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Sliding Mode Tracking Control of a Nonminimum Phase EGR/VGT Diesel Engine

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Abstract

Tracking control of an arbitrary reference has been discussed for 7Th order 2-input 2-output non-minimum phase EGR/VGT diesel engines. To meet strict emission regulations and customer demands, the desired set points, the air-fuel ratio and the ERG flow fraction, determined from a static engine data based on engine speed and the desired fueling rate are transformed into the set points for the two sensor measurement outputs. Applying the sliding mode tracking control theory proposed by Jeong and Utkin, two step design was carried out using the bounded solution of an unstable zero dynamics for the given reference signals. This paper shows through simulations how stabilization of unstable zero dynamics and reference tracking can be accomplished simultaneously.

1. Introduction

For production of passenger car internal combustion engines to meet strict emission regulations and customer demands of improved fuel economy and drive-ability, advanced hardware components are increasingly being considered. However, the basic requirements in terms of peak power, transient response, fuel economy and emissions are often contradictory and require judicious tradeoffs at every stage of the design process. Diesel (compression ignition) engines have a significant advantage over gasoline spark ignition (SI) engines in fuel economy. An effective way to reduce the formation of NO_x during combustion is to recirculate the exhaust gas through the exhaust gas recirculation (EGR) valve into the intake manifold. The exhaust gas acts as a diluents and lowers peak combustion temperature restricting the formation of NO_x. The fraction of EGR recirculated flow must be scheduled on the operating condition (Berry and Burnt 1996). To improve its relative low power density, a diesel engine can be equipped with a turbo-charger which consist of a turbine and a compressor attached to the common shaft. A conventional turbo-charger faces a tradeoff between fast transient response at low engine speeds and high power without over-speeding at high engine speeds. This tradeoff can be adjusted by employing a variable geometry turbo-charger (VGT) (Moody 1986, Watson and Banisoleiman 1988) in order to prevent the back-pressure and improve low fuel economy suffered at low-load, high-speed conditions. For diesel engines equipped with EGR and VGT actuators, Utkin et al. (1998) designed stabilizing controller for this system based on block control principle.

The remainder of this paper is organized as follows; Section 2 describes the typical properties of EGR/VGT diesel engines. Section 3 gives brief review of sliding mode tracking controller for non-minimum phase systems and its relevant bounded solution of unstable systems. In section 4, the tracking control theory is applied for diesel engine to trace the desired output signals. Simulation results appear in section 5 and concluding remarks are given in section 6.

2. Plant Properties

The schematic diagram of EGR/VGT diesel engine is shown in figure 1. Kolmanovsky et al. (1998) has described a nonlinear model for this system. The dynamical model is developed using the conservation of energy and mass and the ideal gas law together with several assumptions and experimentally derived maps. Its overall response including engine speed (N) are mainly governed by fueling rate (W_f).

According to Kolmanovsky et al. (1998), for each quadruple (N, W_f, x_{EGR}, x_{VGT}) there exists a unique equilibrium of the diesel engine model which is asymptotically stable. Typically, diesel engines are equipped with sensors for intake manifold pressure (p_1) and compressor mass flow rate (W_{c1}). Considering these two variables as outputs, the MIMO system with inputs (x_{EGR}, x_{VGT}) was also shown to be non-minimum phase system which has a single right-half plane zero at various operating points (N, W_f, x_{EGR}, x_{VGT}). Our design objective is to regulate the air-fuel ratio and the EGR flow fraction to the set points determined from a static engine data based on engine speed and the desired fueling rate (W_f^d).

$$AFR_{ref} = AFR_{ref}(N, W_f^d),\quad EGR_{ref} = EGR_{ref}(N, W_f^d)$$

Since diesel engines are equipped with sensors for p_1 and W_{c1}, the set points for AFR and EGR may be transformed into the set points for the intake manifold pressure (p_1^d) and the compressor mass flow rate (W_{c1}^d) from the static engine map. We restrict our attention to the linearization of the model at a medium engine speed operating point (N=2000 rpm and W_f=6 kg/hr).

$$\dot{x} = A_{7\times7}\begin{bmatrix} m_1 \\ F_1 \\ p_1 \\ m_2 \\ F_2 \\ p_2 \\ N_{tc} \end{bmatrix} + B_{7\times2}\begin{bmatrix} \chi_{EGR} \\ \chi_{VGT} \end{bmatrix} = Ax + Bu \quad (1)$$

$$y = \begin{bmatrix} p_1 \\ W_{c1} \end{bmatrix} = Cx$$

Thus all state variables as well as control inputs are deviations from the corresponding nominal values. Two system outputs have both relative degree 1 and the zero dynamics with respect to the outputs has one unstable mode i.e. non-minimum phase system.

