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I-PDA Controller Designed by CDM

Arjin Numsomran, Witsarut Sriratana, Prasit Julsereewong, Viriya Kongratana and Kitti Tirasesth

Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand
(Tel: (662) 326-7346-7; Fax: (662) 326-7346-7 Ext. 103; E-mail: ktkitti@kmitl.ac.th)

Abstract

A design of I-PDA controller for the third order plant by CDM is presented in this paper. Using CDM in the controller design procedures, the step responses of the controlled system with the I-PDA controller satisfied both transient and steady state response specifications without adjustment, and also satisfy the requirements of stability, faster response and robustness. The step responses of the controlled system using I-PDA controller are coinciding to the ones using PIDA controller, and the integral gain of the I-PDA controller also equals to the prefilter gain of the PIDA controller designed by CDM. The effect of the disturbances can also be fastly eliminated. The fast step response of the controlled system can be obtained by reducing the equivalent time constant. MATLAB's numerical results show that the desired specifications of the controlled system using I-PDA controller is obtained. Furthermore, the results also show a good robustness that the desired performances of the controlled system have no significant changed when the plant parameters are varied.

1. Introduction

In 1996, S. Jung and R.C. Dorf had proposed a structure of the PIDA (Proportional-Integral-Derivative-Acceleration) controller to be used especially for the third order plant [1]. The design of the PIDA controller using the CDM (Coefficient Diagram Method), which is proposed by Shunji Manabe [2], [3] has been studied and concluded that the PIDA controller designed by CDM can be used more efficiently when compared to the PIDA controller designed by Jung-Dorf technique [4]. But the gain of the prefilter in the controlled system in some cases is rather high and should be carefully considered. In addition, most of the reference input of the industrial plants is a step function. However, such a step change in the manipulated signal may not be desirable in many occasions [5]. Therefore, it may be advantageous to use only the integral (I) control action in the forward path, and move the proportional-derivative-acceleration (PDA) control action to the feedback path so that the PDA control action affect only the feedback signal. That is, the structure of the PIDA controller can be changed to the form of I-PDA controller.

This paper presents a design of the I-PDA controller parameter for the third order plant by CDM. Using the structure of the I-PDA controller in the controlled system, it is not necessary to use the prefilter. The I-PDA controller parameter is designed by CDM so that the controlled system satisfies both transient and steady state response specifications. The parameter of the I-PDA controller is designed based on the stability and the speed of the controlled system, which are defined in the term of the standard stabilty index and the equivalent time constant. When the settling time of the controlled system has been selected, the equivalent time constant is obtained. The stability index and the equivalent time constant specify the coefficients of the characteristic polynomial. These coefficients are related to the controller parameters algebraically in explicit form. Hence, the transient and the steady state performances can be obtained as desired.

The step responses of the controlled system using I-PDA controller and compared to the step responses of the controlled system using PIDA controller designed by both CDM and Jung-Dorf technique are shown by various MATLAB's numerical examples. The step responses of the controlled system using I-PDA controller and the PIDA controller designed by CDM are coincidently. These step responses have no overshoot and reaches the desired settling time without adjustment, and as mentioned in [4], it is better than the step response of the controlled system using PIDA controller designed by Jung-Dorf technique. The integral gain of the I-PDA controller also equals to the prefilter gain of the PIDA controller. Furthermore, the I-PDA controller fastly eliminated the effect of the disturbances. The numerical results also show a good robustness that the desired performances of the controlled system have no significant changed when the plant parameters are varied.

2. Structure of the control system with the I-PDA Controller

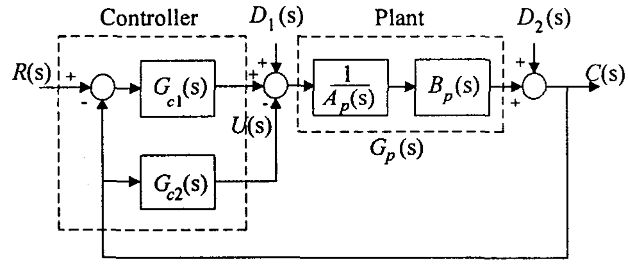

Fig. 1. Structure of the SISO control system.

