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Decentralized Dynamic Controller Design for Uncertain Large-Scale Systems

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Abstract - In this paper, a dynamic output feedback controller design technique for robust decentralized stabilization of uncertain large-scale systems is presented. Based on the Lyapunov method, a sufficient condition for robust stability, is derived in terms of three linear matrix inequalities (LMIs). The solutions of the LMIs can be easily obtained using efficient convex optimization techniques.

1. Introduction

A large-scale dynamical system can be usually characterized by a large number of state variables, system parametric uncertainties, and a complex interaction between subsystems [1]. During a decade, the decentralized stabilization problem of large-scale systems with uncertainties has received considerable attention. Using the Lyapunov method, several controller design method are proposed by [2]-[4]. Recently, However most of the methods have a difficulty to tune parameters for the design of controllers. Also, all the stabilization methods made use of system states for feedback, therefore they can not be applicable when all the states are not available. In fact, it is not reasonable to demand all the states in large-scale systems. So, the static output controller for stabilization of large-scale system is considered in the work of [5]-[6].

This paper is concerned with the design problem of robust decentralized dynamic output feedback controller for the large-scale systems with time-varying parametric uncertainties. A sufficient condition for robust stability of the system is derived in terms of LMIs using Lyapunov method. The LMI approach has been one of the hot spots in the control problem due to its computational advantage and simplicity in solving the addressed problems [7-9]. The controller parameters which satisfy the above LMIs can be easily found by various efficient convex optimization algorithms.

Notations: Through the paper, R^n denotes the n dimensional Euclidean space, $R^{n\times m}$ is the set of all $n\times m$ real matrices, I is the identity matrix with appropriate dimensions, diag{ · } denotes a block diagonal matrix, and $*$ denotes symmetric part. The notation $X>0$ for $X\in R^{n\times n}$, means that the matrix X is symmetric and positive definite.

2. Problem Formulation

Consider a class of uncertain large-scale system composed N interconnected subsystems described by

$$\begin{aligned} S_i:\ \dot{x}_i(t) &= (A_i+\Delta A_i(t))x_i(t)+\sum_{j\neq i}^{N}(A_{ij}+\Delta A_{ij}(t))x_j(t) \\ &\quad +(B_i+\Delta B_i(t))u_i(t), \\ y_i(t) &= C_ix_i(t), \qquad i=1,2,\cdots,N \end{aligned} \tag{1}$$

where $x_i(t)\in R^{n_i}$ is the state vector, $u_i(t)\in R^{m_i}$ is the control vector, and $y_i(t)\in R^{q_i}$ is the output vector. The system matrices A_i, B_i, C_i and A_{ij} are of appropriate dimensions, and $\Delta A_i(t), \Delta A_{ij}(t)$ and $\Delta B_i(t)$ are real-valued matrices representing time-varying parameter uncertainties in the system.

Assume that the triple $(A_i, B_i, C_i), i=1,\cdots,N$ is stabilizable and detectable, and assume that the time-varying uncertainties are of the form

$$\begin{aligned} \Delta A_i(t) &= D_{ai}F_{ai}(t)E_{ai},\ \ \Delta A_{ij}(t)=D_{aij}F_{aij}(t)E_{aij} \\ \Delta B_i(t) &= D_{bi}F_{bi}(t)E_{bi} \end{aligned} \tag{2}$$

where $D_{ai}, D_{aij}, D_{bi}, E_{ai}, E_{aij}$ and E_{bi} are known constant real matrices with appropriate dimensions, and $F_{ai}(t), F_{aij}(t)$ and $F_{bi}(t)$ are unknown matrix functions which are bounded as

$$\begin{aligned} F_{ai}^T(t)F_{ai}(t)\le I,\ \ F_{aij}^T(t)F_{aij}(t)\le I, \\ F_{bi}^T(t)F_{bi}(t)\le I,\ \ \forall i,j\ge 0. \end{aligned} \tag{3}$$

Now, in order to stabilize the system given in (1), let's consider the following dynamic output feedback controllers

$$\begin{aligned} \dot{\xi}_i(t) &= A_{ci}\xi_i(t)+B_{ci}y_i(t) \\ u_i(t) &= C_{ci}\xi_i(t)+D_{ci}y_i(t) \end{aligned} \tag{4}$$

where $\xi_i(t)\in R^{k_i}$, and A_{ci}, B_{ci}, C_{ci} and D_{ci} are constant matrices with proper dimensions.

