Proceedings of the 14th
KACC, Octorber 1999

A Quantization Algorithm without Accumulative Error

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Abstract

In this paper, a quantization algorithm by which the accumulative error can be prevented is presented. In digital control systems, the quantization cannot be avoided because of the finite word length of digital computers. The error due to quantization of the computed values may be tolerable in case of directly using them. In case of using the accumulated values, the error between sum of the original values and that of the quantized values becomes larger as the number of the values to be summed increases. Such an increasing accumulative error is critical for the control of precise NC machines, robots and autonomous vehicles. To solve this problem, a quantization algorithm without the accumulative error is presented. Basically, the algorithm is based on the feedback loop by which the accumulationive of the quantization error can be prevented. The error boundness of the proposed algorithm is proven and a computer simulation is performed to show the validity of the algorithm.

1. Introduction

In digital control systems, the quantization process of control variables is inevitable due to the finite word length of the digital computer. The quantization error itself is neglectable in practical cases with high precision of data type. However, it usually causes large accumulative errors in case of using the quntized values with summation. The summation of the quantized values can be easily found in NC machines, manipulator control[1] or mobile robot navigation[2-4] in which micro processors are interconnected through common buses or communication lines. In the control systems of those machines, the motion commands are computed by the high level computer with high precision data types as 32 bit integer or more and transferred to the low level sub-systems with low precision data types, for example as 16 bit data. Let us assume that the transferred values are incremental posiotion commands at every control instant. In case of summation of those values, the accumulative error between sum of the original values and that of the quantized value become larger as the number of the values to be summed increases. Such increasing accumulative error becomes critical for the control of precise NC machines, robots and autonomous vehicles[5-7]. To solve this problem, a quantization algorithm without the accumulative error is proposed. Basically, the algorithm is based on the feedback loop which prevents the algorithm from accumulating the quantization error. The boundness of the algorithm is proven and a simulation result shows the validity of the algorithm. The paper is organized as follows. In section 2, the problem of the conventional quantization algorithm is briefly stated. The proposed quantization algorithm is presented in section 3 and the simulation results and some discussions are given in section 4. Finally, the conclusion is remarked with the future application of this work.

2. Problem Statements

The quantization function is defined as follows;

$$Q_0(x) = n\delta \tag{1}$$

where $n\delta \leq x < (n+1)\delta$ x, n is natural number and δ is the quantization unit. If a computed value v is quantized by using *Eq.(1)*, the quantized value v^0 can be written by

$$v^0 = Q_0(v) \ . \tag{2}$$

In case of directly using the quantized value v^0, the error between the original value v and the quantized value v^0 is bounded by the quantization unit as

$$v - v_0 < \delta \ . \tag{3}$$

When the value v is accumulated as follows

$$p(k) = p(k-1) + v(k) \quad , \tag{4}$$

the accumulation of the quantized value is also represented by

$$p^0(k) = p^0(k-1) + v^0(k) \quad . \tag{5}$$

In the accumulation of the quantized values, the accumulative error becomes larger as the number of accumulation increases.

$$p - p_0 = \sum_{k=0}^{n} (v(k) - v^0(k)) < n\delta$$

$$\lim_{n\to\infty} (p - p_0) = \sum_{k=0}^{\infty} (v(k) - v^0(k)) < \lim_{n\to\infty} n\delta = \infty \tag{6}$$

We call this algorithm given by *Eq.(2) Type 0* quantization.

3. Proposed Algorithm

3.1 Type 1 Quantization

Theorem 1: *If sequential values, $v(k)$ are quantized by the algorithm as*

$$p(k) = p(k-1) + v(k) \tag{8}$$

$$v_1(k) = Q_1(v_1(k)) = Q_0(p(k)) - Q_0(p(k-1)) \tag{9}$$

the sum of the quantized value has the error bounded by the quantization unit, δ.

$$p_1(k) = p_1(k-1) + v_1(k) \tag{10}$$

$$\lim_{n\to\infty} |p(k) - p_1(k)| < \delta \tag{11}$$

Proof: *Eq.(10)* can be rewritten by

$$\begin{aligned} p_1(k) &= \sum v_1(k) \\ &= \sum (Q_0(p(k)) - Q_0(p(k-1))) \\ &= Q_0(p(k)) \end{aligned} \tag{12}$$

Thus, by substituting *Eq.(12)* into *Eq.(11)*,

$$\lim_{n\to\infty} (p(k) - p_1(k)) = \lim_{n\to\infty} (p(k) - p_0(k)) < \delta$$

From *theorem 1*, the quantization algorithm $Q_1(\cdot)$ shows the bounded accumulative error. However, since it sums the values to be quantized, the computational overflow may happen as the number of sequence increases. We call this algorithm given by *Eqs.(8)-(9) Type 1* quantization which can be depicted by *Fig.1*.

