지능형 디지탈 재설계를 이용한 도립 진자의 디지탈 제어

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Digital control of inverted pendulum by using intelligent digital redesign

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Abstract

This paper presents a simple and new digital redesign algorithm for fuzzy-model-based controllers. In the first stage, a continuous-time TS fuzzy model is constructed for a given continuous-time nonlinear system and a corresponding continuous-time fuzzy-model-based controller is established based on the existing controller synthesis algorithms. In the second stage, the continuous-time fuzzy-model-based controller is converted to equivalent discrete-time fuzzy-model-based controller, aiming at maintaining the property of the analogue controlled system, which are called intelligent digital redesign. Finally, the proposed method is applied to the digital control of inverted pendulum system to shows the effectiveness and the feasibility of the method.

1 Introduction

Most practical systems, such as aircraft, electric motors and robots, are formulated in continuous-time settings. Hence, practical systems or plants are most suitably represented by continuous-time models.

Among various kinds of digital controller design algorithms, digital redesign technique is quite useful when the pre-designed analogue controller is available [1-3]. However, most applications of digital redesign methodologies reported in the literature are aimed at controlling linear systems and there are few research works on the digital redesign of nonlinear systems [4]. Joo *et al.* applied a digital redesign technique to the digital control of the chaotic Chua's circuit with TS fuzzy model and fuzzy-model-based controller [4]. The main idea of intelligent digital redesign is to derive each digitally redesigned control rule for each plant rule of the fuzzy system and construct a global digitally redesigned controller by the aggregation of the local digital controllers with fuzzy inference system. Although this allows the designer to take advantage of classical digital redesign techniques to design digitally redesigned controller for nonlinear systems, it may lead to undesirable and/or inaccurate digital redesign especially since its local digital redesign concept.

In this paper, we develop a new and simple intelligent digital redesign of fuzzy-model-based controller for continuous-time complex dynamical systems considering the global state matching property in mind. The continuous-time TS fuzzy model is first construct to represent the given nonlinear system. The continuous-time fuzzy-model-based controller is designed by using parallel distributed compensation (PDC) technique [5]. An equivalent digital controller is designed, so that the states of the digitally controlled system can globally closely match those of the pre-designed continuous-time controlled system. This new design technique provides a simple and new framework for integration of the fuzzy-model-based control theory and the advanced digital redesign technique for complex nonlinear dynamical systems. To show the effectiveness of the proposed intelligent digital redesign method, we applied it to the inverted pendulum system.

2 TS fuzzy models representing nonlinear systems

We will use the following TS fuzzy model to represent a complex, multi-input multi-output, nonlinear system:

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } F_n^i,\\ &\text{THEN } \dot{\mathbf{x}}_c(t) = \mathbf{A}_i\mathbf{x}_c(t) + \mathbf{B}_i\mathbf{u}_c(t) \quad (i = 1, 2, \cdots q)\end{aligned} \tag{1}$$

where F_j^i $(j = 1, 2, \cdots, n)$ is the fuzzy set of jth premise variable, q is the number of fuzzy rules, $\mathbf{A}_i \in \mathbb{R}^{n\times n}$, $\mathbf{B}_i \in \mathbb{R}^{n\times m}$, $z_1(t), \cdots, z_n(t)$ are the premise variables. The numerical output of the fuzzy system (1) is

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(\mathbf{A}_i x(t) + \mathbf{B}_i u_c(t)) \tag{2}$$

where $\mu_i = \frac{\prod_{j=1}^{n} F_j^i(z_j(t))}{\sum_{i=1}^{q}\prod_{j=1}^{n} F_j^i(z_j(t))}$ and $F_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in F_j^i.

We will use the fuzzy-model-based controller as shown in (3).

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } F_n^i,\\ &\text{THEN } \mathbf{u}_c(t) = -\mathbf{K}_c^i\mathbf{x}_c(t), \quad (i = 1, 2, \cdots, q)\end{aligned} \tag{3}$$

where $\mathbf{K}_c^i$ is the feedback gain in the ith subspace. The defuzzified output of fuzzy model-based controller is given by

$$\mathbf{u}_c(t) = -\sum_{i=1}^{q} \mu_i K_c^i \mathbf{x}_c(t) \tag{4}$$

In order to design a digital controller, digital modeling of continuous-time TS fuzzy model is necessary. The rule structure of the discrete-time TS fuzzy model used in this paper is

$$\begin{aligned}&\text{IF } z_1(kT) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(kT) \text{ is } F_n^i,\\ &\text{THEN } \mathbf{x}_d(kT+T) = \mathbf{G}_i\mathbf{x}_d(t) + \mathbf{H}_i\mathbf{u}_d(t) \quad (i=1,2,\cdots q)\end{aligned} \tag{5}$$

where F_j^i $(j=1,2,\cdots,n)$ is the fuzzy set, q is the number of fuzzy rules, $\mathbf{A}_i \in \mathbb{R}^{n\times n}$, $\mathbf{B}_i \in \mathbb{R}^{n\times m}$, $z_1(t),\cdots,z_n(t)$ are the premise variables. Similarly to the continuous-time case, the global dynamics of system (5) is described by

