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재구성기법을 이용한 칼만필터 기반의 실시간 정밀 GPS 측위기법

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Real-time Precision GPS Positioning Algorithm Based on Reconfiguration Kalman Filter

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Abstract - This paper presents a practical On-The-Fly(OTF) integer ambiguity resolution algorithm for real-time precise positioning with low cost, L_1 single frequency, conventional C/A code GPS receiver. A state reconfiguration scheme is adopted in the Kalman filter to deal with the variation of ambiguity states caused by varying sets of visible GPS satellites. The proposed algorithm reduces the ambiguity search space from the coarse m-level C/A code pseudorange measurements of the conventional C/A code reciever, thereby reducing the computational time.

Simulation results are presented to show that the algorithm achieves a cm-level accuracy.

1. Introduction

For a short time span of data, kinematic carrier phase ambiguity resolution called the On-The-Fly(OTF) is the key to cm-level differential GPS positioning. It is obtainable only when the integer ambiguities is reliably and quickly determined. A carrier phase ambiguity resolution technique with static initialization was developed earlier using high precision, single frequency, C/A code receivers[1]. Use of a high performance C/A code receiver with a narrow correlator technology allows small search space for the correct integer ambiguity solution and makes it possible to carry out the carrier phase ambiguity resolution without static initialization[2].

There are several problems in applying the conventional OTF algorithms developed for post-processing to real-time applications. The degradation of signal-to-noise ratio and the multipath errors, which depend on field environment are main sources of the errors. These kind of errors can be reduced in post-processing by rejection of fault measurements. However, the excessive computational load for integer ambiguity search restricts its use in kinematic positioning applications which require output rates of 1-20Hz.

Besides the problems mentioned above, carrier phase integer ambiguity resolution algorithms for low cost, single frequency, C/A code receivers have to overcome other problems. Deficiency of L_2 frequency information makes it impossible to take advantage of the useful phase measurements combinations, such·as wide-lane. As a result, it increases the search volume, which indicates more integer ambiguity candidate sets to be searched and tested. Because the pseudorange measurements in conventional low cost, C/A code receivers have m-level accuracy, the accuracy of float integer ambiguity solution is also degraded to m-level. The search space constructed from this level of observations is too big to be applicable to real-time integer ambiguity resolution because of its computational load and highly correlated covariance matrix. Most receivers which contain OTF function operate in kinematic mode after setting initial integer ambiguity is resolved in static mode for a few minutes. These multi-epoch OTF algorithms use only carrier phase measurements and can not be applied to the medium or high dynamic maneuvering case. Therefore, in order to satisfy observability requirements, the algorithm needs other types of data such as pseudorange or doppler measurements. Also most of the integer ambiguity resolution algorithms need precise initial condition to construct initial search space from covariance matrix. Precise code data is required to build such a small search space to speed up search time. The variation of the visible GPS satellites causes the variation of the states along with time. This fact poses a challenging problem in maintaining small search space without significant disturbance.

To overcome these problems, we present a new OTF algorithm based on the Kalman filtering with a state reconfiguration scheme. The Kalman filter generates more precise float position than least-squares when the integer ambiguity search is subjected to noises containing bad data with unpredictable errors. The smaller search space constructed from the space covariance matrix of the Kalman filter gives the less integer ambiguity candidate sets. This makes it possible to use m-level pseudorange measurements for initialization of the OTF algorithm with less computational time. Since the state and covariance matrix in Kalman filter contain the past information history, it is possible to set the initial integer ambiguity search space small enough to continue estimating without much disturbance. An efficient reconfiguration scheme is adopted to deal with the variation of integer ambiguity states caused by the varying sets of the visible satellites.

2. Kalman Filter Design

2.1 Kalman Filter for Integer Ambiguity Resolution

The observation equations of double differenced L_1 frequency code pseudorange and integrated carrier phase can be written as

$$\nabla\Delta P_{AB}^{ij}(t) = \nabla\Delta\rho_{AB}^{ij}(t) + \nabla\Delta e_{P_{AB}}^{ij}(t), \tag{1}$$

$$\nabla\Delta\Phi_{AB}^{ij}(t) = \nabla\Delta\rho_{AB}^{ij}(t) + \lambda_1 \cdot \nabla\Delta N_{AB}^{ij} + \nabla\Delta e_{\Phi_{AB}}^{ij}(t), \tag{2}$$

where

$\nabla\Delta(\bullet)_{AB}^{ij} = (\bullet)_B^i - (\bullet)_B^j - (\bullet)_A^i + (\bullet)_A^j$: the double difference operator,

P : code pseudorange measurement,

Φ : integrated carrier phase measurement,

ρ : geometric range between satellite and receiver,

N : integer ambiguity,

λ_1: wave length of L1 frequency(≈19cm),

e_P, e_Φ : errors of pseudorange and carrier phase, respectively,

i,j : satellite identifier,
A,B : receiver identifier.

