

Impact Reduction for Unknown Environment Using Kinematic Redundancy

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Abstract

In this article, a new performance index is proposed to reduce the collision impulsive force by controlling the null motion of redundant manipulators. First, we define the normalized impact ellipsoid in the viewpoint of instantaneous velocity change. Then, we propose a new impact performance index based on velocity direction for null motion to reduce initial impulsive effects. It gives some advantage for the case of unknown environment. The optimization of this index is that the successional impact forces are reduced. The performance of the proposed index is demonstrated by simulation study.

1 Introduction

Generally, a redundant manipulator has more degree of freedoms(DOF) than those required to execute a planned task. In that case, therefore, the manipulator has capability to perform additional tasks by the self-motion control, i.e., collision avoidance, manipulability[10] maximization, impact reduction[2, 4], etc.

There are two principal motions of operation for a manipulator: free space motion and contact/constrained motion. In many practical applications, the task requires the transition phase between the two motion, so the end-effector of manipulator cannot avoid physical contact with an object or an environment. In this situation, the transition of the manipulator from free space motion to constrained motion is crucially important because a severe collision may cause unrecoverable damages on manipulators and environment as well as make the system unstable. Zheng and Hemami[12] initially worked about the effects of impact on the manipulator in this transition phase. According to their works, there are two basic approaches for modeling and analysis of impact:

- The impact event is assumed to have some finite duration of time.(with soft environment)
- The impact event is modeled as an instantaneous phenomenon.(with stiff environment)

For the case of redundant manipulators, Walker[7] has established an instantaneous impulsive modeling for self-motion control using configuration change. Thereafter, many researchers have studied about impact control with this impulsive model[2, 4]. But those works have critical limitations, i.e., they must have at least the vector normal to the environment. However, in some cases, we do not know any information about environment.

In this article, we model the impact as an instantaneous phenomenon. Firstly, the normalized impact ellipsoid which means the directional capability of enduring about

the impact force will be introduced. Next, we propose a new impact performance index based on velocity direction to reduce the impulsive effect under unknown environment condition. Finally, simulations of a planar 3-DOF manipulator are performed to illustrate the effectiveness of the proposed index.

2 Basic modeling

Modeling of redundant manipulators

For general n -DOF serial manipulators, the kinematic relations are given as:

$$\dot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \quad \mathbf{J}(\mathbf{q}) = \frac{\partial \mathbf{k}(\mathbf{q})}{\partial \mathbf{q}} \quad (1)$$

$$\ddot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$$

where $\mathbf{k}(\cdot) : \mathfrak{R}^n \rightarrow \mathfrak{R}^m$ is the forward kinematics and $\mathbf{J} \in \mathfrak{R}^{m \times n}$ is the manipulator Jacobian matrix. With $m < n$, for kinematically redundant manipulators.

The equations of motions can be expressed by

$$\boldsymbol{\tau}_c = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) - \mathbf{J}(\mathbf{q})^T \mathbf{F}, \quad (2)$$

where $\boldsymbol{\tau}_c \in \mathfrak{R}^n$ is joint command torque vector; $\mathbf{M}(\mathbf{q}) \in \mathfrak{R}^{n \times n}$ is joint inertia matrix; $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathfrak{R}^{n \times n}$ is the Coriolis and centrifugal matrix; $\mathbf{g}(\mathbf{q}) \in \mathfrak{R}^n$ is gravitational torque vector; $\mathbf{F} \in \mathfrak{R}^m$ is the external force exerted by the environment.

