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Robust Adaptive Control of a Nonholonomic Mobile Robot

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Abstract

The main stream of researches on the mobile robot is planning motions of the mobile robot under nonholonomic constraints while only considering kinematic model of a mobile robot. These researches, however, assume that there is some kind of dynamic controller which can produce perfectly the same velocity that is necessary for the kinematic controller. Moreover, there are little results about the problem of integrating the nonholonomic kinematic controller and the dynamic controller for a mobile robot. Also the literature on the robustness of the controller in the presence of uncertainties or external disturbances in the dynamical model of a mobile robot is very few.
Thus, in this paper, the robust adaptive controller which can achieve velocity tracking while considering not only kinematic model but also dynamic model of the mobile robot is proposed. The stability of the dynamic system will be shown through the Lyapunov method.

1. Introduction

Mobile robot has been the object of many researchers because of its usefulness in many applications. The main stream of researches on the mobile robot is planning motions of the mobile robot. But a mobile robot suffers nonholonomic constraints. That is, the robot can only move in the direction normal to the axis of the driving wheels. To overcome this constraints, much has been written about solving the problem of motion under nonholonomic constraints using the kinematic model of a mobile robot. Those methods, however, assume that there is some kind of dynamic controller which can produce perfectly the same velocity that is necessary for the kinematic controller. Some nonlinear feedback controllers have been proposed to solve these problems.[1][2][5]. All these controllers consider only the kinematic model (e.g. steering system) of the mobile robot, and 'perfect velocity' tracking is assumed to generate the actual vehicle control inputs. But it is not easy to realize 'perfect velocity' tracking.
Although there have been a few researches in which the dynamics of the mobile robot is considered.[3][4], but in these method, a perfect knowledge about the mobile robot parameter is necessary. Generally, this requirement cannot be fulfilled. For example, friction is very difficult to model by conventional techniques.
There are little results about the problem of integrating the nonholonomic kinematic controller and the dynamic controller for a mobile robot. Moreover, the literature on the robustness of control scheme in the presence of uncertainties in the dynamical model of such system is very few.
By the way, there are many possible methods we can use when the knowledge about the robot is not complete, like adaptive control, robust control and so on.[6][7] Thus, in this paper, the robust adaptive controller which can achieve velocity tracking while considering not only kinematics but also dynamics of the mobile robot will be designed. The proposed controller can compensate the uncertainty or external disturbances by robust technique.

2. A Nonholonomic Mobile Robot

A mobile robot having an n-dimensional configurations space C with generalized coordinates $(q_1, \dots, q_n)$ and subject to m constraints can be described by following equation.

$$M(q)\ddot{q}+ V_m(q,\dot{q})\dot{q}+F(\dot{q})+G(q)+\tau_d=B(q)\tau-A^T(q)\lambda \quad (1)$$

where $M(q)\in R^{n\times n}$ is a symmetric, positive definite inertia matrix, $V_m(q,\dot{q})\in R^{n\times n}$ is the centripetal and coriolis matrix, $F(\dot{q})\in R^{n\times 1}$ denotes the surface friction, $G(q)\in R^{n\times 1}$ is the gravitational vector τ_d denotes bounded unknown disturbances including unstructured unmodeled dynamics, $B(q)\in R^{n\times r}$ is the input transition matrix, $\tau\in R^{n\times 1}$, is the input vector, $A(q)\in R^{m\times n}$ is the matrix associated with the constraints, and $\lambda\in R^{m\times 1}$ is the vector of constraint forces. All kinematic equality constraints can be expressed as follows:

$$A(q)\dot{q}=0 \quad (2)$$

Then, $S(q)$ is a full rank matrix (n-m) formed by a set of smooth and linearly independent vector fields spanning the null space of $A(q)$:

$$S^T(q)A^T(q)=0 \quad (3)$$

According to the (2) and (3), it is possible to find an auxiliary vector time function $v(t)\in R^{n-m}$ such that for all t, next equation holds.

$$\dot{q}=S(q)v(t) \quad (4)$$

where S(q) is given by

$$S(q)=\begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \quad (5)$$

Kinematic equations of motion (4) can be described in terms of its linear velocity and angular velocity.

$$v=\begin{bmatrix} v \\ w \end{bmatrix}=\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix}=\begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (7)$$

where $|v_1|\le V_{\max}$, $|v_2|\le W_{\max}$.

System (7) is called the steering system of the mobile robot. A mobile robot in 2-D plane is shown in Fig.1.