In kinematic GPS precise positioning system, the rover dynamics are often described by a constant velocity model and the carrier phase ambiguities are treated as parameters of real number[4,5]. The system dynamics and the nonlinear measurement equations of the extended Kalman filter can be represented as

$$X_{k+1} = F_k \cdot X_k + \varpi_k , \qquad \omega_k \sim N(0,Q_k) \tag{3}$$

$$z_k = h(X_k) + \upsilon_k , \qquad \upsilon_k \sim N(0,R_k) \tag{4}$$

where

$$X_k = [x_k, y_k, z_k, \dot{x}_k, \dot{y}_k, \dot{z}_k, n_{21}, n_{32}, \cdots, n_{m(m-1)}]^T,$$

$$F_k = \begin{bmatrix} I_{3\times3} & \Delta t \cdot I_{3\times3} & \underline{0}_{3\times(m-1)} \\ \underline{0}_{3\times3} & I_{3\times3} & \underline{0}_{3\times(m-1)} \\ \underline{0}_{(m-1)\times3} & \underline{0}_{(m-1)\times3} & I_{(m-1)\times(m-1)} \end{bmatrix},$$

$$z_k = \begin{bmatrix} \nabla\Delta P_{AB}{}^T & \frac{\nabla\Delta\Phi_{AB}{}^T}{\lambda_1} \end{bmatrix}^T .$$

The state prediction and the correction equations are given by

$$\hat{X}_{k+1} = F_k \hat{X}_k \tag{5a}$$

$$P_{k+1} = F_k P_k F_k{}^T + Q_k \tag{5b}$$

$$\hat{X}_k = \hat{X}_k^- + K_k (z_k - \hat{z}_k) \tag{6a}$$

$$K_k = P_k H_k{}^T (H_k P_k^- H_k{}^T + R_k)^{-1} \tag{6b}$$

$$P_k = (I - K_k H_k) P_k^- \tag{6c}$$

where H is a design matrix of Kalman filter, Q means process noise covariance, and R represents measurement noise covariance.

The use of Kalman filter in the carrier phase ambiguity resolution have several advantages compared to least squares. The float ambiguity solution of Kalman filter is more accurate than that of least squares and the state covariance matrix becomes smaller along with time. consequently, the search space becomes smaller and the number of integer ambiguity sets to be tested decreases resulting in much less computational load. This is due to the fact that the covariance matrix of Kalman filter contains all information of past data, i.e., the use of Kalman filter for one epoch data is equivalent to that of least squares for multi-epoch data. In addition, measurement residuals in correction process directly indicates the reliability factor of the solution. These advantages make it possible to resolve integer ambiguities without static initialization.

2.2 Reconfiguration Scheme

In double difference observations, one major consideration is to define the observation differencing strategy associated with particular satellites for a given baseline. Suppose the common satellites observed by the reference and roving receivers are 1, 2,..., and m. From this data set the user can define differencing strategies in either of the two fashions as described in Table 1[3,7]. Fixed-reference satellite differencing, indicated by set 1, usually generates smaller bias errors than the scheme with set 2. But the former method has a potential problem when an epoch of data is encountered in which the data from the reference satellite 1 is absent. This causes a serious problem in real-time kinematic positioning application. In this paper, the sequential differencing approach(set 2) is selected because it is known to lead faster ambiguity resolution. Nevertheless the late method requires special treatment when the data from several satellites previously exist disappears or data from new visible satellites emerges. This causes a change of integer ambiguity states in Kalman filter associated with the lost or new satellites. In this paper, a reconfiguration Kalman filter scheme to reconstruct the optimal state and covariance matrix is introduced.

<u>Problem</u> : Let X and P_x be a vector of random variables and its covariance matrix, respectively. Then Y, a linear combination of X, is also a vector of random variables whose covariance matrix(P_Y) is given by

$$Y = A \cdot X \tag{7-1}$$

$$P_Y = A \cdot P_X \cdot A^T \tag{7-2}$$

The problem is to find a transformation matrix A.