Modeling of impulsive contact

We can derive the model of impact dynamics from the basic assumption, that is, impact occurs at time t and lasts for an infinitesimal period, δt , of time. Under this assumption, we integrate both sides of Eq. (2) over δt as follows

$$\int_t^{t+\delta t} \boldsymbol{\tau}_c dt = \int_t^{t+\delta t} [\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{J}(\mathbf{q})^T \mathbf{F}] dt. \quad (3)$$

Then, because all linear and angular velocities remain finite, and there are no changes in positions or orientations of any bodies in the system as $\delta t \rightarrow 0$, the integrals involving $\boldsymbol{\tau}_c$ and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ in Eq. (3) become zero. So we can rewrite Eq. (3) as the following equivalent equation.

$$\mathbf{J}^T(\mathbf{q}) \lim_{\delta t \rightarrow 0} \int_t^{t+\delta t} \mathbf{F} dt = \mathbf{M}(\mathbf{q}) \lim_{\delta t \rightarrow 0} \int_t^{t+\delta t} \ddot{\mathbf{q}} dt \quad (4)$$

$$\mathbf{M}^{-1} \mathbf{J}^T \mathbf{F}_{imp} = \lim_{\delta t \rightarrow 0} [\dot{\mathbf{q}}(t + \delta t) - \dot{\mathbf{q}}(t)] \quad (5)$$

$$\mathbf{J} \mathbf{M}^{-1} \mathbf{J}^T \mathbf{F}_{imp} = \mathbf{J} \Delta \dot{\mathbf{q}} = \Delta \dot{\mathbf{p}} \quad (6)$$

$$\mathbf{F}_{imp} = (\mathbf{J} \mathbf{M}^{-1} \mathbf{J}^T)^{-1} \Delta \dot{\mathbf{p}} = \mathbf{\Lambda}(\mathbf{q}) \Delta \dot{\mathbf{p}}, \quad (7)$$

where $\Lambda = (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)^{-1} \in \mathfrak{R}^{m \times m}$ is the pseudo-inertia matrix[3].

The discontinuity of velocity will be happened at the moment of impact with the environments of infinite or sufficiently large mass. The jump in velocity has to be specified through a so-called *restitution rule* which relates post- and pre-impact velocities. This rule is expressed as:

$$(\dot{\mathbf{p}} + \Delta\dot{\mathbf{p}})^T \mathbf{n} = -e\dot{\mathbf{p}}^T \mathbf{n}, \quad (8)$$

where \mathbf{n} is the unit vector normal to the plane of contact between the end-effector and the environment, and e is the constant coefficient of restitution denoting the type of collision taking place. For instance, elastic impacts correspond to $e = 1$, purely inelastic percussions correspond to $e = 0$, and in general $e \in [0, 1]$.

Finally, we note that the impulsive force functions only in the direction of normal to the contact plane. Therefore, we can write

$$\mathbf{F}_{imp} = F_{imp}\mathbf{n}. \quad (9)$$

From Eqs. (7), (8), and (9), we can derive the following equation.

$$F_{imp} = -\frac{[(1+e)\dot{\mathbf{p}}^T \mathbf{n}]}{\mathbf{n}^T \Lambda^{-1} \mathbf{n}}, \quad (10)$$

this equation defines the magnitude of the impulsive force in terms of three controllable parameters, *i.e.*, the manipulator's configuration(\mathbf{q}) at impact, the pre-impact velocity($\dot{\mathbf{p}}$) of end-effector, and the unit vector(\mathbf{n}) normal to the plane of collision impact.

3 Impact performance index

Many previous researchers used the denominator of Eq. (10) as the performance index to resolve null motion of redundant manipulators, impact reduction. If we maximize this index, the magnitude of impact force is minimized.

$$m(\mathbf{q}) = \mathbf{n}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T) \mathbf{n}. \quad (11)$$

For Eq.(11), there are two assumptions that the task velocity($\dot{\mathbf{p}}$) is already decided and the restitution constant(e) is given. Also, the unit vector(\mathbf{n}), which is normal to the contact plane, is supposed to be known. Therefore, the only controllable parameter is the configuration(\mathbf{q}) of manipulator at the moment of impact.

The null desired velocity, which is to optimize a scalar potential function $m(\mathbf{q})$, is assigned by gradient projection method type[9] as follows:

$$\dot{\mathbf{n}}_d = \kappa \nabla m(\mathbf{q}), \quad (12)$$

where κ denotes the rate of convergence factor.