After defining the error as in (8), the velocity command in equation (9) can be used while guaranteeing the asymptotic stability of the steering system if dynamics of a mobile robot is ignored.[1]

$$e_p=T_e(q_r-q),\ \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_r-x \\ y_r-y \\ \theta_r-\theta \end{bmatrix} \quad (8)$$

$$v_c=\begin{bmatrix} v_r\cos e_3+k_1e_1 \\ w_r+k_2v_re_2+k_3v_r\sin e_3 \end{bmatrix},\ v_c=f_c(e_p, v_r, K) \quad (9)$$

The dynamics of a nonholonomic mobile robot is as in equation (1). To develop a dynamic controller, the dynamics of a mobile robot is transformed into a more appropriate form. If we differentiate (3) and substitute the results in (1), and multiplying (1) by S^T, then complete equations of motion of the nonholonomic mobile robots are given by

$$\dot{q}=Sv \quad (10)$$

$$S^TMS\dot{v}+S^T(M\dot{S}+V_mS)v+S^TF+S^T\tau_d=S^TB\tau \quad (11)$$

With appropriate definitions, (11) can be rewritten as follwos:

$$\overline{M}(q)\dot{v}+\overline{V_m}(q,\dot{q})v+\overline{F}(v)+\overline{\tau_d}=\overline{B}\tau \quad (12)$$
$$\overline{\tau}\equiv\overline{B}\tau$$

There are several properties hold for this system as stated in [3].

Boundedness : $\overline{M}(q)$ and the norm of the $\overline{V_m}(q,\dot{q})$, are bounded.

Skew-symmetry: $\dot{\overline{M}}-2\overline{V_m}$ is skew symmetric.

The proof for skew-symmetry property is straightforward and omitted here. These properties enable the introduction of general nonlinear control method like Lyapunov direct method. $G(q)$ has been deleted because the motion of a mobile robot is constrained to the horizontal plane.

3. Controller Design

Under assumption that the perfect velocity tracking is possible, the input command v_c in (9) can achieve mobile robot navigation problem. Thus here we design the dynamic controller which guarantees v approaches v_c as $t\to\infty$. The objective is to track the desired velocity v_c in the presence of uncertainties of the dynamic model or external disturbances. We can write the components of equation (12) as follows :

$$\overline{M}(q)=\overline{M_o}(q)+\triangle\overline{M}(q) \quad (13)$$

$$\overline{V_m}(q,\dot{q})=\overline{V_{mo}}(q,\dot{q})+\triangle\overline{V_m}(q,\dot{q}) \quad (14)$$

where subscripts o represent nominal part of the system and $\triangle$ represents unknown parts. Then equation (12) can be rewritten as follows:

$$\overline{M_o}(q)\dot{v}+\overline{V_{mo}}(q,\dot{q})v+\eta=\overline{\tau} \quad (15)$$

where η is lumped uncertainty.

$$\eta=\triangle\overline{M}(q)+\triangle\overline{V_m}(q,\dot{q})+\overline{F}(v)+\overline{\tau_d} \quad (16)$$

Entire friction is lumped into the uncertainty because it is hard to modeling. Without loss of generality, we can state that every components of equation (16) is bounded by unknown coefficients.

$\|S\|\le d_1$, $\|\dot{S}\|\le d_2\|\dot{\theta}\|$, $\|\triangle\overline{M}\|\le m_1$,

$\|V_m\|\le v_1\|\dot{q}\|$, $\|F\|\le f_1\|\dot{q}\|+f_2$, $\|\tau_d\|\le\tau_1$

where d_1, d_2, m_1, v_1, f_1, f_2, are positive constants.

Then we can make an assumption that η is bounded by the norm of the following form.

$$\|\eta\|\le c_0+c_1\|\dot{q}\|+c_2\|\dot{q}\|\|v\|+c_3\|\dot{\theta}\|=\mu^T\psi=\rho \quad (17)$$

where $\mu=[c_0\ c_1\ c_2\ c_3]\in R^4$ is an unknown nonnegative constant vector and $\psi=[1\ \ \|\dot{q}\|\ \ \|\dot{q}\|\|v\|\ \ \|\dot{\theta}\|\ ]^T$

Theorem 1

If η is bounded as in (17), and if the control law (18) ~ (20) is applied to the system (12), then the error between v of the system and v_c in [9] is globally uniformly ultimately bounded.

$$\overline{\tau}=K_4e+\overline{V_{mo}}v_c+\overline{M_o}\dot{v}_c+\triangle U \quad (18)$$

$$\triangle U=-\hat{\rho}\frac{e}{\gamma(\|e\|)} \quad (19)$$

$$\dot{\hat{\mu}}=\Gamma\left(\frac{\psi\|e\|^2}{\gamma(\|e\|)}-\sigma\hat{\mu}\right) \quad (20)$$

where K_4 in (18) is positive gain and $\triangle U$ is a robust control input. $\hat{\rho}=\widehat{\mu}^T\psi$ and $\hat{\mu}$ is adapted by adaptation law (20). $\Gamma\in R^{4\times4}$ in (20) is a positive constant adaptation gain matrix and $\sigma\in R^{4\times4}$ is a positive definite constant diagonal matrix.