3. Sliding Mode Tracking Control Theory

Recently Jeong and Utkin (1998) developed a sliding mode tracking control methodology for an arbitrary reference signal of multivariable systems with unstable zero dynamics. The proposed approach does not require an exosystem and is applicable to any relative degree systems. It is based on the block control principle utilizing some of states as fictitious control and has the two step design procedure as the sliding mode theory. In the following, a brief review of sliding mode tracking methodology is presented.

3.1 Outline of Methodology

Consider the n-th order nonlinear systems with the same number m of inputs and outputs.

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})u, \quad \mathbf{x} \in R^n, u \in R^m \qquad (2)$$
$$y = h(\mathbf{x}), \quad y \in R^m$$

where $\mathrm{rank}(g(\mathbf{x}))=m \le n$, $u=\mathrm{col}(u_1,\dots,u_m)$, $y=\mathrm{col}(y_1,\dots,y_m)=\mathrm{col}(h_1(\mathbf{x}),\dots,h_m(\mathbf{x}))$, $g(\mathbf{x})=[g_1(\mathbf{x}),\dots,g_m(\mathbf{x})]$ in which $f(\mathbf{x})$, $g_1(\mathbf{x}),\dots,g_m(\mathbf{x})$ are smooth vector fields and $h_1(\mathbf{x}),\dots,h_m(\mathbf{x})$ are smooth functions defined on an open set of $\mathbf{R}^n$.

Let us first briefly review the sliding mode theory and block control principle utilizing some of the components of the state vector as a fictitious control (Luk'yanov 1993, Luk'yanov and Dodds 1996). Suppose the above system can be decomposed into two subsystems of the following form, so-called *regular form* through a nonlinear transformation

$$\dot{z} = q(z, x), \quad z \in R^{n-m} \qquad (3a)$$

$$\dot{x} = b(z, x) + a(z, x)u, \quad x \in R^m \qquad (3b)$$

where $a(z,x)$ has rank m and the upper equation (3a) does not depend on the control input u. Following the regular form design approach, one assumes that u is a discontinuous control enforcing sliding mode in the manifold $s(\mathbf{x})=0$ with m selected switching surfaces denoted by the vector $s(\mathbf{x})=[s_1(\mathbf{x}),\dots,s_m(\mathbf{x})]^T$. Generally speaking, after sliding mode occurs on $s(\mathbf{x})=0$, m components of the state vector may be found as functions of the remaining $(n-m)$ ones, $x=s_o(z)$. As a result, the sliding mode equation on the manifold $s(\mathbf{x})=x-s_o(z)=0$ is

$$\dot{z} = q(z, s_o(z)) \qquad (4)$$

The sliding dynamics (4) are determined only by the state z. The desired behavior may be achieved by a suitable choice of the function $x=s_o(z)$, with state x regarded as a fictitious control for the upper subsystem (3a). To confine the state trajectory to the selected manifold $s(\mathbf{x})=0$, the discontinuous control

$$u = -M\, sign(s(\mathbf{x})) \qquad (5)$$

may be employed with gain matrix M satisfying the existence condition of sliding mode (Utkin 1992). Then, the state is steered to the manifold $s(\mathbf{x})=0$ within finite time. Thus, the design reduces to an $(n-m)$ dimensional problem of a proper selection of the function $x=s_o(z)$ and the design of a control u enforcing the sliding mode in the manifold $s(\mathbf{x})=0$.

If the state x is reduced to zero by some control u, then the equation (3a) on the surface $x=0$, $\dot{z} = q(z,0)$ are referred to *zero dynamics* with respect to the state x. When the zero dynamics are unstable, the system is called *non-minimum phase*.