<u>Example 1 (lost satellite)</u> : Suppose that the common satellites observed by two stations are 1, 2, 3, 4, and 6 at epoch t_0 and then satellite 3 is lost at epoch t_1. By sequential differencing method, the double differenced integer ambiguities at t_0 and t_1 are written as

$$N_0 = [n_{21}, n_{32}, n_{43}, n_{64}]^T , \qquad N_1 = [n_{21}, n_{42}, n_{64}]^T . \tag{8}$$

In Eqs (8), the new element, n_{42}, at epoch t_1 can be expressed by the existing elements of state at epoch t_0 as

$$n_{42} = n_4 - n_2 = (n_4 - n_3) + (n_3 - n_2) = n_{43} + n_{32} . \tag{9}$$

From Eqs. (8) and Eq. (9), a matrix A, which transforms N_0 to N_1, can be defined as follows·

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In general case, a transformation matrix, A, has following the structure:

$$A = \begin{bmatrix} \mathrm{O} & & & & & & & \\ & 1 & & & & & & \\ & & \mathrm{O} & & & & & \\ & & & 1 & \Lambda & 1 & & \\ & & & & & & \mathrm{O} & & \\ & & & & & & & 1 & \\ & & & & & & & & \mathrm{O} \end{bmatrix}.$$

Using Eqs (7-1) and (7-2), the predicted integer ambiguity states and corresponding covariance matrix in Kalman filter at epoch t_1 can be obtained.

<u>Example 2 (new satellite encountered)</u> : Suppose that the common satellites are 1, 2, 4, and 6 at epoch t_1 and then new satellite 5 is encountered at epoch t_2. The integer ambiguity state at t_2 is

$$N_2 = [n_{21}, n_{42}, n_{54}, n_{65}]^T . \tag{10}$$

If this integer ambiguity state in Eq. (10) is compared with the state at epoch t_1, there are two new integer ambiguities, n_{54} and n_{65}. This means that one satellite is associated with two elements of integer ambiguity state. But if we devide satellites into two groups as existing and new satellites groups and reorder the sequence of double differencing, it is possible to reduce the two new elements of state to one as follows

$$N_2 = [n_{21}, n_{42}, n_{64}, n_{36}]^T \quad (11)$$

Expanding this result to a general case when n new satellites are encountered, the state and the covariance matrix at that epoch can be expressed as

$$\begin{bmatrix} Y \\ Y_{new} \end{bmatrix}, \quad \begin{bmatrix} P_y & 0 \\ 0 & \alpha \cdot I_{n \times n} \end{bmatrix}. \quad (12)$$

where Y_{new} denotes the double differenced integer ambiguity set associated with the new satellites and α is the predefined value for initialization. The flow chart of the presented algorithm is presented in Figure 1. The field test results of this proposed algorithm are presented in References[8,9].

3. Simulation

The proposed algorithm has been tested via simulation to compare its performance with a least-squares OTF algorithm. Figure 2 depicts the flow chart of the simulation based on RINEX input/output format files. The use of RINEX format in simulation makes it possible to reduce the redundant effort and time of conversion from simulation program to that of post-processing. It is assumed that the reference receiver is located at the (-3060929.0, 4055654.0, 3842493.0) in meters in ECEF coordinates and the roving receiver is located at 100 meters apart from the reference in north and east directions, respectively, and 20 meters in vertical direction above the horizon. The realistic random and bias range error components such as ionospheric and tropospheric delay, multipath and thermal noise are added to the range and phase measurements. More than seven satellites are visible to the user during the simulation period(Figure 3(a)). Figures 3(b)~3(d) show the result of static mode simulation. The diagonal components of covariance matrix is used as a check flag whether the search step is needed; i.e., if the search time is larger than the predefined threshold value, the algorithm skips the search and outputs the float solution. It is seen in the figure that, the positioning output of Kalman filter converges to the truth without much disturbance when the number of satellites change.

The second part of the simulation was performed for a kinematic case where roving part moves at speed about 4m/s along the path of a slanted rectangle. The simulation results are depicted in Figure 4. While the roving receiver moves, the algorithm performs the integer ambiguity resolving process without static initialization. As shown in the figure, after 100 seconds, Kalman filter resolves the integer ambiguity correctly, and the position error is reduced to a cm-level. The root-mean-squared positioning errors of Kalman filter are 0.18 and 0.29 in meters in static and kinematic simulations, respectively, compared to 2.13 and 2.48 obtained from the least-squares.