Then the problem is to find the parameters of the dynamic controller (4) such that the resulting closed-loop system is robustly stable for all admissible uncertainties.

3. Decentralized Dynamic Controller Design

In this section, we establish a sufficient condition for robust stability of the system (1) with dynamic output feedback controller (4). Then we derive three LMIs in order to find the controller parameter matrix which satisfies the sufficient condition.

Now, the augmented state vector is defined as

$$z_i(t)=\begin{bmatrix} x_i(t) \\ \xi_i(t) \end{bmatrix} \tag{5}$$

and also is defined controller parameter matrices $K_i \in R^{(m_i+k_i)\times(m_i+k_i)}$ as

$$K_i=\begin{bmatrix} D_{ci} & C_{ci} \\ B_{ci} & A_{ci} \end{bmatrix} \tag{6}$$

The closed-loop system of (1) with controller (4) can be described in the form of

$$\dot{z}_i(t)= \hat{A}_i z_i(t)+\sum_{j\neq i}^{N} \hat{A}_{ij} x_j(t). \tag{7}$$

Then,

$$\begin{aligned} \hat{A}_i &= \overline{A}_i + \overline{B}_i K_i \overline{C}_i + \overline{D}_{ai} F_{ai}(t)\, \overline{E}_{ai} \\ &\quad + \overline{D}_{bi} F_{bi}(t)\, \overline{E}_{bi} K_i \overline{C}_i, \\ \hat{A}_{ij} &= \overline{A}_{ij} + \overline{D}_{aij} F_{aij}(t) E_{aij}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} &\overline{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{B}_i=\begin{bmatrix} B_i & 0 \\ 0 & I \end{bmatrix}, \quad \overline{C}_i=\begin{bmatrix} C_i & 0 \\ 0 & I \end{bmatrix}, \\ &\overline{A}_{ij} = \begin{bmatrix} A_{ij} \\ 0 \end{bmatrix}, \quad \overline{D}_{ai}=\begin{bmatrix} D_{ai} \\ 0 \end{bmatrix}, \quad \overline{D}_{aij}=\begin{bmatrix} D_{aij} \\ 0 \end{bmatrix}, \\ &\overline{D}_{bi} = \begin{bmatrix} D_{bi} \\ 0 \end{bmatrix}; \quad \overline{E}_{ai}=[E_{ai}\ 0], \quad \overline{E}_{bi}=[E_{bi}\ 0]. \end{aligned} \tag{9}$$

Now, we have the following theorem.

Theorem 1. Let's define $[W_{i1}^T\ W_{i2}^T]^T$ and W_{i3} which are orthogonal complement of $[B_i^T\ E_{bi}^T]^T$ and C_i^T, respectively, and $\beta=(N-1)^{1/2}$ Then, the stabilizing controllers, K_i, which guarantee the robust stability of system (1) exist, if there exist $X_i>0$ and $Y_i>0$ for $i=1,2,\cdots,N$ satisfying the LMIs:

$$\overline{W}_{xi}^T \begin{bmatrix} X_iA_i^T+A_iX_i & 0 & D_{ai} & D_{bi} & D_{di} & \beta X_i & X_iE_{ai}^T & X_iE_{di}^T & A_{di} \\ * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} \overline{W}_{xi} < 0 \tag{10}$$

$$\overline{W}_{yi}^T \begin{bmatrix} \begin{pmatrix} A_i^TY_i \\ +Y_iA_i \end{pmatrix} & Y_iD_{ai} & Y_iD_{bi} & Y_iD_{di} & \beta I & E_{ai}^T & E_{di}^T & Y_iA_{di} \\ * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} \overline{W}_{yi} < 0 \tag{11}$$

$$\begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix} \geq 0, \tag{12}$$

where

$$\overline{W}_{xi}=\begin{bmatrix} W_{i1} & 0 \\ W_{i2} & 0 \\ 0 & I \end{bmatrix}, \quad \overline{W}_{yi}=\begin{bmatrix} W_{i3} & 0 \\ 0 & I \end{bmatrix}.$$

Sketch of Proof: Consider the following Lyapunov function for the system (7) as

$$V(t) := \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N} z_i^T(t) P_i z_i(t) \tag{13}$$

where P_i is a positive-definite matrix.