The structure of the SISO (Single-Input-Single-Output) control system using I-PDA controller is shown in Fig. 1. $R(s)$ is the reference input, $C(s)$ is the controlled output, $D_1(s)$ and $D_2(s)$ are the process step disturbance and the output step disturbance to be applied to the system. $G_{c1}(s)$ and $G_{c2}(s)$ are the I control action and the PDA control action, $G_p(s)$ is the transfer function of the third order plant. From Fig. 1, the transfer functions of the I-PDA controller and the third order plant are

$$G_{c1}(s) = \frac{K_i}{s}, \quad G_{c2}(s) = K_p + k_d s + k_a s^2 . \quad (1)$$

and

$$G_p(s) = B_p(s)/A_p(s)$$
$$B_p(s) = K, \quad A_p(s) = \prod_{l=1}^{3}(s + p_l). \quad (2)$$

where K_i, K_p, K_d and K_a are respectively the integral gain, proportional gain, derivative gain and acceleration gain of the I-PDA controller. For the third order plant, K is the gain, p_i is a real or complex poles with negative real part. Closed-loop transfer function of the controlled system is

$$\frac{C(s)}{R(s)}=\frac{G_{c1}(s)G_p(s)}{1+G_{c1}(s)G_p(s)+G_{c2}(s)G_p(s)}. \quad (3)$$

The characteristic polynomial is

$$\begin{aligned}P(s)&=1+G_{c1}(s)G_p(s)+G_{c2}(s)G_p(s)\\&=s^4+(p_1+p_2+p_3+KK_a)s^3\\&+(p_1p_2+p_2p_3+p_3p_1+KK_d)s^2\\&+(p_1p_2p_3+KK_p)s+KK_i.\end{aligned} \quad (4)$$

3. CDM Design Procedures

The CDM is used to design the controller so that the step response of the controlled system satisfies both transient and steady state response specifications, and also satisfy the requirements of stability, faster response and robustness. Generally, the order of the controller designed by CDM is less than the order of the plant [3]. However, when using the I-PDA controller for the third order plant, the order of the controller is equal to the order of the plant, but the integrator of the $G_{c1}(s)$ virtually makes the plant to be fourth order. Thus CDM condition is satisfied.
The polynomials form of the controller and the plant are generally be respectively written in the form [3]

$$\begin{aligned}A_c(s)&=l_\lambda s^\lambda+l_{\lambda-1}s^{\lambda-1}+\dots+l_0\\B_c(s)&=k_\lambda s^\lambda+k_{\lambda-1}s^{\lambda-1}+\dots+k_0\end{aligned} \quad (5)$$

and

$$\begin{aligned}A_p(s)&=p_ks^k+p_{k-1}s^{k-1}+\dots+p_0\\B_p(s)&=q_ms^m+q_{m-1}s^{m-1}+\dots+q_0,\end{aligned} \quad (6)$$

where $\lambda<k$ and $m<k$.
The characteristic polynomial of the closed-loop controlled system shown in Fig. 1 can be given in the following form

$$\begin{aligned}P(s)&=a_ns^n+a_{n-1}s^{n-1}+\dots+a_1s+a_0\\&=\sum_{i=0}^{n}a_is^i,\end{aligned} \quad (7)$$

where $a_0, a_1,\dots\dots,a_n$ are the real coefficients.
The stability index γ_i, the equivalent time constant τ and the stability limit γ_i^* are defined as follows:

$$\gamma_i=\frac{a_i^2}{a_{i+1}a_{i-1}}, \quad (8)$$

$$\tau=\frac{a_1}{a_0}, \quad (9)$$

$$\gamma_i^*=\frac{1}{\gamma_{i+1}}+\frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0,\gamma_n=\infty, \quad (10)$$

where $i=1,\dots,n-1$.
To meet the specifications, equivalent time constant τ and the standard values of the stability index γ_i are chosen as

$$t_s=2.5\tau\sim3\tau. \quad (11)$$

$$\gamma_{n-1}=\dots=\gamma_3=\gamma_2=2,\gamma_1=2.5. \quad (12)$$

In general, the settling time $t_s=2.5\tau$, and the stability index $\gamma_1=2.5$, $\gamma_2=\gamma_3=2$ are strongly recommended due to the stability and the step response requirement. However, it is not necessary to always define $t_s=2.5\tau$ and $\gamma_4\sim\gamma_{n-1}=2$. Then the condition for the stability index can be relaxed to

$$\gamma_i>1.5\gamma_i^*. \quad (13)$$

The standard values of the stability index γ_i in (12) can be used if the following condition in (14) is satisfied.