Now let us turn to the tracking problem. The block control principle (Drakunov *et al.* 1990) introduced in sliding mode approach greatly simplifies the stabilization problem especially when it is transformed into a regular form. This principle may be applied for tracking of minimum phase systems but it can not be directly applied to non-minimum phase systems since a tracking problem implies stabilization of unstable zero dynamics as well as signal following.

Let us assume that $x^o \in \mathrm{R}^m$ is the desired trajectory. Suppose the bounded solution $z^o(t)$ of unstable zero dynamics with respect to x is available on the trajectory $x=x^o$. Then, tracking can be accomplished within finite time without an error as follows. Define the deviation $\triangle z=z-z^o$ and $\triangle x=x-x^o$, where $\triangle x$ means the tracking error. *Mismatch dynamics* may be obtained from (2)

$$\dot{\triangle z} = [q(z^o + \triangle z, x^o + \triangle x) - q(z^o, x^o)] \qquad (6a)$$

$$\dot{\triangle x} = b(z,x) + a(z,x)u - \dot{x}^o \qquad (6b)$$

Now it can be seen from (6) that the tracking problem making $x \to x^o$ yields the stabilization problem $\triangle z \to 0$ and $\triangle x \to 0$.

Applying the sliding mode technique described above, the stabilization of the upper mismatch dynamics (6a) can be achieved by the fictitious control $\triangle x = f_s(\cdot)$. Accordingly, define the sliding surface

$$s = \triangle x - f_s(\triangle z, z^o, x^o) \qquad (7)$$

with $f_s(0, z^o, x^o)=0$. The real control enforcing sliding mode along the sliding surface $s=0$ may be selected as discontinuous function of type (5).

Note that, when the mismatch dynamics are stabilized, the term $[\cdot]$ in (6a) vanishes. Thus, the mismatch $\triangle z \to 0$ and no steady state error on the surface $s=0$ can be expected from (7) with $f_s(0, z^o, x^o)=0$. Since the bounded solution $z^o(t)$ of (3a) is available, by assumption, on the trajectory $x=x^o$, an arbitrary smooth signal can be tracked even though the upper (zero) dynamics are unstable. This is a natural extension of the concept of the set point neutralization.

3.2 Bounded Solution of Unstable Linear Systems

Let us consider the linear system

$$\dot{z}(t) = Az(t) + Bu(t) \quad (8)$$

where matrix A has no eigenvalues on imaginary axis. When A is stable, the solution satisfying the initial condition is in the following form

$$z(t) = e^{At}z(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \quad (9)$$

which is bounded if input u is bounded. If all eigenvalues of A have positive real part, the bounded solution satisfying the following boundary condition can also be obtained

$$z(T) = 0, \quad t \leq T \leq \infty \quad (10)$$

Theorem 1: The bounded solution of system (8) all modes of which are unstable, satisfying boundary condition (10) with T=constant is

$$z^o(t) = -\int_t^T e^{A(t-\tau)}Bu(\tau)\,d\tau \quad (11)$$

Proof: Differentiating (9) shows that it is indeed the solution of the unstable system (8).

$$\dot{z}^o(t) = -Ae^{At}\int_t^T e^{-A\tau}Bu(\tau)d\tau - e^{At}[0 - e^{-At}Bu(t)]$$
$$= Az^o(t) + Bu(t)$$

The boundedness can be seen from the facts that A is unstable and $(t-\tau)<0$. ◇

Generally the system matrix A has both stable and unstable modes. Without loss of generality, one can transform the system (8) into a block diagonal modal form via a suitable similarity transformation $z_m = Wz$

$$\dot{z}_m = A_m z_m + B_m u = \begin{pmatrix} A_s & 0 \\ 0 & A_u \end{pmatrix}\begin{bmatrix} z_s \\ z_u \end{bmatrix} + \begin{pmatrix} B_s \\ B_u \end{pmatrix} u \quad (12)$$

where z_s (z_u) is the state corresponding to the stable (unstable) mode, i.e, the matrix A_s (A_u) has only open left (right) half plane eigenvalues, respectively.