Mostly, there exist coupling effects between the task motion dynamics and the null motion dynamics without using adequate pseudo inverse and decomposition. Some researches[5, 6] have done for the dynamic decoupling of task and null motion and shown good results. In our works, the systematic decomposition method is used, that is, the kinematically decomposed joint space decomposition(KD-JSD) method by Park[6].

Normalized impact ellipsoid

It's clear that, we can easily visualize the characteristics of manipulators kinematics and dynamics through the geometrical representation, *i.e.*, many kinds of ellipsoid structure[1, 8, 10, 11]. Among them, Walker proposed the generalized impact ellipsoid[8] and explained it from the viewpoint of the change of kinetic energy. The generalized ellipsoid in in \mathfrak{R}^m is described by

$$\mathbf{f}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T) \mathbf{f} \leq 1. \quad (\mathbf{f} \in \mathfrak{R}^m). \quad (13)$$

He explained, from Eq.(6), that the quantity $\mathbf{F}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T) \mathbf{F}$, for a given \mathbf{F} , is the change in kinetic energy $\Delta\dot{\mathbf{q}}^T \mathbf{M} \Delta\dot{\mathbf{q}}$ of the robot due to the collision. However, strictly it does not mean change of kinetic energy. Therefore, the generalized impact ellipsoid is not enough to explain physical meanings.

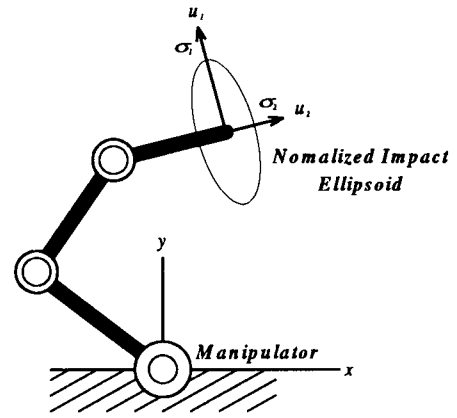


Fig. 1: Normalized impact ellipsoid

Now, consider the normalized impact ellipsoid in \mathfrak{R}^m is defined as:

$$\mathbf{n}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T) \mathbf{n} \leq 1. \quad (\mathbf{n} \in \mathfrak{R}^m), \quad (14)$$

where \mathbf{n} is the directional unit vector. This ellipsoid is almost the same as the generalized impact ellipsoid, but it gives some physical meanings. This normalized inertia ellipsoid means the instantaneous changes in the \mathbf{n} -directional task velocities by effect of the \mathbf{n} -directional unit impact force. From Eqs. (6) and (9), it follows that

$$\left| \frac{\mathbf{n}^T \Delta\dot{\mathbf{p}}}{F_{imp}} \right| = \mathbf{n}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T) \mathbf{n}. \quad (15)$$

As it was seen in Eq. (8), for the given environment and velocity, the change of the normal directional velocity is determined. Consequently, the large change of velocity means the small effect of impact.

We can construct the size and shape of the normalized impact ellipsoid via the same method as Walker[8], that is, we use the singular value decomposition(SVD) of $\mathbf{J}^{+T} \mathbf{M}^{1/2}$ for an appropriate pseudoinverse. The SVD of $\mathbf{J}^{+T} \mathbf{M}^{1/2}$ is given by

$$\mathbf{J}^{+T} \mathbf{M}^{1/2} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \quad (16)$$

where $\mathbf{\Sigma} = [\text{diag}\{\sigma_1, \dots, \sigma_m\} [\mathbf{0}_{m \times (n-m)}]]$.

4 A new impact performance index

Until now, the null motion of kinematically redundant manipulator has been resolved in impact situation by the impact performance index in Eq. (11). But, in those cases, we must know at least the normal vector to the environment. Therefore, if we don't have environment information of wall, we cannot use them. As we already knew, however, the impact force also relates to the task velocity. So, if we use this additional characteristic of impact, we can accomplish impact reduction, even the environment is unknown.