$$p_k/p_{k-1}>\tau/(\gamma_{n-1}\gamma_{n-2}\cdots\gamma_1), \quad (14)$$

where p_k and p_{k-1} are the coefficients of the plant at k^{th} and $(k-1)^{th}$, respectively.
If (14) is not satisfied, the γ_{n-1} is first increased, then γ_{n-2} and so on, until (14) is satisfied. From (8)-(10), the coefficient a_i and the characteristic polynomial $P(s)$ are

$$a_i=a_0\tau^i\frac{1}{\gamma_{i-1}\cdots\gamma_2^{i-2}\gamma_1^{i-1}}=a_0\tau^i\prod_{j=1}^{i-1}\frac{1}{(\gamma_{i-j})^j}. \quad (15)$$

$$P(s)=a_0\left\{\left[\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^j}\right)(\tau s)^i\right]+\tau s+1\right\}. \quad (16)$$

From (7) and (16), the characteristic polynomial $P(s)$ is

$$P(s)=a_4s^4+a_3s^3+a_2s^2+a_1s+a_0. \quad (17)$$

The design procedures for the I-PDA controller by CDM are summarized as follows:

1) Determine the equivalent time constant τ from the desired settling time t_s.
2) Determine the proper values of the stability index γ_i from the standard values of the stability index in (12).
3) Equate the $P(s)$ in the form of (4) of each plant with I-PDA controller to the $P(s)$ obtained from (17) using τ and γ_i found from 1) and 2). Hence, the I-PDA controller parameters can be obtained.

4. Numerical Examples

In this section, the MATLAB's numerical examples of the controlled system using I-PDA controller are compared to the.step responses of the same system with the PIDA controller designed by CDM and Jung-Dorf technique.

Example of the type 0 plant
To compare the step responses of the controlled system using I-PDA controller to the ones using PIDA controller from [1] and [4], the type 0 plant from [1] is used here.

$$G_p(s)=\frac{1}{(s+1)(s+3)(s+6)}.$$

The desired specifications for step input are

$$P.O.\le5\%,\quad t_s(\pm2\%)\le2\text{ sec},\quad e_{ss}(t)=0.$$

When the settling time $t_s=2$ sec. is required, $\tau=0.8$ sec. ($t_s=2.5\tau$) and $\gamma_3=\gamma_2=2,\gamma_1=2.5$. Hence $P(s)$ is

$$P(s)=s^4+12.5s^3+78.125s^2+244.14s+305.18.$$

Equated the above $P(s)$ to the $P(s)$ of the plant with the I-PDA controller in the form of (4), the I-PDA controller is

$$G_{c1}(s)=\frac{305.18}{s},\quad G_{c2}(s)=226.14+51.125s+2.5s^2.$$

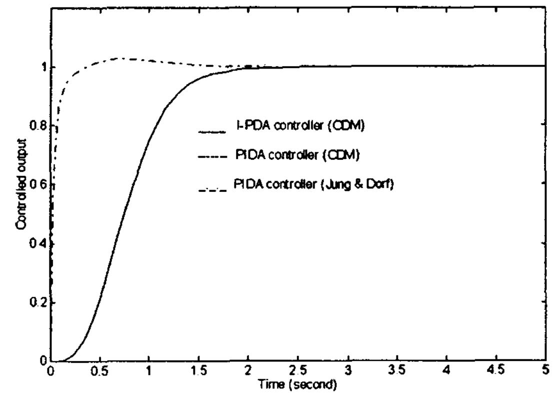

Fig. 2. Step responses of the type 0 plant.

The step response with no overshoot of the controlled system using I-PDA controller is compared to the step responses using PIDA controller designed by CDM and Jung-Dorf technique for t_s = 2 sec. These step responses are shown in Fig. 2. From these results, the step response of the controlled system using I-PDA controller is coincide to the ones using PIDA controller designed by CDM.

Example of the type 1 plant

The effect of the process step disturbance and the output step disturbane on the step responses of the controlled system using I-PDA controller is investigated using the transfer of the type 1 plant from [1].

$$G_p(s)=\frac{1}{s(s+1)(s+7)}.$$

The desired specifications for step input are

$$P.O. \le 5\ \%,\quad t_s(\pm 2\%) \le 2 \text{ sec},\quad e_{ss}(t)=0.$$

When the required settling time t_s = 2 sec., τ = 0.8 sec. (t_s = 2.5τ) and $\gamma_3=\gamma_2=2, \gamma_1=2.5$. Hence,