$$\mathbf{x}_d(kT+T) = \sum_{i=1}^{q}\mu_i(\mathbf{G}_i\mathbf{x}_d(kT) + \mathbf{H}_i\mathbf{u}_d(kT)) \tag{6}$$

where, $F_j^i(z_j(kT))$ is the grade of membership of $z_j(kT)$ in F_j^i. The corresponding discrete-time fuzzy-model-based controller has the following structure:

$$\begin{aligned}&\text{IF } z_1(kT) \text{ is } F_1^i \text{ and } \cdots \text{ and } z_n(kT) \text{ is } F_n^i,\\ &\text{THEN } \mathbf{u}_d(kT) = -\mathbf{K}_d^i\mathbf{x}_d(t), \quad (i=1,2,\cdots,q)\end{aligned} \tag{7}$$

where $\mathbf{K}_d^i$ is the feedback gain in the ith subspace. The defuzzified output of fuzzy model-based controller is given by

$$\mathbf{u}_d(kT) = -\sum_{i=1}^{q}\mu_i\mathbf{K}_d^i\mathbf{x}_d(kT) \tag{8}$$

3 Digital redesign of state feedback fuzzy-model-based controllers

Consider the following linear system

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}\mathbf{x}_c(t) + \mathbf{B}\mathbf{u}_c(t) \tag{9}$$

Let the system in (9) with a piecewise constant control law $\mathbf{u}_d(t)$ be

$$\begin{aligned}&\dot{\mathbf{x}}_d(t) = \mathbf{A}\mathbf{x}_d(t) + \mathbf{B}\mathbf{u}_d(t)\\ &\mathbf{u}_d(t) = \mathbf{u}_d(kT) = -\mathbf{K}_d\mathbf{x}_d(kT)\\ &\text{for } kT \le t < kT+T\end{aligned} \tag{10}$$

The closed-loop sampled-data system becomes

$$\dot{\mathbf{x}}_d(t) = \mathbf{A}\mathbf{x}_d(t) - \mathbf{B}\mathbf{K}_d\mathbf{x}_d(kT) \tag{11}$$

and the discrete-time state $\mathbf{x}_c(kT)$ is evaluated as

$$\mathbf{x}_c(kT+T) = \mathbf{G}\mathbf{x}_c(kT) - \int_{kT}^{kT+T}\exp^{\mathbf{A}(kT+T-\lambda)}\mathbf{B}\mathbf{K}_c\mathbf{x}_c(\lambda)d\lambda \tag{12}$$

The corresponding discrete-time closed-loop system (11) is

$$\mathbf{x}_d(kT+T) = (\mathbf{G} - \mathbf{H}\mathbf{K}_d)\mathbf{x}_d(kT) \tag{13}$$

where $\mathbf{G} = \Phi(kT+T,kT) = \exp^{\mathbf{A}T}$, $\Phi(t_1,t_2)$ is the state transition matrix of the continuous-time system, $\mathbf{H} = \int_{kT}^{kT+T}\Phi(kT+T,kT)\mathbf{B}\mathbf{u}_c(\tau)d\tau = (\mathbf{G}-\mathbf{I}_n)\mathbf{A}^{-1}\mathbf{B}$.

By using the block-pulse function method in [1], the integral term in (12) is approximated as

$$\begin{aligned}&\int_{kT}^{kT+T}\exp^{\mathbf{A}(kT+T-\lambda)}\mathbf{B}\mathbf{K}_c\mathbf{x}_c(\lambda)d\lambda\\ &\quad\cong \mathbf{H}\mathbf{K}_c\tfrac{1}{2}\left[\mathbf{x}_c(kT) + \mathbf{x}_d(kT+T)\right]\end{aligned} \tag{14}$$

Substituting (14) into (12) yeilds

$$\mathbf{x}_c(kT+T) \cong \left[\mathbf{I}_n + \frac{1}{2}\mathbf{H}\mathbf{K}_c\right]^{-1}\left[\mathbf{G} - \frac{1}{2}\mathbf{H}\mathbf{K}_c\right]\mathbf{x}_c(kT) \tag{15}$$

Here, our goal is to make the state $\mathbf{x}_c(kT)$ in (15) closely match the state $\mathbf{x}_d(kT)$ in (13).

$$\mathbf{G} - \mathbf{H}\mathbf{K}_d \cong \left[\mathbf{I}_n + \frac{1}{2}\mathbf{H}\mathbf{K}_c\right]^{-1}\left[\mathbf{G} - \frac{1}{2}\mathbf{H}\mathbf{K}_c\right] \tag{16}$$

Solving the above equation gives

$$\mathbf{K}_d \cong \frac{1}{2}\left[\mathbf{I}_m + \frac{1}{2}\mathbf{K}_c\mathbf{H}\right]^{-1}\mathbf{K}_c(\mathbf{I} + \mathbf{G}) \tag{17}$$