(proof)

Error in velocity is defined as follow.

$$e=v_c-v \quad (21)$$
$$\dot{e}=\dot{v}_c-\dot{v} \quad (22)$$

Let us consider the following Lyapunov candidate.

$$V=\frac{1}{2}e^T\overline{M}_oe+\frac{1}{2}\tilde{\mu}^T\Gamma^{-1}\tilde{\mu} \quad (23)$$

where $\tilde{\mu}=\hat{\mu}-\mu$

If we differentiate V, then $\dot{V}$ is

$$\dot{V}=e^T\overline{M}_o\dot{e}+\frac{1}{2}e^T\dot{\overline{M}}_oe+\tilde{\mu}^T\Gamma^{-1}\dot{\hat{\mu}} \quad (24)$$

If we multiply equation (22) by $\overline{M}_o$ and use equation (15), (18) and (19), we can obtain following results.

$$\begin{aligned}\dot{V} &= e^T(-K_4e+\eta+\triangle U)+\tilde{\mu}^T\Gamma^{-1}\dot{\hat{\mu}}\\ &=-e^TK_4e+e^T(\eta-\hat{\rho}\frac{e}{\gamma(\|e\|)})+\tilde{\mu}^T\Gamma^{-1}\dot{\hat{\mu}}\end{aligned} \quad (25)$$

The bound for $\dot{V}$ is obtained by taking norm of right hand side of (25) and using (17).

$$\begin{aligned}\dot{V} &\le -e^TK_4e-\hat{\rho}\frac{\|e\|^2}{\gamma(\|e\|)}+\|e\|\|\eta\|+\tilde{\mu}^T\Gamma^{-1}\dot{\hat{\mu}}\\ &=-e^TK_4e-\hat{\rho}\frac{\|e\|^2}{\gamma(\|e\|)}+\|e\|\rho+\tilde{\mu}^T\Gamma^{-1}\dot{\hat{\mu}}\\ &=-e^TK_4e-\tilde{\rho}\frac{\|e\|^2}{\gamma(\|e\|)}\\ &\quad+\frac{\|e\|}{\gamma(\|e\|)}(\gamma(\|e\|)-\|e\|)\mu^T\psi+\tilde{\mu}^T(\frac{\|e\|^2}{\gamma(\|e\|)}\psi-\sigma\hat{\mu})\\ &=-e^TK_4e-\tilde{\mu}^T\sigma\hat{\mu}+w(\rho,\|e\|)\end{aligned} \quad (26)$$

where $w(\rho,\|e\|)=\frac{\|e\|}{\gamma(\|e\|)}(\gamma(\|e\|)-\|e\|)\mu^T\psi$ The following relationship holds: $\frac{1}{2}(\tilde{\mu}+\mu)^T\sigma(\tilde{\mu}+\mu)\ge 0$ Thus $\tilde{\mu}^T\sigma\tilde{\mu}+\tilde{\mu}^T\sigma\mu\ge\frac{1}{2}(\tilde{\mu}^T\sigma\tilde{\mu}-\mu^T\sigma\mu)$ then above inequality becomes as following.

$$\begin{aligned}\dot{V} &\le -e^TK_4e-\frac{1}{2}(\tilde{\mu}^T\sigma\tilde{\mu}-\mu^T\sigma\mu)+w(\rho,\|e\|)\\ &=-\frac{1}{2}z^TQz+\overline{w}(\rho,\|e\|)\\ &\le-\frac{1}{2}\lambda_{\min}(Q)\|z\|^2+\overline{w}(\rho,\|e\|)\end{aligned} \quad (27)$$

where $z=[e^T\tilde{\mu}^T]^T$, $\lambda_{\min}(\cdot)$ represent minimum eigenvalue of its argument, $Q=\begin{pmatrix}2K_4 & 0\\ 0 & \sigma\end{pmatrix}$ and $\overline{w}(\rho,\|e\|)=\frac{1}{2}\mu^T\sigma\mu+w(\rho,\|e\|)$. From (23),

