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Position Control of AS/RS Stacker Crane By Using Gain-Scheduled Control Method in Automated High Rack Warehouse System

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Abstract

An automated storage and retrieval machinery for high rack warehouse systems is developed in order to stack the various kinds of productions. However, according to increase in the rack height, the long lead time should be taken. In stacker crane systems, the variations of the lifting height and the load generate the vibration of lifting machine, and it makes a position control to be difficult. Therefore, the reduction of vibration will be important factor for saving the lead time and the damage of productions. This paper deals with a position control of stacker crane in automated high rack warehouse system by using a gain-scheduled control algorithm via a LMI method, where the variations of elastic coefficient of the stacker crane's post are considered.

1. Introduction

Recently, manufacturing systems introduce CIM (Computer Integrated Manufacturing) and JIT (Just In Time) systems for satisfying the various needs of customer. To do this, we need the full information of manufacturing system and the uniform concept of logistics. Also quick delivery service and precise control are required in these systems. In all cases, the automated warehouse systems have been located on the central position in manufacturing system and logistics system[1].

For the throughput capacity of warehouse, many types of automated warehouse systems have been developed in order to meet various requirements. A stacker crane functions as a key component in the automated warehouse system are required to move quickly with minimum residual vibration[2].

On the other hands, the dynamics of stacker crane systems are highly varied over the operating ranges of the linearized model. This fact makes it difficult for a single linear time invariant(LTI) controller to achieve the desired closed-loop design specifications in parameter variations.

To overcome these facts, robustness with respect to model uncertainties could be achieved by using a H_∞ controller synthesis. But, it is often impossible to achieve high performance over the entire operating range with a single robust LTI controller[3]. When the parameter values are measured in real time, it is then desirable to use controllers that incorporate such measurements to adjust the current operating conditions. Such controllers are said to be scheduled by the parameter measurements. This control strategy, called a gain-scheduled method[4], typically achieves higher performance in the face of large variations in operating conditions.

In this paper, we design a gain scheduled controller to achieve the precise position control of stacker crane system. First, we present a linearized model which depends on parameter variations, and show that the parameter variation is bounded in a large level. Second, we show a gain scheduled control algorithm for parameter affined system and apply the method to the stacker crane system. Last, we verify the efficiency of the presented gain scheduled controller for the position control of stacker crane system.

2. Modelling of stacker crane

The configuration of stacker crane, which is used in high rack automated warehouse, is shown in Fig. 1 and its model is also shown[5] in Fig. 2. The stacker crane can store and retrieve the productions by moving the fork left and right in center of rack system, and have the functions of travelling, lifting and forking.

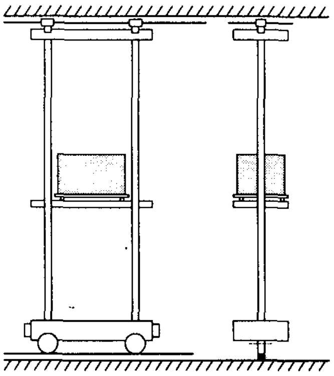

Fig. 1 Configuration

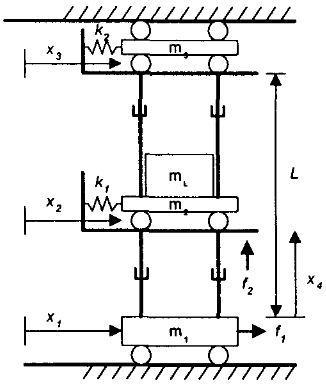

Fig. 2 model

Moreover, the stacker cranes consist of one pair of post, which made by rectangular or circle shape steel, and the upper and the lower part of its post are attached to the wheels and it is used to travel the crane. In some cases, the stacker crane lifts a production within 4~30[m], and the weight to carry reach to few tonnage.

For the stacker crane model in Fig. 2, we assume that the weight of crane's post is concentrated mass, and the mass in each part is given as m_1, m_2 and m_3. Then the following model for travelling can be obtained.

$$m_{13}\ddot{x}_1 + k_1(x_1 - x_2) + B_1\dot{x}_1 = f_1 \qquad (1a)$$

$$m_{23}\ddot{x}_2 + k_1(x_2 - x_1) + k_2(x_2 - x_3) = 0 \qquad (1b)$$

$$m_2\ddot{x}_3 + k_2(x_3 - x_2) + B_2 x_3 = 0 \qquad (1c)$$

where,

$$m_{13} = m_1 + m_2 + m_L + m_3,\quad m_{23} = m_2 + m_L + m_3,$$

$$k_1 = 2\left(\frac{3EI}{x_4^3}\right),\quad k_2 = 2\left(\frac{3EI}{(L - x_4)^3}\right)$$

