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Identification of Volterra Kernels of Nonlinear System Having Backlash Type Nonlinearity

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Abstract

The authors have recently developed a new method for identification of Volterra kernels of nonlinear systems by use of pseudorandom M-sequence and correlation technique. And it is shown that nonlinear systems which can be expressed by Volterra series expansion are well identified by use of this method.

However, there exist many nonlinear systems which can not be expressed by Volterra series mathematically. A nonlinear system having backlash type nonliear element is one of those systems , since backlash type nonlinear element has multi-valued function between its input and output.

Since Volterra kernel expression of nonlinear system is one of the most useful representations of nonlinear dynamical systems, it is of interest how the method of Volterra kernel identification can be applied to such backlash type nonlinear system.

The authors have investigated the effect of application of Volterra kernel identification to those nonlinear systems which, accurately speaking, is difficult to express by use of Volterra kernel expression. A pseudorandom M-sequence is applied to a nonlinear backlash-type system, and the crosscorrelation function is measured and Volterra kernels are obtained.

The comparison of actual output and the estimated output by use of measured Volterra kernels show that we can still use Volterra kernel representation for those backlash-type nonlinear systerms.

1. Introduction

The nonlinear systems with backlash element which has multi-valued function between the input and output are generally thought difficult to indentify. There are many researchers who have tried to indentify this kind of system by various methods.

Simpson and Power[3] have shown that a mehod of identification developed for a class of system containing a zero-memory nonlinearity is applicable to certain types of nonlinear elements with memory.

The extension of the Weierstrass theorem to the Volterra functionals was made in 1910 by Maurice Fréchet who showed that any continuous functional can be represented by a series of functionals of integer order whose convergence is uniform in all compact sets of continuous functions.

Therefore, Volterra kernel expansion method can only be used for those nonlinear systems which are continuous for input-output relation, and single valued, theoretically. The nonlinear systems having backlash type element are multi-valued as far as input-output relationship is concerned, so these nonlinear systems are not suited to Volterra kernel representation.

However, Volterra kernel expression of nonlinear system is one of the most useful method for representing nonlinear systems, we would like to know what would be the result if we apply Volterra kernel identification method to nonlinear system having backlash element.

In this paper, we apply the method of the application of Volterra series with M-sequences to the identification of a nonlinear system consisting of backlash characteristics. The system shown in Fig.1, second-order system plus backlash nonlinearity is investigated.

In this method, M-sequences are applied to the nonlinear system and the crosscorrelation function between the inputs and the outputs are measured. From the crosscorrelation function ,we can get not only the linear impulse response of the linear part of the system, but also cross-sections of high-order Volterra kernel up to 3rd order of nonlinear system.[6]

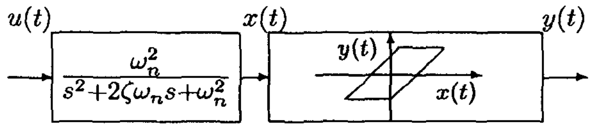

Fig.1 : Nonlinear system consisting of backlash nonlinearity

2. Identification of Volterra Kernels

One of the solutions of the identification ploblem of a nonliear system is based on the measurement of Volterra kernels. Consider the identification of a nonlinear system which can be described as follows,

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \cdots \tau_i)$$
$$\times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2 \cdots d\tau_i \quad (1)$$

where $u(t)$ is the input, and $y(t)$ is the output of the nonlinear system, and $g_i(\tau_1, \tau_2, ...)$ is called Volterra kernel of i-th order. When $i = 1$, Eqn.(1) shows linear systerm.

In order to get the Volterra kernels $g_i(\tau_1, \tau_2, ...)$, we use an M-sequance as an input to the nonlinear system. The crosscorrelation function $\phi_{uy}(\tau)$ between the input $u(t)$ and the output $y(t)$ can be written as,

$$\phi_{uy}(\tau) = \overline{u(t-\tau)y(t)}$$
$$= \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \cdots \tau_i)$$
$$\times \overline{u(t-\tau)u(t-\tau_1)\cdots u(t-\tau_i)} d\tau_1 d\tau_2 \cdots d\tau_i \quad (2)$$

where $\overline{}$ denotes time average.Usually the n-th moment of $u(t)$ is difficult to obtain. But when we use M-sequence, we can get n-th moment of $u(t)$ easily .Namely, the $(i+1)$th moment of the input M-sequence $u(t)$ can be written as

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} = \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)} \end{cases} \quad (3)$$

where N is the period of the M-sequence. When we use the M-sequence with the degree greater than 16, $1/N$ is in the order of 10^{-5}. So Eqn.(3) can be approximated as a set of impulses which appear at certain τ's.