When the zero dynamics (3a) are unstable, the block control principle is not applicable in the case of output tracking. But if non-causality is allowed, i.e. if the future signal of the input x^o is available, one can obtain the corresponding bounded solution z^o of linear zero dynamics due to theorem 1. Tracking problem is one of the case because x^o is the desired trajectory we want to trace.

4. Tracking Controller Design

According to the design procedure, system (1) is transformed into the following form

$$\begin{aligned}
\dot{Z}_s &= A_s Z_s && + B_{s1}y_1 + B_{s2}y_2 \\
\dot{z}_u &= A_u z_u && + B_{u1}y_1 + B_{u2}y_2 \\
\dot{y}_1 &= A_{11}Z_s + A_{12}z_u && + A_{13}y_1 + A_{14}y_2 + B_{11}u_1 \\
\dot{y}_2 &= A_{21}Z_s + A_{22}z_u && + A_{23}y_1 + A_{24}y_2 + B_{21}u_1 + B_{22}u_2
\end{aligned} \quad (13)$$

where Z_s represents a stable mode vector and z_u is an unstable mode. It is obtained in the following steps. First, state p_1 is moved to the sixth position and then states p_1 and N_{tc} is replaced with system outputs p_1 and W_{cl} since both of the relative degree of system outputs are 1. Second, it is transformed into regular form using a nonsingular matrix

$$T = \begin{pmatrix} I_5 & -B_1B_2^{-1} \\ 0 & I_2 \end{pmatrix} \quad (14)$$

where B_2 is the last two rows and B_1 is the remaining matrix of B. Third, the zero dynamics are decomposed into stable / unstable modal form. Finally, unstable mode of zero dynamics

$$\dot{z}_u = A_u z_u + B_{u1}y_1 + B_{u2}y_2 \quad (15)$$

is moved to the lowest position of zero dynamics. Our reference signals, y_1^d and y_2^d, are chosen as in figure 2. To simplify the design, the bounded solution of only unstable mode is numerically calculated via convolution integral. It may be approximately obtained in advance for the given desired trajectory as

$$z_u^o(t) = -\int_t^{t+T} e^{A_u(t-\tau)}[B_{u1}y_1^d(\tau) + B_{u2}y_2^d(\tau)]\,d\tau \quad (16)$$

Where T may be called *non-causal actuation time* which decides the accuracy of the approximate solution. The corresponding mismatch dynamics are

$$\begin{aligned}
\dot{Z}_s &= A_s Z_s && && + B_{s1}y_1 + B_{s2}y_2 \\
\Delta\dot{z}_u &= && A_u\Delta z_u && + B_{u1}\Delta y_1 + B_{u2}\Delta y_2 \\
\Delta\dot{y}_1 &= A_{11}Z_s && + A_{12}\Delta z_u && + A_{13}\Delta y_1 + A_{14}\Delta y_2 + B_{11}u_1 \\
& && + (A_{12}z_u^o && + A_{13}y_1^d + A_{14}y_2^d - \dot{y}_1^d) \\
\Delta\dot{y}_2 &= A_{21}Z_s && + A_{22}\Delta z_u && + A_{23}\Delta y_1 + A_{24}\Delta y_2 + B_{21}u_1 + B_{22}u_2 \\
& && + (A_{22}z_u^o && + A_{23}y_1^d + A_{24}y_2^d - \dot{y}_2^d)
\end{aligned} \quad (17)$$

where mismatch variables are $\Delta z_u = z_u - z_u^o$, $\Delta y_1 = y_1 - y_1^d$ and $\Delta y_2 = y_2 - y_2^d$. Utilizing Δy_1 as a fictitious control for the unstable Δz_u zero dynamics, taking the sliding surfaces as

$$\begin{aligned} s_1 &= \Delta y_1 + k_1\Delta z_u \\ s_2 &= \Delta y_2 \end{aligned} \quad (18)$$

and control enforcing the surfaces s_1 and s_2 to zero may stabilize the mismatch dynamics (17).

Note that stable Z_s dynamics are left as free motions and Z_s in Δy_1 and Δy_2 dynamics may be regarded as a kind of disturbance, which can be rejected because it is located in the input channel, i.e, it satisfies the so-called matching conditions.

