(\text{mod}4) > 0) \end{pmatrix}$$

3) n even, n_1 odd

$$r_0 \le x_0 + x_1 - \frac{1-(-1)^{x_0+x_1}}{2}$$

4) n odd n_1 even

$$r_0 \le x_0 + x_1 - \frac{1+(-1)^{x_0+x_1}}{2}$$

5) n odd n_1 odd

$$r_0 \le x_0 + x - 1 - 1 \quad (c_{n_1} - 1 > 0 \quad \text{if} \quad n - n_1 = 0(\text{mod}4) > 0)$$

with $x_0 = \lfloor (n+1)/2 \rfloor$, $x_1 = \lfloor (n_1+1)/2 \rfloor$

III. Main Results

In the case of constant gain stabilization, $C(s) = k$ so that the closed loop characteristic polynomial $\delta(s) = f_0(s) + kf_1(s)$.

Our objective is to obtain a sufficient condition for the existence of $k > 0$ or $k < 0$ for which the characteristic polynomial of the closed loop system is stable, i.e., $\delta(s,k)$ is Hurwitz. Since using Lemma 1 & 2 it is easy to prove the main results. we skip the proof. It is a position to present the main results for the existence of a constant gain controller.

Theorem 1: *Consider a unity negative feedback control system for which a given plant with the minimum phase transfer function $\frac{f_1(s)}{f_0(s)}$ and a controller k are in cascade. The constant gain controller $k > 0$ exists if the condition*

1) $\pi_0 + \tau_0 \le n_1 + 1$

holds and one of the following inequalities hold which corresponds to the given numbers n and n_1:

2) n even, n_1 even

$$r_0 \le x_0 + x_1 - \begin{cases} 1 & \text{if } n = 2\,(\mathrm{mod}4) > 0 \text{ and } n_1 = 0 \\ 0 & otherwise(c_{n_1-1} < 0 \text{ if } n - n_1 = 2(\mathrm{mod}4)) \end{cases}$$

3) n even, n_1 odd

$$r_0 \le x_0 + x_1 - \frac{1+(-1)^{x_0+x_1}}{2}$$

4) n odd n_1 even

$$r_0 \le x_0 + x_1 - \frac{1-(-1)^{x_0+x_1}}{2}$$

5) n odd n_1 odd

$$r_0 \le x_0 + x - 1 \quad (c_{n_1-1} < 0 \quad \text{if} \quad n - n_1 = 2(\mathrm{mod}4))$$

with $x_0 = \lfloor (n+1)/2 \rfloor$, $x_1 = \lfloor (n_1+1)/2 \rfloor$

Theorem 2: *Consider a unity negative feedback control system for which a given plant with the minimum phase transfer function $\frac{f_1(s)}{f_0(s)}$ and a controller k are in cascade. The constant gain controller $k < 0$ exists if the condition*

1) $\pi_0 + \tau_0 = 0$

holds and one of the following inequalities hold which corresponds to the given numbers n and n_1:

2) n even, n_1 even

$$r_0 \le x_0 + x_1 - \begin{cases} 1 & \text{if } n = 2\ (\mathrm{mod}4) > 0 \text{ and } n_1 = 0 \\ 0 & otherwise(c_{n_1-1} < 0 \ \text{if } n - n_1 = 2(\mathrm{mod}4)) \end{cases}$$

3) n even, n_1 odd

$$r_0 \le x_0 + x_1 - \frac{1+(-1)^{x_0+x_1}}{2}$$

4) n odd n_1 even

$$r_0 \le x_0 + x_1 - \frac{1-(-1)^{x_0+x_1}}{2}$$

5) n odd n_1 odd

$$r_0 \le x_0 + x - 1 \quad (c_{n_1-1} < 0 \quad \text{if} \quad n - n_1 = 2(\mathrm{mod}4))$$

with $x_0 = \lfloor (n+1)/2 \rfloor$, $x_1 = \lfloor (n_1+1)/2 \rfloor$

Ⅵ. Examples

To see the validity of Theorems 1 & 2, consider a following plant with transfer function .

$$\frac{f_1(s)}{f_0(s)} = \frac{(s+A)(s^2+Bs+C)(s+D)}{(s+E)(s^2+Fs+G)(s^2+Hs+J)} \quad \text{for any}$$

$A, B, C, D, E, F, G, J > 0$

Since $f_0(s)$ has no form of $(s^2 - Ks - L)$or $s^2 - M$ for any $K.L > 0$ $f_0(s)$ is normal. the number $\pi_0 = 0$ because $f_0(s)$ has roots at the origin and $\tau_0 = 0$ because $f_0(s)$ has no forms of three factors $(s^2 + Ks - L)$or $Ks^2 - L$ or $Ks - L$ for any $K, L > 0$. In addition, $f_1(s)$ is stable. The given transfer function has the minimum phase property. Now, it is straightforward that the condition (4) in Theorem 1 is satisfied. Therefore there exists $k > 0$ for which the closed loop characteristic polynomial is stable. To apply this example to the Theorem 2, it is easy to verify that the example above satisfies the conditions in Theorem 2. Hence we conclude that there exist both a $k > 0$ and $k < 0$ for which the characteristic polynomial of the closed loop system associated with the example is stable.

V. Conclusions.

In this paper, we presented a sufficient condition for the existence of a constant controller analytically. The main results are based on Lehnigk's lemmas. By the example we showed the validity of the main results To see the validity of Theorems 1 & 2, consider a following plant with transfer function

$$\frac{f_1(s)}{f_0(s)} = \frac{(s+A)(s^2+Bs+C)(s+D)}{(s+E)(s^2+Fs+G)(s^2+Hs+J)} \quad \text{for any}$$

$A, B, C, D, E, F, G, J > 0$

Since $f_0(s)$ has no form of (s^2-Ks-L) or s^2-M for any $K.L>0$ $f_0(s)$ is normal. the number $\pi_0=0$ because $f_0(s)$ has roots at the origin and $\tau_0=0$ because $f_0(s)$ has no forms of three factors (s^2+Ks-L) or Ks^2-L or $Ks-L$ for any $K, L>0$. In addition, $f_1(s)$ is stable. The given transfer function has the minimum phase property. Now, it is straightforward that the condition (4) in Theorem 1 is satisfied. Therefore there exists $k>0$ for which the closed loop characteristic polynomial is stable. To apply this example to the Theorem 2, it is easy to verify that the example above satisfies the conditions in Theorem 2. Hence we conclude that there exist both a $k>0$ and $k<0$ for which the characteristic polynomial of the closed loop system associated with the example is stable.

Reference

[1] W. R. Evans, *Control system Dynamics*, Mcgraw-Hill,New York, 1954.

[2] S. H. Lehnigk, *Stability Theorems for Linear Motions with an Introduction to Liapunov's Direct Method*, Prentice Hall, Englewood Cliffs, NJ, 1966.

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