Let us consider the case where we measure i-th Volterra kernel. Then for any integer $k_{i1}^{(j)}, k_{i2}^{(j)}, \cdots k_{i,i-1}^{(j)}$ (suppose $k_{i1}^{(j)} < k_{i2}^{(j)} < \cdots, k_{ii-1}^{(j)}$), there exists a unique $k_{ii}^{(j)} (mod\ N)$ such that

$$u(t)u(t+k_{i1}^{(j)})u(t+k_{i2}^{(j)})\cdots u(t+k_{i,i-1}^{(j)}) = u(t+k_{ii}^{(j)}) \quad (4)$$

where j is the number of a group $(k_{i1}, k_{i2}, \cdots, k_{i,i-1})$ for which Eqn.(4) holds. We assume that total number of those groups is m_i(that is, $j = 1, 2, \cdots, m_i$). Therefore Eqn.(3) becomes unity when

$$\tau_1 = \tau - k_{i1}^{(j)}, \tau_2 = \tau - k_{i2}^{(j)}, \cdots \tau_i = \tau - k_{ii}^{(j)} \quad (5)$$

Therefore Eqn.(2) becomes

$$\phi_{uy}(\tau) \simeq \sum_{i=1}^{\infty}\sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)}) \quad (6)$$

Since $g_i(\tau_1, \tau_2, \cdots \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)})$ in Eqn.(6) appear in the crosscorrelation function $\phi_{uy}(\tau)$ when $\tau > k_{ii}^{(j)}$.

In order to obtain the Volterra kernels from Eqn.(6) $k_{ii}^{(j)}$ in Eqn.(6) are sufficiently apart from each other. For this to be realized, we have to select suitable M-sequences, which make the appearance of each cross-section of Volterra kernels sufficiently apart each other. Some of those usable M-sequences are shown in reference (7).

When we measure Volterra kernels up to 3rd order, the crosscorrelation function $\phi_{uy}(\tau)$ becomes,

$$\phi_{uy}(\tau) = \Delta t g_1(\tau) + F(\tau)$$
$$+2(\Delta t)^2 \sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}, \tau - k_{22}^{(j)})$$
$$+6(\Delta t)^3 \sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}, \tau - k_{32}^{(j)}, \tau - k_{33}^{(j)}) \quad (7)$$

where

$$F(\tau) = (\Delta t)^3 g_3(\tau, \tau, \tau) + 3(\Delta t)^3 \sum_{q=1}^{m_1} g_3(\tau, q, q) \quad (8)$$

In general case, we have,

$$\phi_{uy}(\tau) = \Delta t g_1(\tau) + F(\tau)$$
$$+ \sum_{i=2}^{\infty} i!(\Delta t)^i \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)}) \quad (9)$$

Here the function $F(\tau)$ is the function of τ and sum of the odd order Volterra kernels when some of its argument are equal. Since $F(\tau)$ appears together with $g_1(\tau)$ in a overlapped manner, $F(\tau)$ must be calculated from the odd order Volterra kernels and be subtracted from the measured $g_1(\tau)$ in order to obtain the accurate $g_1(\tau)$.

3. Identification of a System Having Backlash Type Nonlinearity

Backlash type nonlinear element exists in a mechanical transmission systems. An interval of transmission is a kind of nonlinear backlash characteristic. When the main tooth wheeler has made a change of direction, the following wheeler won't change until the interval has been used up. In the iron magnetic material, the magnetic material hysteresis is also a kind of nonlinear backlash charactericitic.