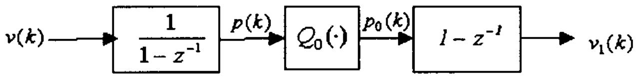

Fig.1 Type 1 Quantization Algorithm

3.2. Type 2 Quantization

To prevent the overflow problem of the *type 1* quantization, the second algorithm is presented based on the following theorem.

Theorem 2 *If sequential values, $v(k)$ are quantized by Eqs.(13)-(14),*

$$z(k) = z(k-1) + v(k) - v_2(k-1) \tag{13}$$

$$v_2(k) = Q_2(v(k)) = Q_0(z(k)) \tag{14}$$

the sum of the quantized values has the bounded error within the quantization unit δ as

$$p_2(k) = p_2(k-1) + v_2(k) \tag{15}$$

$$\lim_{n\to\infty} |p(k) - p_2(k)| < \delta \ . \tag{16}$$

Proof: From the characteristics of the quantization of *Eq.(1)*, the following equation is satisfied.

$$Q_0(x_0 + y) = Q_0(x) + Q_0(y) = x_0 + y_0 \tag{17}$$

Eq.(17) can be derived as follows:

$$\begin{aligned} Q_0(x_0 + y) &= Q_0(n\delta + m\delta + \eta) \\ &= n\delta + m\delta \\ &= x_0 + y_0 \end{aligned} \tag{18}$$

where $x_0 = n\delta$, $y = m\delta + \eta$ and $0 \le \eta < \delta$. Next, *Eq.(13)* is rewritten by

$$z(k) = z(k-1) + v(k) - z_0(k-1) \tag{19}$$

and from *Eq.(19)*,

$$z(k) = \sum v(k) - \sum z_0(k-1) \tag{20}$$

and rewriting *Eq.(14)* using *Eq.(20)* and *Eq.(18)*,

$$v_2(k) = Q_0(\sum v(k) - \sum z_0(k-1)) \tag{21}$$
$$= Q_0(\sum v(k)) - \sum z_0(k-1)$$
$$= Q_0(\sum v(k)) - \sum v_2(k-1) \tag{22}$$

And from *Eqs.(21)-(22)*,

$$\sum v_2(k) = Q_0(\sum v(k)) \tag{23}$$
$$= Q_0(p(k)) = p_0(k) \tag{24}$$

Thus, from *Eqs.(15)* and *(24)*, *Eq.(16)* can be proven by following equations:

$$\lim_{n\to\infty}(p(k) - p_2(k)) = \lim_{n\to\infty}(p(k) - \sum v_2(k))$$
$$= \lim_{n\to\infty}(p(k) - p_0(k)) < \delta \tag{25}$$

We call this algorithm given by *Eqs.(13)-(14)*, *type 2* quantization which can be depicted by *Fig.2*. As shown in *Eq.(2)*, the *type 2* algorithm has inherently the feedback loop which prevents the overflow of the internal variable *z(k)*. Furthermore, from *Eq.(24)*, it can be seen that the *type 2* algorithm is just equivalent to the *type 1* algorithm.

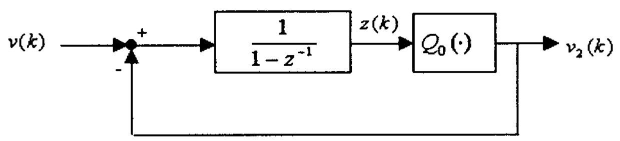

Fig.2 *type 2* Quantization Algorithm

Now, let us investigate the overflow problem of the *type 2* quantization algorithm $Q_2(\cdot)$.

Theorem 3 *If sequential values, $v(k)$ are quantized by the algorithm given by Eq.(13)-(14), the quantization algorithm $Q_2(\cdot)$ does not cause the overflow problem.*

Proof: *Eq.(19)* yields to the following equation,

$$z(k) - v(k) = z(k-1) - z_0(k-1) < \delta \tag{26}$$

The *theorem 3* can be proven since $z(k)$ is bounded by the following equation which is derived form *Eq.(26)*.

$$z(k) < v(k) + \delta \ . \tag{27}$$

4. Simulation

To investigate the validity of the proposed quantization algorithm, the computer simulation was performed for motion planning of a mobile robot with smooth velocity profile[7].