In [4], ***intelligent digital redesign*** is first introduced to find the digitally redesigned fuzzy-model-based controller for complex nonlinear systems and successfully applied to the digital control of chaotic Chua's circuit. They apply the conventional digital redesign techniques to each compensator in fuzzy controller rules. That is, the digitally redesigned controller cannot consider the global state matching between the digital controlled system and analogue controlled system. In this paper, the following digital controller is proposed. The detailed derivation of the digital control law (18) is omitted due to the limitation of pages.

$$\begin{aligned}&\mathbf{u}_d(t) = \mathbf{u}_d(kT)\\ &= \tfrac{1}{2}\left[\textstyle\sum_{i=1}^{q}\mu_i\mathbf{I}_n + \tfrac{1}{2}\mathbf{K}_c^i\mathbf{H}_i\right]^{-1}\left[\textstyle\sum_{i=1}^{q}\mu_i\mathbf{K}_c^i\right]\left[\textstyle\sum_{i=1}^{q}\mu_i\mathbf{I}_n + \mathbf{G}\right]\end{aligned} \tag{18}$$

4 Simulation

To show the effectiveness of the proposed method in designing an digital FMBC, familiar inverted pendulum system is used

$$\begin{aligned}&\dot{x}_1 = x_2\\ &\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}\end{aligned} \tag{19}$$

where, $x_1(rad)$ is the angle of the pendulum from vertical axis, $x_2(rad/s)$ is the angular velocity, $g = 9.8m/s^2$ is the acceleration due to gravity, $m = 2.0kg$ is the mass of the pendulum, $a = 1/(m+M)$, $M = 8.0kg$ the mass of the cart, $2l = 1.0m$ is the length of the pendulum, and $u(N)$ is the force applied to the cart. In [6], a TS fuzzy model to approximate the above system is obtained as follows:

$$\begin{aligned}&\text{Rule 1:}\quad \text{IF } x_1 \text{ is } F_1^1, \text{ THEN } \dot{\mathbf{x}}_c = \mathbf{A}_1\mathbf{x}_c + \mathbf{B}_1\mathbf{u}_c\\ &\text{Rule 2:}\quad \text{IF } x_1 \text{ is } F_1^2, \text{ THEN } \dot{\mathbf{x}}_c = \mathbf{A}_2\mathbf{x}_c + \mathbf{B}_2\mathbf{u}_c\end{aligned} \tag{20}$$

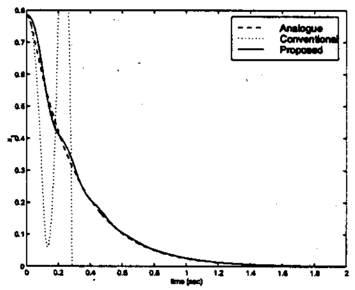

Figure 1: Comparison of $x_1(t)$

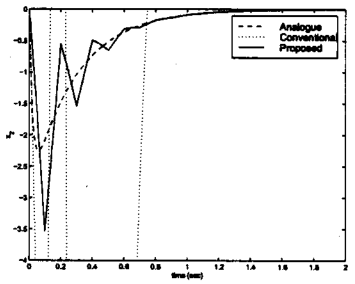

Figure 2: Comparison of $x_2(t)$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 9.36 & 0 \end{bmatrix}$$
$$\mathbf{B}_1 = \begin{bmatrix} 0 & -0.1765 \end{bmatrix}^T, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & -0.0052 \end{bmatrix}^T$$

and the membership functions are given by $F_1^1(x_1(t)) = \max(\min(\frac{x_1+1.57}{1.57}, \frac{1.57-x_1}{1.57}), 0)$ and $F_1^2(x_1(t)) = 1 - F_1^1(x_1(t))$. For digital control, sampling time $T_s = 0.01(sec)$. Figure1 shows the state $x_1(t)$ by the proposed method (solid line), the conventional method (dotted line), and the analogue controller (dashed line), respectively. Figure2 shows the state $x_2(t)$ by the proposed method (solid line), the conventional method (dotted line), and the analogue controller (dashed line), respectively. Finally, Figure3 shows the control input by the proposed method (solid line), the conventional method (dotted line), and the analogue controller.

5 Conclusion

A simple and new intelligent digital redesign technique that consider the global state matching is proposed in this paper. The design of digital controller is based on TS fuzzy model with fuzzy-model-based controller. An advantage of the proposed method over the existing conventional digital redesign technique is that the proposed method can consider the global state matching property and the digital redesign

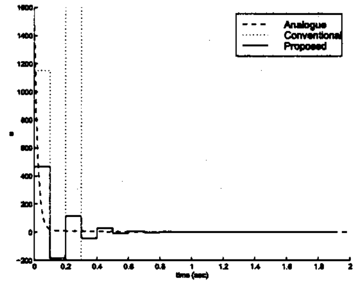

Figure 3: Comparison of $u(t)$

techniques for linear systems can easily applied to complex nonlinear systems. The proposed method has been successfully applied to the digital control of inverted pendulum system.

6 Acknowledgement

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