We use here Volterra kernel identification method to identify the system having backlash type nonlinearity. The system used in the simulaton is shown in Fig.1. As is well known, when the input to a backlash element is sinusoidal signal,

$$x(t) = Xsin\omega t \qquad (10)$$

the output of backlash element will be as follows:

$$y(t) = \begin{cases} k_0(Xsin\omega t - a) & 0 < \omega t < \frac{\pi}{2} \\ k_0(X - a) & \frac{\pi}{2} < \omega t < (x - \beta) \\ k_0(Xsin\omega t + a) & (x - \beta) < \omega t < \pi \end{cases} \qquad (11)$$

where $\beta = arcsin(1 - \frac{2a}{X})$, ρ is the slope of the inclined lines of the backlash charactoristic curve and $k_0 = tg\rho$.

The parameters of the simulated system are as follows:

$\zeta = 0.4$
$\omega_n = 1.0$
$| 2a |= 0.8$: backlash width
$\rho = \pi/2$: $dt = 0.5s$: sampling time

4. Results of Simulation

As the input u(t), 12 degree M-sequence with the characteristic polynomial ($f(x) = 15341$ in octal motation) is used. The first, second and third order Volterra kernels of the system are measured by calculating the crosscorrelation between the input and the output.

Fig.2 shows the first order Volterra kernel $g_1(\tau)$thus measured. As you see here, the response is not so smooth as the response of single-valued nonlinear system, because of the multi-valued characteristic of the backlash.

Fig.3 shows the second order Volterra kernal $g_2(\tau_1, \tau_2)$, which are measured by M-sequence correlation method.

Fig.4 shows one of the crosssection of the third Volterra kernel $g_3(\tau_1, \tau_2, \tau_3)$, when $\tau_3 = 1\Delta t$.

Fig.5 shows one of the crosssection of the third Volterra kernel $g_3(\tau_1, \tau_2, \tau_3)$, when $\tau_3 = 2\Delta t$.

Fig.6 shows the simulation result, where the solid line is the output calculated by use of up to second order Volterra kernels, the big dotted line is the output calculated by use of 1st order Volterra kernel and the small dotted line is the actual output. Here you see that the calculated output by use of 1st or up to 2nd order Volterra kernels are not enough for estimating the output.

Fig.7 shows the result of comparison of the actual output (dotted line) and the output (solid line) calculated by use of up to third order Volterra kernels, showing relatively good agreement between them. From these results of simulation we can say the use of Volterra kernel indentification method is effective even for backlash type nonlinear system.

5. Conclusion

In this paper, the authors investigated the effect of application of M-sequence correlation method of Volterra kernel identification to backlash type nonlinear system. Here M-sequences are applied to a nonlinear system and the crosscorrelation function between the input and the output are measured. From the crosscorrelation function, we obtain not only the linear impulse response, but also cross-sections of 3rd order Volterra kernels of backlash element nonlinear system. From comparison of the simulation results , we can still use the method of Volterra kernels for indentification of backlash type nonlinear system.

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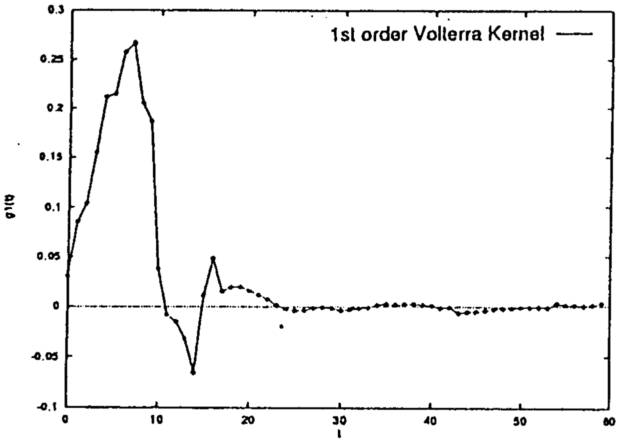