4.1. Motion planning

In mobile robot navigation[5], the command wheel velocities are computed in the main computer by path tracking control algorithm and then transferred to wheel velocity controllers through the quantization. The quantization unit is determined by the word length of the wheel controller's CPU. In robotic systems, the jerk constrained motion guarantees a smooth and stable motion as in *Fig.3*.

4.2. Simulation results

Fig.4 shows the trajectories of the accumulation of the quantized values of the velocities as shown in *Fig.3*.

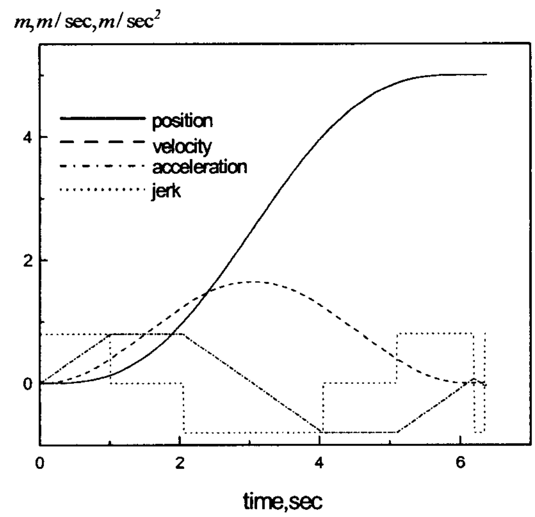

Fig.3 Smooth Motion Planning

The dotted line is the trajectory of the accumulation of the *type 0* quantization and the solid line is that of the *type 2*. Fig.5 shows that the proposed quantization has no accumulative error while the quantization of *type 0* shows the increasing accumulative error.

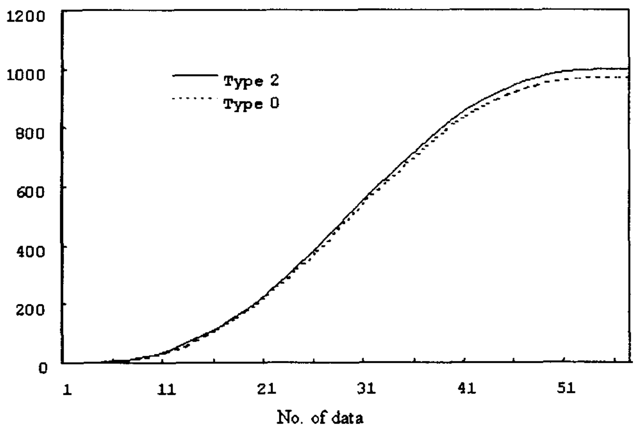

Fig.4 Trajectories of accumulation of quantized values

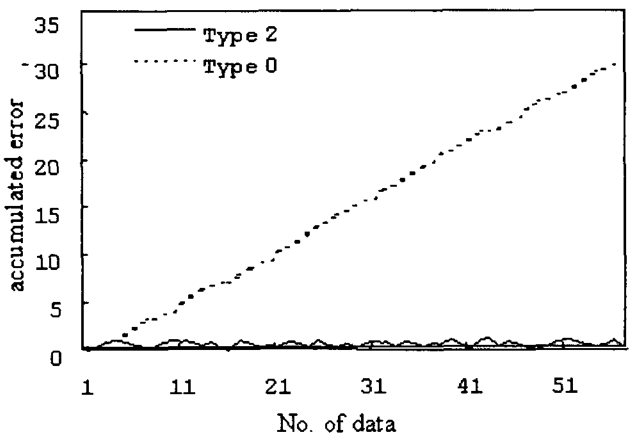

Fig.5 Trajectories of accumulated quantized errors

5. Concluding Remarks

In this paper, we proposed two types of quantization algorithm which have no accumulative quantization error. Particularly the type 2 algorithm has a feedback loop which can prevent the overflow of the internal parameter. Through the mathematical proofs, the similarity of type 1 and 2 algorithms and the error boundness of the both algorithms are shown. The simulation results confirm the effectiveness of the algorithm.

Acknowledgement

This work was supported by the Korea Science and Engineering Foundation(KOSEF) through the RRC-ACCT at Sun Moon University.

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