$V(z)=\frac{1}{2}z^TPz\le\frac{1}{2}\lambda_{\max}(P)\|e\|^2$ where $\lambda_{\max}(\cdot)$ represent maximum eigenvalue of its argument and $P=\begin{pmatrix}\overline{M}_o & 0\\ 0 & \Gamma^{-1}\end{pmatrix}$. Thus,

$$\begin{aligned}\dot{V} &\le -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V+\overline{w}(\rho,\|e\|)\\ &\le-\zeta V+\overline{w}(\rho,\|e\|)\end{aligned} \quad (28)$$

The differential inequality (28) has the following solution:

$$\begin{aligned}V(t,z(t)) &\le\frac{\overline{w}(\rho,\|e\|)}{\zeta}\\ &\quad+[V(t_0,z(t_0))-\frac{\overline{w}(\rho,\|e\|)}{\zeta}]e^{-\zeta(t-t_0)}\end{aligned} \quad (29)$$

Since $V(t,z(t))\ge\frac{1}{2}e^T\overline{M}_oe\ge\frac{1}{2}\lambda_{\min}(\overline{M}_o)\|e\|^2$ and $V(t,z(t))\ge\frac{1}{2}\tilde{\mu}^T\Gamma^{-1}\tilde{\mu}\ge\frac{1}{2}\lambda_{\min}(\Gamma^{-1})\|\tilde{\mu}\|^2$, e and μ are bounded as

$$\|e\|\le[\frac{2V}{\lambda_{\min}(\overline{M}_o)}]^{\frac{1}{2}},\ \|\tilde{\mu}\|\le[\frac{2V}{\lambda_{\min}(\Gamma^{-1})}]^{\frac{1}{2}} \quad (30)$$

Consequently, the error between the velocity v of the dynamic controller and v_c of the kinematic controller is globally uniformly ultimately bounded.

4. Simulation

Reference cart tracking is simulated in this paper.
Reference cart is moving according to the equation (28).

$$\dot{x}=v_r\cos\theta,\quad \dot{y}=v_r\sin\theta,\quad \dot{\theta}=w_rt \quad (28)$$

where $v_r=0.5m/$secand $w_r=0.125rad/$sec

The trajectory of reference cart is shown in Fig.2. k_1 is 10, k_2 is 8, k_3 is 20, and k_4 is 60. Maximum values for linear velocity and angular velocity, $v_{\max}$, $w_{\max}$ are set to $10m/$sec and $5rad/$sec respectively.

Resulting trajectory of a mobile robot is shown in Fig. 3. The error of x and y are also shown in Fig. 4. and Fig 5. As one can see from Fig. 6 and Fig. 7, the velocity of a robot approaches the velocity v_c.

5. Conclusion

Until now, most research on the mobile root problem has ignored dynamics. Even in the case dynamics is used, the exact knowledge was necessary - that is almost impossible. Thus, in this paper, by using robust adaptive control technique, a dynamic controller for a mobile robot that can track the desired velocity without exact knowledge about the model. The proposed method also can overcome the model uncertainties or external disturbance. Boundedness of the velocity error is proven based on the Lyapunov method.

7. References

[1] Y. Kanayama, Y. Kimura, T. Noguchi, "A Stable Tracking Control for an Autonomous Mobile Robot", Proc. IEEE Int. Conf. on Robotics and Automation, vol. 1, pp. 384-389, May 1990.

[2] C. Samson, "Time-varying Feedback Stabilization of Car-like Wheeled Mobile Robots," The Int. Journal of Robotics Research, vol. 12, no. 1, Feb 1993.

[3] R. Fierro and F. L. Lewis, "Control of A Nonholonomic Mobile Robot: Backstepping Kinematics into Dynamics," Proc. 34th Conf. on Decision and Control, New Orleans, LA, 1995, pp3805-3810.

[4] J. M. Yang, J. H. Kim, "Sliding Mode Motion Control

of Nonholonomic Mobile Robots," IEEE. Control. Syst. Mag, vol. 19, No. 2, pp. 15-23, April, 1999.
[5] C. Canudas de Wit, H. Khennouf, C. Samson and O. J. Sordalen, "Nonlinear Control Design for Mobile Robots," in Recent Trends in Mobile Robots, Y. F. Zheng ed., pp. 121-156, World Scientific, 1993.
[6] K. S. Narendra, A. M. Annaswamy, Stable Adaptive Systems, Reading, 1989, Prentice Hall
[7] J. H. Shin, J. J. Lee, "Robust Adaptive Control for Robot Manipulators using Regressor-based Form," Proc. IEEE. Int. Conf. SMC, San Antonio, TX, USA, 1994, pp2063-2068.