$$P(s)=s^4+12.5s^3+78.125s^2+244.14s+305.18.$$

The I-PDA controller is obtained as

$$G_{c1}(s)=\frac{305.18}{s},\quad G_{c2}(s)=244.14+71.125s+4.5s^2.$$

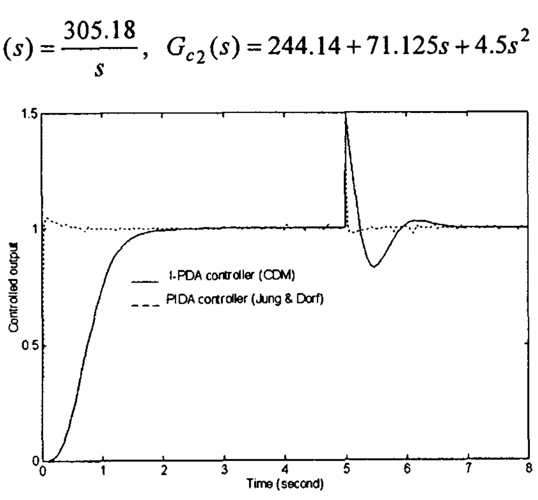

Fig. 3. Step responses of the type 1 plant.

Figure 3 shows the step responses of the controlled system using I-PDA controller and the ones using PIDA controller designed by Jung-Dorf technique, with the 50% process step disturbance and the 50% output step disturbance are applied at t =3 sec. and t =5 sec., respectively. The effect of the process step disturbance is small when compared to the effect of the output step disturbance, which gives a severe effect to the step response at the initial state. The effect of the disturbances is fastly rejected.

Example of the type 2 plant

The step responses of the controlled system with three different equivalent time constant τ are studied here.

$$G_p(s)=\frac{1}{s^2(s+1)}.$$

The desired specifications for step input are

$$P.O. \le 5\ \%,\quad t_s(\pm 2\%) \le 2 \text{ sec},\quad e_{ss}(t)=0.$$

When t_s = 2 sec., τ = 0.8 sec. and $\gamma_3=\gamma_2=2, \gamma_1=2.5$,

$$P(s)=s^4+12.5s^3+78.125s^2+244.14s+305.18.$$

With the same procedures, the I-PDA controller is

$$G_{c1}(s)=\frac{305.18}{s},\quad G_{c2}(s)=244.14+78.125s+11.5s^2.$$

When t_s is changed to t_s = 1.5 sec. and t_s = 1 sec., τ = 0.6 sec. and τ = 0.4 sec., respectively, while the stability index remains the same. The I-PDA controller are obtained as

$$G_{c1}(s)=\frac{964.506}{s}, G_{c2}(s)=578.704+138.889s+15.667s^2.$$

and

$$G_{c1}(s)=\frac{4{,}883}{s},\quad G_{c2}(s)=1{,}953+312.5s+24s^2.$$

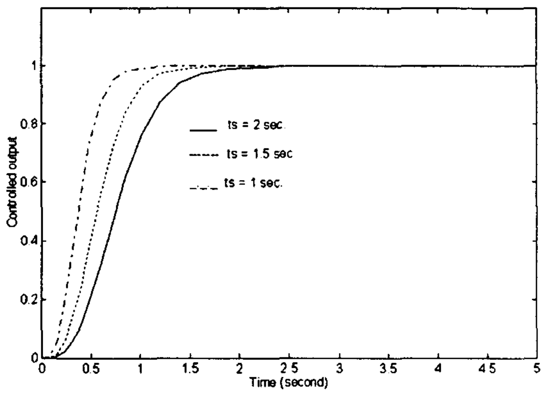

Fig. 4. Step responses of the type 2 plant.

The step responses with no overshoot of the controlled system using I-PDA controller for the three values of τ are shown in Fig. 4. When t_s is decreased from 2 sec. to 1 sec., the integral gain of the I-PDA controller is increased.

Example of an AC induction motor model

The proposed controller applied to the third order plant for the simplified position control of an AC induction motor model [6] is illustrated here. The step responses of the controlled system using I-PDA controller are compared to the step response of the same model using PIDA controller [1]. From (15) and (16) in [1],

$$G_p(s) = \frac{K_I K_t}{s\left(Js^2 + \left(f + K_P K_t\right)s + K_I K_t\right)} = \frac{168.0436}{s\left(s^2 + 25.921s + 168.0436\right)},$$

where K_P, K_I are PI controller gains, K_t is motor constant. The motor parameters used are as follows: J = 0.305, f = 0.2725, K_P = 14.0242, K_I = 94.1637 and K_t = 0.5443 [6]. The desired specifications for step input are