E : elastic coefficient of material($= 2.1\times10^{10}$)

I : second moment of area (case of iron pipe $I = \dfrac{\pi(d_2^4 - d_1^4)}{64}$, d_2 :outer diameter, d_1 :inner diameter)

The dynamic equation for lifting is now obtained as follows:

$$m_{2L}\ddot{x}_4 + B_3\dot{x}_4 = f_2 \qquad (1d)$$

where $m_{2L} = m_2 + m_L$.

In above equations, the stacker crane has following properties:

(1) effect of the production's weight m_L.

(2) variation of elastic coefficients k_1, k_2 by lifting variable x_4.

(3) difficulty in measuring variable x_2.

These effects make it difficult to the precise position control and the reduction of vibration in stacker crane.

Based on the above equations, we have a state space equation as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (2a)$$

$$y(t) = Cx(t) \qquad (2b)$$

where,

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1}{m_{13}} & \frac{k_1}{m_{13}} & 0 & 0 & \frac{-B_1}{m_{13}} & 0 & 0 & 0 \\ \frac{k_1}{m_{23}} & \frac{-k_1-k_2}{m_{23}} & \frac{k_2}{m_{23}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{k_2}{m_3} & \frac{-k_2}{m_3} & 0 & 0 & 0 & \frac{-B_2}{m_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-B_3}{m_{2L}} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_{13}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_{2L}} \end{bmatrix},\quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix},\quad u = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To show the properties of the stacker crane, we consider the parameters as in Table 1.

Table 1 Parameters of stacker crane

Symbol	Value	Symbol	Value
m_1	150[kg]	B_3	100
m_2	30[kg]	L	20[m]
m_L	0~50[kg]	E	2.1×10^6 kgf/cm²
m_3	20[kg]	d_1	0.1[m]
B_1, B_2	100	d_2	0.09[m]

Fig. 3 and 4 show the frequency responses of x_2 and x_3 in variation of x_4.

Define $\Delta k_i = k_i - k_{ni}$, $i = 1, 2$, where k_{ni} and x_{ni} denote nominal values of k_i and x_i, respectively.

Fig. 5 illustrates the variations of elastic coefficient $|\Delta k_1|$ and $|\Delta k_2|$ in $x_4 = 10$[m]. From these results, we know that the variation of elastic coefficients is changed as a exponential function, and it effects to the large deviations in the given system.

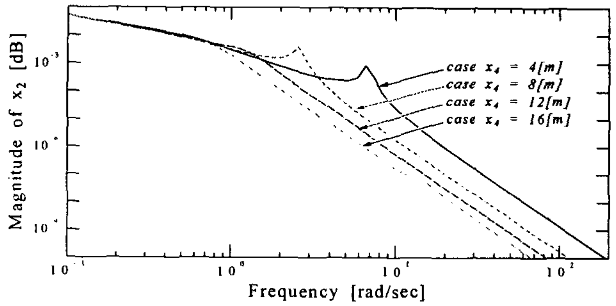

Fig. 3 Freq. response of x_2 in the variations of x_4

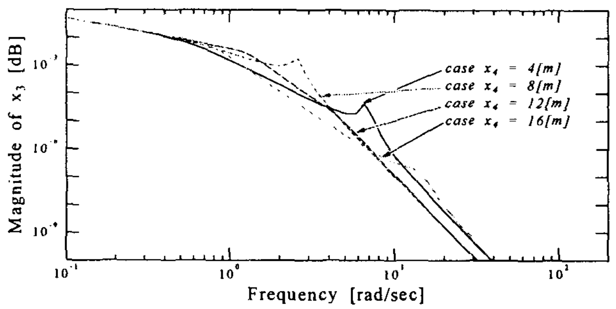

Fig. 4 Freq. response of x_3 in the variations of x_4

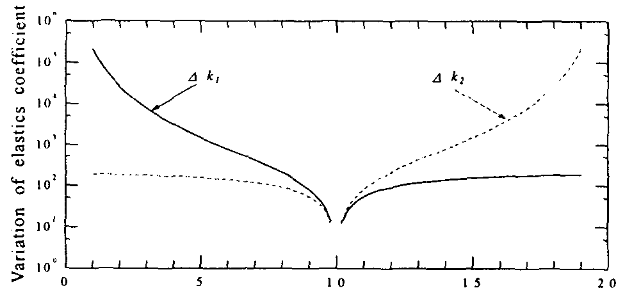

Fig. 5 Variation of elastic coefficient in $x_{m4}=10$ [m]