Impact performance index based on velocity direction

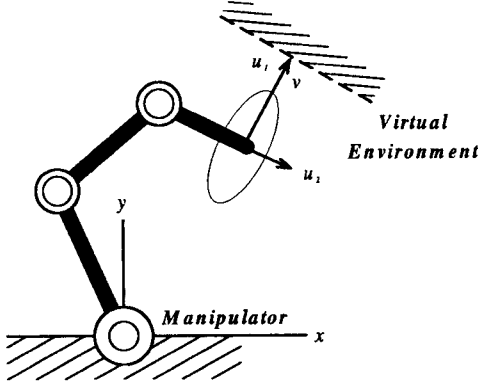


Fig. 2: The schematic diagram of impact performance index based on velocity direction

Now, replace the direction vector \mathbf{n} of Eq. (11) by the task velocity direction of manipulators with the assumption that we know the wall is lying in velocity direction like Fig. 2. Then, we define Eq. (17) as *the impact performance index based on velocity direction*.

$$m_v(\mathbf{q}) = \mathbf{v}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)\mathbf{v}, \quad (\dot{\mathbf{p}}_d - \dot{\mathbf{p}} = \mathbf{0}) \quad (17)$$

where $\mathbf{v} = \frac{\dot{\mathbf{p}}}{\|\dot{\mathbf{p}}\|}$ is the task velocity direction of manipulators. In the above index, the fact that \mathbf{v} is a function of \mathbf{q} and $\dot{\mathbf{q}}$ limits its usage for general case. However, in the perfect tracking case ($\dot{\mathbf{p}}_d - \dot{\mathbf{p}} = \mathbf{0}$), \mathbf{v} is well defined as function of \mathbf{q} for a given trajectory. So far, the performance of task motion control has been sufficiently well-done, so it is reasonable to assume those conditions. Under these assumption, Eq. (17) is a function of configuration(\mathbf{q}) only. The proposed impact performance index does not give a perfect effect and solution, but it is possible to give some intuition when we do not know the environment geometry. If the normal vector to the environment have much misalignment with the velocity direction, then the inertia effect increases, whereas the magnitude of velocity decreases. Therefore, the total effect of impact force is not severe.

Now, consider the implementation of the proposed index. At the starting point, the proposed index has a discontinuity because the velocity direction is not defined. To resolve this problem, let the initial direction as the direction from task start point to task end point. And, the task trajectory may has some stopping point, then we use the last velocity direction.

Table 1: Parameter of three-link planar redundant manipulator

Link No.	ℓ_i (m)	h_i (m)	m_i (kg)	I_i (kgm ²)
1	0.350	0.141	7.86	0.112
2	0.350	0.150	4.10	0.067
3	0.260	0.155	2.35	0.035

Change of the direction after the first impact

If, after the occurrence of first impact, the perfect tracking is not satisfied, we cannot apply the proposed impact performance index any more. In that case, however, we can get the geometry information of environment via a force-torque sensor. Therefore, it is possible to use this additional information for reducing the succession of impact forces and improving the force control performance. After the first impact, we change the velocity direction as the measured force direction which is the same as the vector normal to the environment surface(\mathbf{n}) as follows

$$m_v(\mathbf{q}) = \mathbf{n}^T (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)\mathbf{n}. \quad (18)$$

5 Simulation study

In this section, we illustrate the impact reduction property of the proposed impact performance index by presenting computer simulations of some examples. The simulated manipulator is a rigid three-link planar redundant manipulator without gravity. The dynamic and kinematic parameters are computed from the CAD drawing of POSTECH DDArm-II which is already constructed, and those are addressed in Table 1. The surface of environment is modeled as a spring with stiffness $K_s = 2 \times 10^5 N/m$ to represent a hard surface.

The sampling frequency is assumed as 1KHz and the integration step set to 5 times faster than sampling frequency to emulate the continuous system. The task control system is simulated with position and velocity feedback gain matrix $\mathbf{K}_p = \text{diag}\{150, 150\}$ and $\mathbf{K}_d = \text{diag}\{30, 30\}$, respectively. And the null velocity feedback gain constant is used as $K_n = 50$.