$$P.O. \le 5\ \%, \quad t_s(\pm 2\%) \le 2 \text{ sec}, \quad e_{ss}(t) = 0.$$

To compare the step response of an AC induction motor model using I-PDA controller with the step response of the same model using PIDA controller designed by Jung-Dorf technique. For the selected t_s = 1.18 sec., τ = 0.472 sec., and to satisfy (14), $\gamma_3 = 2.5, \gamma_2 = 2, \gamma_1 = 2.5$, hence

$$P(s) = s^4 + 26.483s^3 + 280.541s^2 + 1{,}486s + 3{,}148.$$

The I-PDA controller is obtained as

$$G_{c1}(s) = \frac{18.734}{s}, \quad G_{c2}(s) = 8.842 + 0.669s + 0.0033s^2.$$

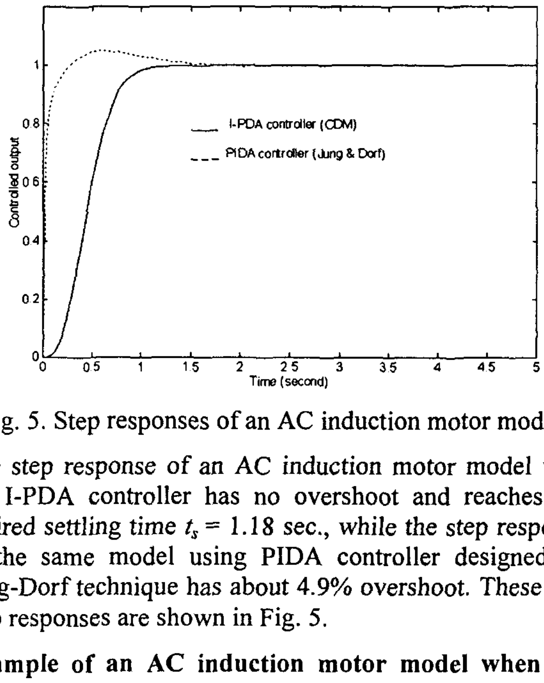

Fig. 5. Step responses of an AC induction motor model.

The step response of an AC induction motor model with the I-PDA controller has no overshoot and reaches the desired settling time t_s = 1.18 sec., while the step response of the same model using PIDA controller designed by Jung-Dorf technique has about 4.9% overshoot. These two step responses are shown in Fig. 5.

Example of an AC induction motor model when the parameter J are varied

In order to investigate that the plant with parameter variation could also be made robust by the I-PDA controller, the parameter J of an AC induction motor model in the previous example is varied to $0.5J$ and $2J$, respectively, while the parameters of the I-PDA controller remain unchanged. The transfer functions of an AC induction motor model when J is varied become

$$G_p(s) = \frac{336.0872}{s\left(s^2 + 51.8420s + 336.0872\right)};\ (J \to 0.5J),$$

$$G_p(s) = \frac{84.0218}{s\left(s^2 + 12.9605s + 84.0218\right)};\ (J \to 2J).$$

Figure 6 shows that the step response of the controlled system with nominal parameter has no significant changed when compared to the step responses with the parameter variation for t_s = 1 sec. This implies that the performances of the controlled system have no significant changed.

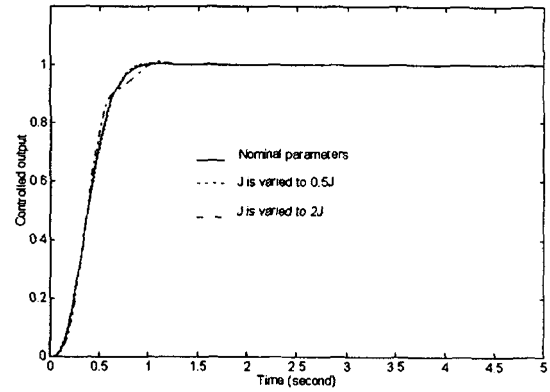

Fig. 6. Step responses of an AC induction motor model with nominal parameters and J varied.

5. Conclusions

The design procedures of the I-PDA controller by CDM for the third order plant have been proposed in this paper. With the I-PDA controller, the step response of the controlled system satisfies both transient and steady state response specifications, and also satisfies the requirements of stability, robustness and fast response. The I-PDA controller fastly eliminated the effect of the disturbances. The numerical results also show a good robustness that when the parameters of plant are varied, there is no significant change on the step responses of the controlled system. The performances of the controlled system using I-PDA controller is better than the performances of the controlled system using PIDA controller designed by Jung-Dorf technique in the sense that the step response has no overshoot and reaches the desired settling time without adjustment. However, the step response of the controlled system using PIDA controller designed by Jung-Dorf technique in most cases give faster rise time but require gain adjustment to meet the desired specifications.

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