3. Design of Gain-Scheduled Controller

3.1 Gain-Scheduled controller

Consider a linear parameter-varying system as

$$P(.,p) = \begin{cases} \dot{x} = A(p)x + B_1(p)w + B_2u \\ z = C_1(p)x + D_{11}(p)w + D_{12}u \\ y = C_2x + D_{21}w + D_{22}u \end{cases} \quad (3)$$

where $p(t) = (p_1(t), \cdots, p_n(t))$, $\underline{p_i} \le p_i(t) \le \overline{p_i}$ is a time-varying vector of physical parameters: $A(\cdot)$, $B_1(\cdot), C_1(\cdot), D_{11}(\cdot)$ are affine functions of $p(t)$.

When $p(t)$ takes values in a box of R^n with corners $\{\Pi_i\}_{i=1}^{N} (N=2^n)$, the plant system matrix ranges in a matrix polytope with vertices $S(\Pi_i)$

$$S(p) = \begin{pmatrix} A(p) & B_1(p) & B_2 \\ C_1(p) & D_{11}(p) & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} \quad (4)$$

where convex decomposition is given as

$$p(t) = \alpha_1\Pi_1 + \cdots + \alpha_N\Pi_N, \quad \alpha_i \ge 0, \ \Sigma_{i=1}^{N}\alpha_i = 1 \quad (5)$$

and the system matrix $S(p)$ is described by

$$S(p) = \alpha_i S(\Pi_1) + \cdots + \alpha_N S(\Pi_N) \quad (6)$$

Then, the gain-scheduled controller is given as follows:

$$K(.,p) = \begin{cases} \dot{\zeta} = A_K(p)\zeta + B_K(p)y \\ u = C_K(p)\zeta + D_K(p)y \end{cases} \quad (7)$$

where convex decomposition is given by current parameter value $p(t)$ as $p(t) = \sum_{i=1}^{n} \alpha_i \Pi_i$. The values of $A_K(p)$, $B_K(p)$, $\cdots$ are derived from the values $A_K(\Pi_i), B_K(\Pi_i)$, $\cdots$ at the corners of the parameter box by

$$\begin{pmatrix} A_K(p) & B_K(p) \\ C_K(p) & D_K(p) \end{pmatrix} = \sum_{i=1}^{N} \alpha_i \begin{pmatrix} A_K(\Pi_i) & B_K(\Pi_i) \\ C_K(\Pi_i) & D_K(\Pi_i) \end{pmatrix} \quad (8)$$

Therefore, the controller state-space matrices at the operating point $p(t)$ are obtained by convex interpolation of the LTI vertex controllers as

$$K_i = \begin{pmatrix} A_K(\Pi_i) & B_K(\Pi_i) \\ C_K(\Pi_i) & D_K(\Pi_i) \end{pmatrix}$$

The gain scheduled controller is given by the following theorem.

Theorem 1:[4] The gain-scheduling problem is solvable if and only if there exist pairs of symmetric matrices (R, S) in $\mathrm{R}^{n\times n}$ and (L, J) in $\mathrm{R}^{p\times p}$ such that

$$N_R^T \begin{pmatrix} A_iR + RA_i^T & RC_{1i}^T & B_{1i} \\ C_{1i}R & -\gamma I & D_{11i} \\ B_1^T & D_{11i}^T & -\gamma I \end{pmatrix} N_R < 0, \quad i = 1, \cdots, N \quad (9)$$

$$N_S^T \begin{pmatrix} A_i^TS + SA_i & SB_{1i} & C_{1i}^T \\ B_{1i}^TS & -\gamma I & D_{11i} \\ C_{1i} & D_{11i} & -\gamma I \end{pmatrix} N_S < 0, \quad i = 1, \cdots, N \quad (10)$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \ge 0 \quad (11)$$

where

$$\begin{pmatrix} A_i & B_{1i} \\ C_{1i} & D_{11i} \end{pmatrix} = \begin{pmatrix} A(\Pi_i) & B_1(\Pi_i) \\ C_1(\Pi_i) & D_{11}(\Pi_i) \end{pmatrix} \quad (12)$$

and N_R and N_S denote bases of the null space of $(B_2^T, D_{12}^T, 0)$ and $(C_2, D_{21}, 0)$, respectively. ■