5. Simulation Results

The bounded solution z_u^o is shown in figure 3. From the figures 2 and 3, it may be expected that noncausal actuation about 0.15 sec is necessary to track the desired signal accurately, i.e. control action must applied before the output needs to move. Considering practical application to alleviate the chattering of mechanical actuators, instead of sign function in (4) a sigmoid function, *tanh(as)*, will be used in simulation study, which is a kind of continuous approximation of discontinuous function. The benefit of this function is that it may smoothen the chattering and the magnitude of its derivative can be limited by the parameter a.

Control gains used for each control input are M_1=-0.8 and M_2=-0.8 and the sliding surface gain in (18) is

k_1=-0.2. Simulation results of non-causal actuation, the time of which was 1 sec to enhance visibility, are shown in figure 3.
Non-causal actuation result, Fig. 3 show that outputs keep tracking arbitrary reference signals exactly, they are almost indistinguishable in the figure. And unstable mode z_u is enforced to follow the bounded trajectory z_u^o.

6. Concluding Remarks

Tracking control of an arbitrary reference has been discussed for EGR/VGT diesel engines. The desired set points, the air-fuel ratio and the EGR flow fraction, are transformed into the set points for the intake manifold pressure and the compressor mass flow rate. Sliding mode tracking control theory proposed by Jeong and Utkin was applied to MIMO non-minimum phase diesel engine. Two step design was carried out using the bounded solution of an unstable zero dynamics and stabilizing the corresponding mismatch dynamics for the given reference signals. The results of non-causal actuation about 0.15 sec show that outputs keep tracking arbitrary reference signals almost perfectly. Unstable mode information necessary for control algorithm may be estimated. Since original systems is observable, siding mode observer may be designed for the estimation of this mode.

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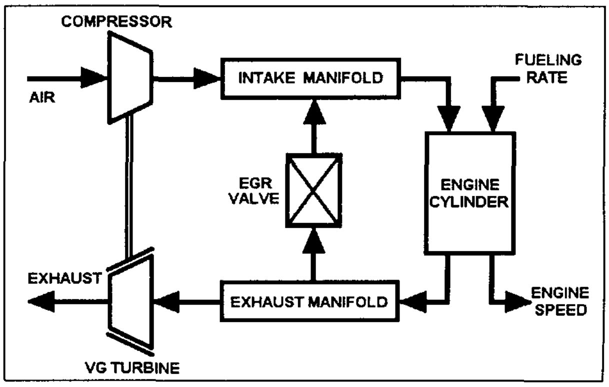

Fig. 1 Schematics of EGR/VGT diesel engine

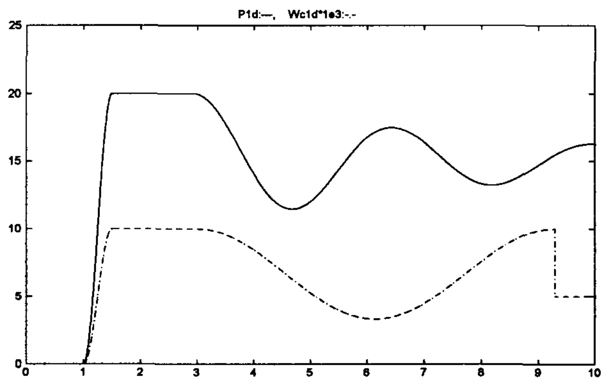

Fig. 2 Reference signals

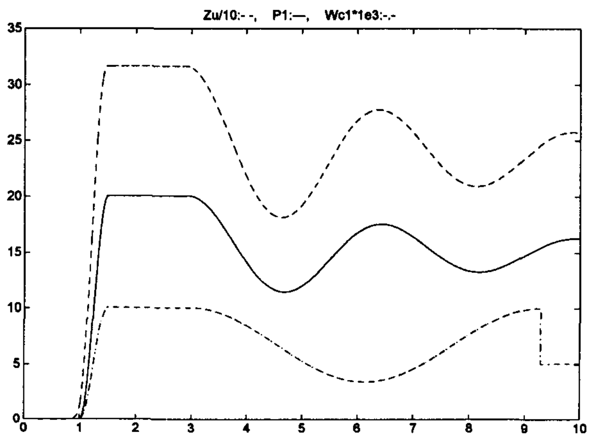

Fig. 3 Outputs of non-causal actuation