4. Conclusions

In this paper, a new OTF algorithm for precise GPS positioning is proposed and evaluated. The new algorithm is obtained from the conventional Kalman filter with reconfiguration scheme to deal with the variation of states correspond to the varying sets of the visible satellites. This algorithm is ideally suited for precise real-time positioning with a low cost, single frequency, conventional C/A code receiver in a wide range of dynamics.

[References]

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Table 1. Example of double difference set.

Set 1	Set 2
1-2	2-1
1-3	3-2
:	:
1-m	m-(m-1)

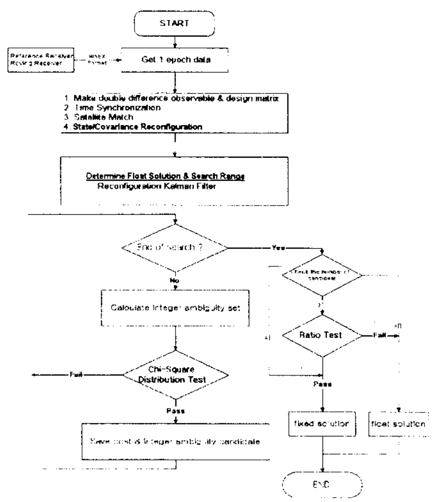


Fig. 1. Flowchart of Reconfiguration Kalman Filter

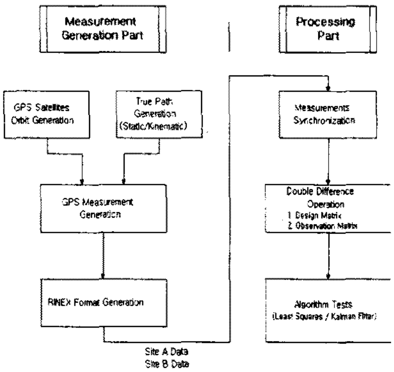
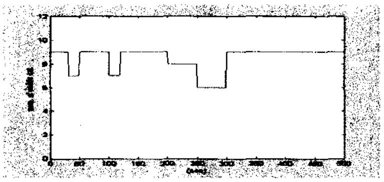
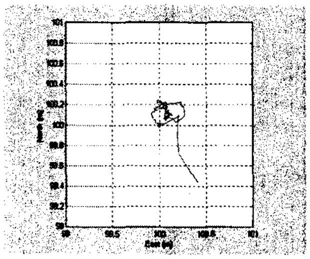


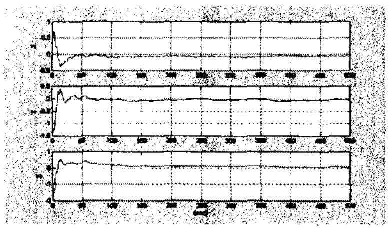
Fig. 2. Flowchart of Simulation



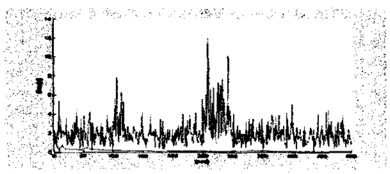
(a)



(b)

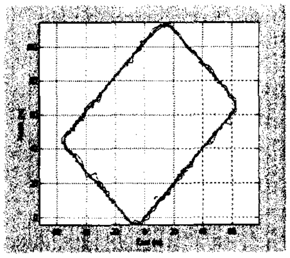


(c)

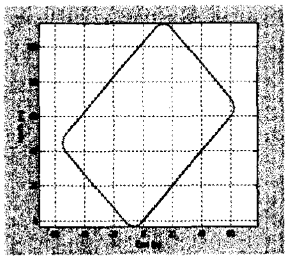


(d)

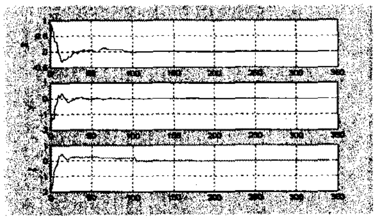
Fig. 3. Static mode simulation results
(a) Variation of the number of the visible satellites
(b) Two dimensional positioning result of Kalman filter
(c) Positioning error of Kalman filter(ECEF)
(d) Difference in errors between the LSE and the Kalman filter



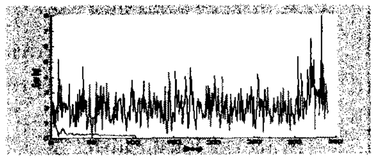
(a)



(b)



(c)



(d)

Fig. 4. Kinematic mode simulation results
(a) Two dimensional positioning result of LSE
(b) Two dimensional positioning result of Kalman filter
(c) Position errors of Kalman filter
(d) Difference in errors between the LSE and the Kalman filter