3.2 Design of gain-scheduling controller

We consider the lift distance variation as $x_4 \in [0.5, 19.5]$. Further, the parameter variation m_L and the modelling error are considered as a system's uncertainties. For these uncertainties, we select a weighting function as

$$W(.,k) = \begin{cases} \dot{x}_w = A_w(k)x_w + B_w(k)e \\ z = C_w(k)x_w + D_w(k)e \end{cases} \qquad (13)$$

The augmented system with weighting functions are given as

$$P_{aug}(.,k) = \begin{cases} \dot{x} = A(k)x + B_1(k)w + B_2u \\ z = C_1(k)x + D_{11}(k)w + D_{12}u \\ e = C_2x + D_{21}w + D_{22}u \end{cases} \qquad (14)$$

where,

$$A(k) = \begin{bmatrix} A_p(k) & 0 \\ -B_w(k)C & A_w(k) \end{bmatrix}, \quad B_1(k) = \begin{bmatrix} 0 \\ B_w(k) \end{bmatrix},$$

$$B_2 = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad C_1(k) = [\,-D_w(k)C_p \;\; C_w(k)\,],$$

$$C_2 = [\,-C_p \;\; 0\,], \quad D_{11}(k) = D_w(k), \quad D_{12} = 0, \quad D_{21} = I,$$

$$D_{22} = 0$$

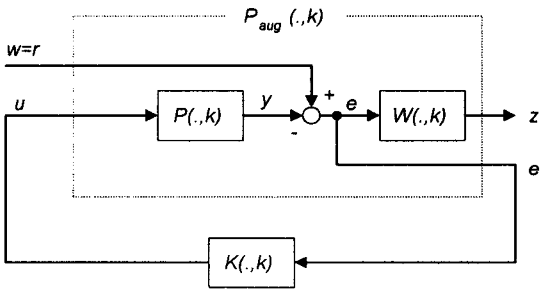

Fig. 6 Gain-scheduled control system

4. simulation results

In simulation, we consider the weighting function as in Fig. 7, and change the lifting distance 8 to 10[m] with a fixed load $m_L = 40kg$. The simulation results are shown in Fig. 8 and 9.

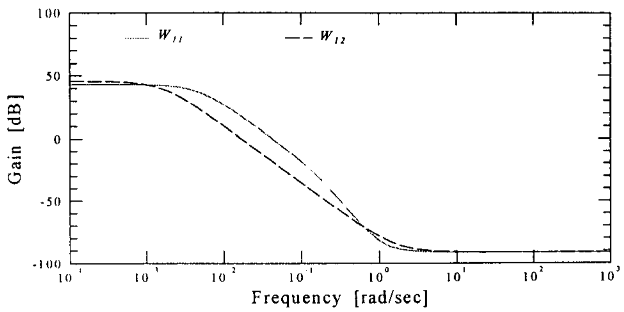

Fig. 7 Weight functions

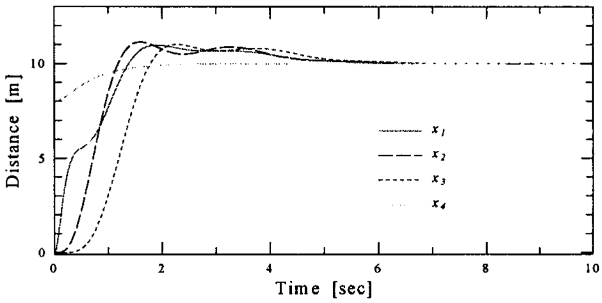

Fig. 8 Time response in case $x_4(0) = 8$[m]

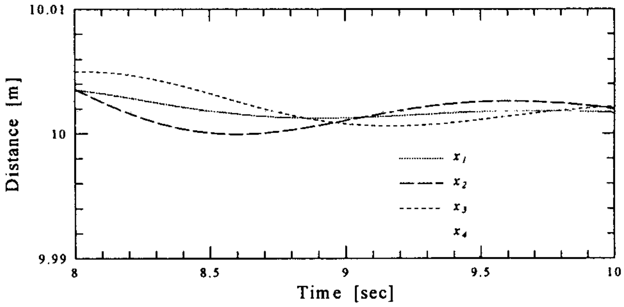

Fig. 9 Time response in case $x_4(0) = 8$[m]

5. Conclusions

In this paper, we deal with a gain scheduled controller for precise position control of stacker crane in high rack warehouse system. By simulation results, we have verified the efficiency of the applied gain scheduled controller.

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