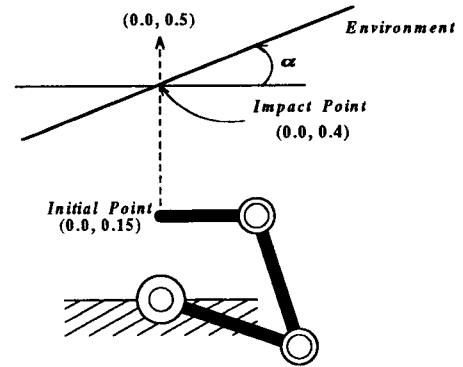


Fig. 3: Desired trajectory and initial configuration of Manipulator

As shown in Fig. 3, the simulation is executed by following line trajectory. The initial point is $p(0) = (0.0, 0.15)m$. And the initial configuration represents the maximum value of performance index. For the proposed index, $q(0) = [-0.575139, 2.242696, 1.504732]^T rad$ mean the maximizing pose about the given task direction. The rate of convergence factor, κ , is set to 30.

The wall is modeled by $x - y + 0.4 = 0$, so the vector normal to the wall is $n = [-\sqrt{2}, -\sqrt{2}]^T$. The execution time is 1.5s. We executed for several α , i.e., $0, \frac{\pi}{6}, \frac{\pi}{4}$, and $\frac{\pi}{3}$. For all the cases, the pre-impact velocities are about $0.4m/s$.

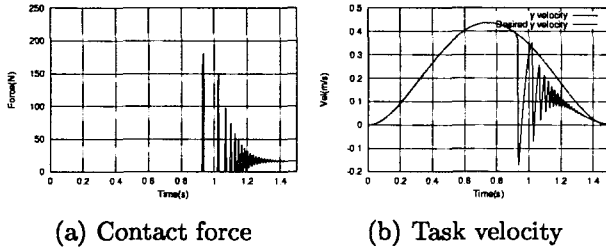


Fig. 4: Simulation result using impact performance index with velocity direction for $\alpha = \frac{\pi}{4}$

As shown in Fig. 4(b), by our assumption, the perfect tracking is almost satisfied before the first impact.

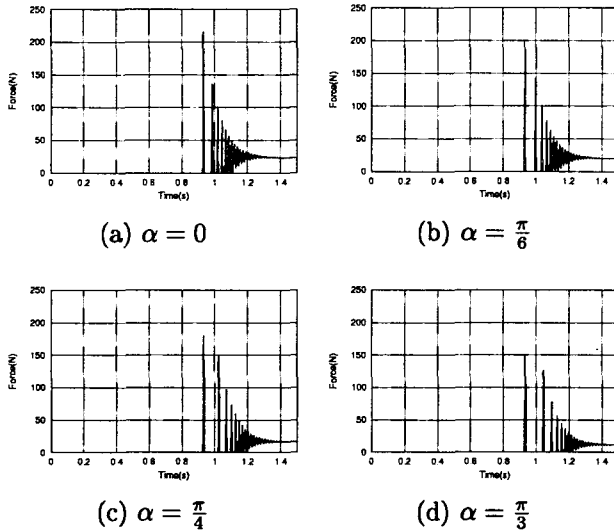


Fig. 5: Contact forces for various α in simulation results

Also, the results of the proposed index is very nice as shown in Fig. 5. As the misalignment between the velocity direction and the normal vector to the environment increases, the first impact force decreases as expected. This property is not always satisfied, because it depends on the task inertia distribution, that is, when the manipulator goes close to the geometric singular points, the task inertia is extremely sensitive to its direction. In these cases, however, another serious problem happens as we already knew. Currently, we are investigating about this property.

6 Concluding remarks

In this article, the null motion control of kinematically redundant manipulator to reduce collision impulsive effects has been considered. First, the normalized impact ellipsoid, which means the directional capability of enduring about the impact force, has been introduced and compared with the generalized impact ellipsoid by Walker.

For unknown environment case, a new impact performance index based on velocity direction was proposed. It needs some assumptions, but those are generally acceptable as shown in simulations. And the change of index using a force-torque sensor has some advantage to reduce a continual forces. The effectiveness of the proposed index was verified via simulation study.

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