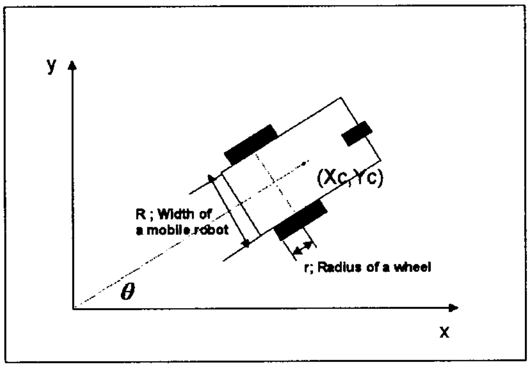

Figure 1. A mobile robot in 2D plane

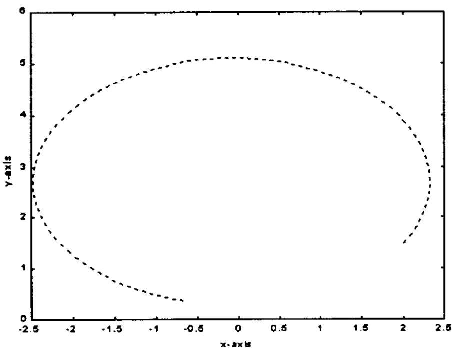

Figure 2. Trajectory of a reference cart

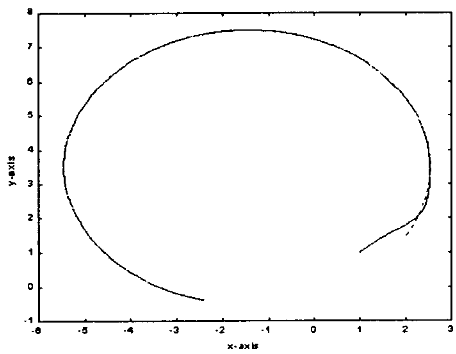

Figure 3. Resulting trajectory (dotted line is the reference trajectory)

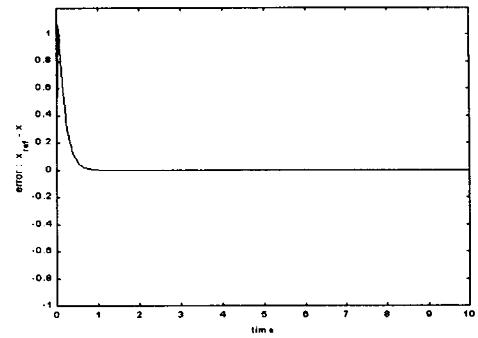

Figure 4. Error in x position value

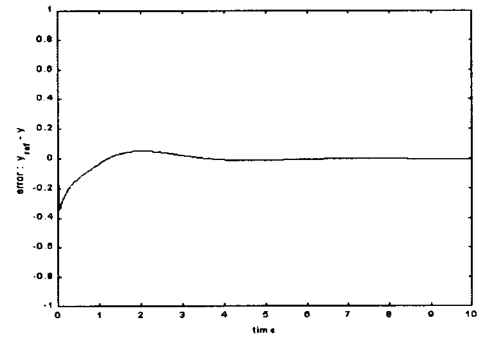

Figure 5. Error in y position value

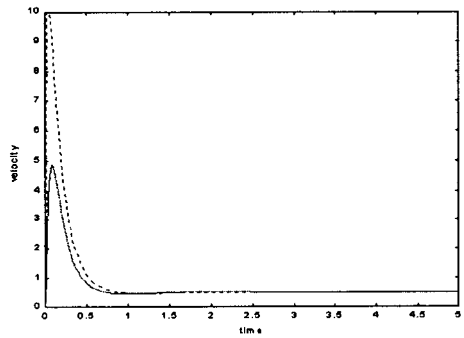

Figure 6. Linear velocity(dotted line is the desired linear velocity)

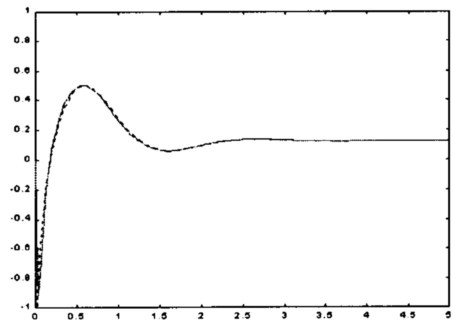

Figure 7. Angular velocity(dotted line is the desired angular velocity)