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On a functional central limit theorem for the multivariate linear process generated by positively dependent random vectors

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ABSTRACT

A functional central limit theorem is obtained for a stationary multivariate linear process of the form $X_t = \sum_{u=0}^{\infty} A_u Z_{t-u}$, where $\{Z_t\}$ is a sequence of strictly stationary m-dimensional linearly positive quadrant dependent random vectors with $EZ_t = O$ and $E\|Z_t\|^2 < \infty$ and $\{A_u\}$ is a sequence of coefficient matrices with $\sum_{u=0}^{\infty}\|A_u\| < \infty$ and $\sum_{u=0}^{\infty} A_u \neq O_{m\times m}$.

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1. INTRODUCTION AND MAIN RESULT

Lehmman(1966) introduced a simple and natural definition of positive dependence : A sequence $\{Y_t,\ t=1,2,\cdots\}$ of random variables is said to be pairwise positive quadrant dependent (pairwise PQD) if for any real y_i, y_j and $i \neq j$, $P\{Y_i > y_i,\ Y_j > y_j\} \geq P\{Y_i > y_i\} P\{Y_j > y_j\}$. A concept stronger than PQD was introduced by Newman(1984) : A sequence $\{Y_t\}$ of random variables is said to be linearly positive quadrant dependent (LPQD) if for any nonempty disjoint subsets $A, B \subset \{1,2,\cdots\}$ and positive r_j's $\sum_{i\in A} r_i Y_i$ and $\sum_{j\in B} r_j Y_j$ are PQD. Two m-dimensional random vectors Z_1, Z_2 are said to be positive quadrant dependent (PQD) if for any real vectors z_1, z_2

$$P(Z_1 > z_1,\ Z_2 > z_2) \geq P(Z_1 > z_1)\,P(Z_2 > z_2)$$

Let $\{Z_1, Z_2, \cdots, Z_t\}$ be a sequence of m-dimensional random vectors. We call that $\{Z_1, Z_2, \cdots, Z_t\}$ is linearly positive quadrant dependent if for any nonempty disjoint

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subsets $A, B \subset \{1, \cdots, t\}$ and for any matrix a_r with positive components,

$$\sum_{s \in A} a_s Z_s \text{ and } \sum_{r \in B} a_r Z_r \text{ are PQD.}$$

Let $\{X_t,\ t=0, \pm 1, \cdots\}$ be an m-dimensional linear process of the form

$$X_t = \sum_{u=0}^{\infty} A_u Z_{t-u} \tag{1}$$

defined on a probability space $(\Omega, \mathcal{F}, P)$, where $\{Z_t\}$ is a sequence of stationary m-dimensional LPQD random vectors with $EZ_t = 0$, $E\|Z_t\|^2 < \infty$ and positive definite covariance matrix $\Gamma : m \times m$. Throughout we shall assume that

$$\sum_{u=0}^{\infty} \|A_u\| < \infty \text{ and } \sum_{u=0}^{\infty} A_u \neq O_{m \times m}, \tag{2}$$

where for any $m \times m$, $m \geq 1$, matrix $A = (a_{ij})$, $\|A\| = \sum_{i=1}^{m} \sum_{j=1}^{m} |a_{ij}|$ and $O_{m \times m}$ denotes the $m \times m$ zero matrix.
Further, let

$$T = \left(\sum_{j=0}^{\infty} A_j\right) \Gamma \left(\sum_{j=0}^{\infty} A_j'\right),$$

where the prime denotes transpose, and the matrix $\Gamma = (\sigma_{kj})$ with

$$\sigma_{kj} = E(Z_{1k} Z_{1j}) + \sum_{i=2}^{\infty} (E(Z_{1k} Z_{ij}) + E(Z_{1j} Z_{ik})). \tag{3}$$

Let $S_n = \sum_{i=1}^{n} X_i$, $(n \geq 0)\ (S_0 = O)$, and define, for $n \geq 1$, the stochastic process ξ_n by

$$\xi_n(u) = n^{-\frac{1}{2}} T^{-\frac{1}{2}} [S_r + (\nu - r) X_{r+1}], \quad r \leq \nu < r+1, \tag{4}$$

where $r = 0, 1, \cdots, n-1$.
Fakhre-Zakeri and Lee(1993) derived a functional central limit theorem for m-dimensional linear process generated by i.i.d. random vectors.
In this paper we consider a functional central limit theorem for LPQD random vectors and prove a functional central limit theorem for an m-dimensional linear process generated by m-dimensional LPQD random vectors.

Theorem. Let $\{Z_t,\ t = 1, 2, \cdots\}$ be a stationary LPQD sequence of m-dimensional random vectors with $E(Z_t) = 0$, $E\|Z_t\|^2 < \infty$ and positive definite covariance matrix Γ as in (3) and let ξ_n be as in (4). Assume that

$$E\|Z_1\|^2 + 2\sum_{t=2}^{\infty} \sum_{i=1}^{m} E(Z_{1i} Z_{ti}) = \sigma^2 < \infty,$$

$$\sum_{t=n+1}^{\infty} E(Z_{1i} Z_{ti}) = O(n^{-\rho}) \text{ for some } \rho > 0,$$

and

$$E\|Z_t\|^s < \infty \text{ for some } s > 2.$$

Then, as $n \to \infty$,

$$\xi_n \Rightarrow W^m,$$

where $\Rightarrow$ indicates weak convergence and W^m denotes an m-dimensional Wiener proess on $C^m[0,1]$, the space of all continuous functions f defined on [0, 1] into R^m equipped with the norm $\| f \|_\infty = \max_{1 \le i \le m} \sup_{0 \le t \le 1} |f_i(t)|$.

Remark. For $m=1$ Kim and Baek(2000) showed that the process xi$_n$ converges weakly to W^1.

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