Fig.2:Volterra kernel of first order

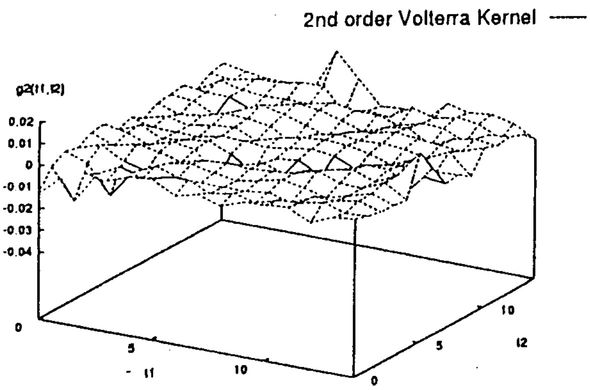

Fig.3:Volterra kernel of second order

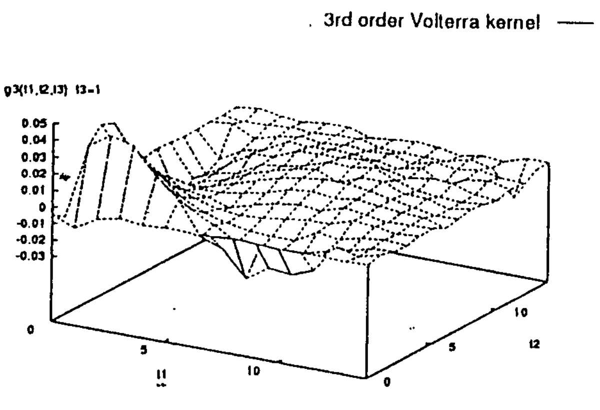

Fig.4:Volterra kernel of third order for $\tau_3 = 1\Delta t$

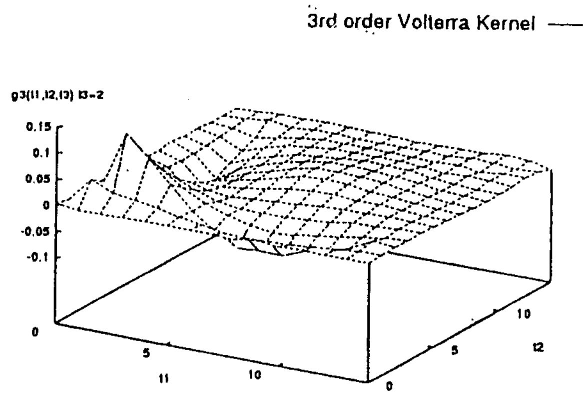

Fig.5:Volterra kernel of third order $\tau_3 = 2\Delta t$

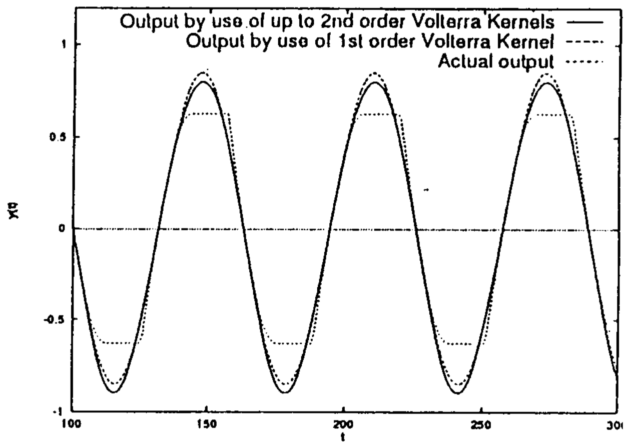

Fig.6 : Comparison of actual output with estimated ones by use of only first order and up to second order Volterra kernels

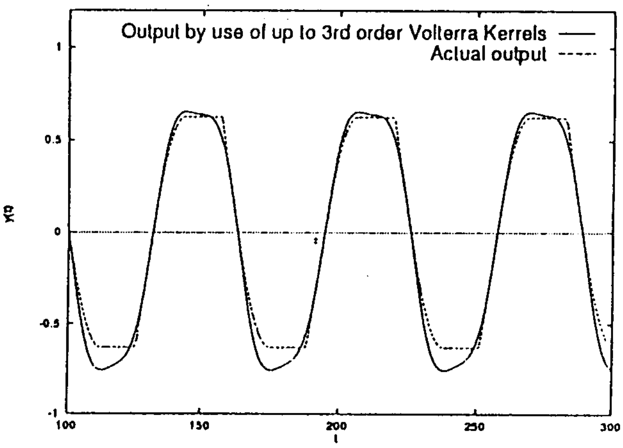

Fig.7 : Comparison of actual output with estimated one by use of up to third order Volterra kernels