From the time-derivative and utilizing some matrix inequalities, we can get the following sufficient condition for stability of the closed-loop system:

$$\Psi_i + \Sigma_i \Pi_i K_i \Theta_i^T + \Theta_i K_i^T \Pi_i^T \Sigma_i^T < 0 \tag{14}$$

where

$$\Sigma_i = \mathrm{diag}\{P_i, I, I, I, I, I, I, I, I\}, \quad \Theta_i=[\ \overline{C}_i\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$$

$$\Pi_i=[\ \overline{B}_i^T\ \ \overline{E}_{bi}^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$$

$$\Psi_i=\begin{bmatrix} \overline{A}_i^TP_i+P_i\overline{A}_i & 0 & P_i\overline{D}_{ai} & P_i\overline{D}_{bi} & P_i\overline{D}_{di} & \beta H & \overline{E}_{ai}^T & \overline{E}_{di}^T & P_i\overline{A}_{di} \\ * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix}.$$

and

$$\overline{D}_{di}=\begin{bmatrix} D_{di} & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{E}_{di}=\begin{bmatrix} E_{di} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{A}_{di}=\begin{bmatrix} A_{di} \\ 0 \end{bmatrix}, \quad H=\begin{bmatrix} I \\ 0 \end{bmatrix}.$$

By elimination lemma [8], the inequality (14) is equivalent to

$$\overline{\Pi}_i^T\, \hat{\Psi}_i\, \overline{\Pi}_i < 0, \tag{15}$$

$$\overline{\Theta}_i^T\, \Psi_i\, \overline{\Theta}_i < 0 \tag{16}$$

where $\hat{\Psi}_i=\Sigma_i^{-1}\Psi_i\Sigma_i$ and $\overline{\Pi}_i$ and $\overline{\Theta}_i$ are any matrices whose columns form bases of the null spaces of Π_i and Θ_i, respectively.

Then we can choose $\overline{\Pi}_i$ and $\overline{\Theta}_i$ as

$$\overline{\Pi}_i=\begin{bmatrix} W_{i1} & 0 \\ 0 & 0 \\ W_{i2} & 0 \\ 0 & I \end{bmatrix}, \quad \overline{\Theta}_i=\begin{bmatrix} W_{i3} & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}. \tag{17}$$

To simplify the condition (15) and (16), we shall partition P_i and P_i^{-1} as

$$P_i = \begin{bmatrix} Y_i & N_i \\ N_i^T & \dagger \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} X_i & M_i \\ M_i^T & \dagger \end{bmatrix} \qquad (18)$$

where $\dagger$ means irrelevant, and $X_i, Y_i \in R^{n_i \times n_i}$, $M_i, N_i \in R^{n_i \times k_i}$ satisfy

$$M_i N_i^T = I - X_i Y_i. \qquad (19)$$

By along similar lines to Gahinet and Apkarian [8], the conditions (15) and (16) are simplified to (10)-(11).

Remark 1. To find the controller parameter matrix K_i for subsystem S_i: first find a solution, (X_i, Y_i), of the LMIs (10), (11) and (12); and second find two full-column-rank matrices, $M_i, N_i \in R^{n_i x k_i}$, satisfying Eq. (19). Then we can find the unique P_i from the following relation [8]

$$\begin{bmatrix} Y_i & I \\ N_i^T & 0 \end{bmatrix} = P_i \begin{bmatrix} I & X_i \\ 0 & M_i^T \end{bmatrix}.$$

From this P_i, the controller parameter matrix of subsystem S_i, K_i can easily be obtained by solving the LMI (14).

Remark 2. If $Rank(I - X_i Y_i) = k_i < n$ for some X_i and Y_i satisfying (10)-(12), then the order of robust controllers given in (4), for subsystem S_i is k_i [8-9].

4. Conclusion

In this paper, we have developed an dynamic output feedback controller design for large-scale systems with time-varying uncertainties. We have obtained a sufficient condition for the existence of the controller in terms of three LMI's using Lyapunov method.

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