2000년도 대한전기학회 하계학술대회 논문집 2000.7.17-20

# 이산 시간 TS 퍼지 모델 기반 제어기 설계

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# Design of Discrete-Time TS Fuzzy-Model-Based Controller

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## Abstract

In this paper, a control technique of Takagi-Sugeno (TS) fuzzy systems with parametric uncertainties is developed. The uncertain TS fuzzy system is represented as an uncertain multiple linear system. The control problem of TS fuzzy system is converted into the stabilization problem of a uncertain multiple linear system. A sufficient condition for robust stabilization is obtained in terms of linear matrix inequalities (LMI). A Design example is illustrated to show the effectiveness of the proposed method.

## 1 Introduction

Many frameworks in real world have hard nonlinearity and uncertainty, so a lot of control techniques have been developed and the fuzzy control is one of the major nonlinear control theories. However, the main drawback of fuzzy control is that it is difficult to analyze the stability of a fuzzy system. The Takagi-Sugeno (TS) fuzzy model is widely used, since it is possible to apply the systematic linear control theory to design a controller.

Cao et. al. develop the switching type controller design technique by applying the multiple linear system theory [4]. However, the uncertain matrices in their paper does not represent real uncertainties in the plant model. Thus, it needs to consider real uncertainties in multiple linear system approaches for fuzzy controller design.

In this paper, We first develop the uncertain multiple linear system which represents the discrete-time TS fuzzy system with parametric uncertainties. The sufficient condition of robust stabilization with guaranteed-cost is derived and formulated in linear matrix inequalities (LMI) framework. The advantage of the studied results in this paper are verified from the computer simulation of the truck trailer system with parametric uncertainties.

## 2 Preliminaries

Consider a discrete-time uncertain nonlinear system of the form:

$$x(t+1) = f(x(t)) + \Delta f(x(t)) + (g(x(t)) + \Delta g(x(t)))u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $f(x(t))$ and $g(x(t))$ are nonlinear vector functions, $\Delta f(x(t))$ and $\Delta g(x(t))$ are uncertain nonlinear vector functions. The uncertain nonlinear system (1) can be modeled as the following TS fuzzy system:

Plant Rule $i$

If $x_1(t)$ is $\Gamma_1^i$ and $\cdots$ and $x_n(t)$ is $\Gamma_n^i$
THEN $x(t+1) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)$ (2)

where $\Gamma_j^i$ $(j = 1, \ldots, n, \quad i = 1, \ldots, q)$ is the fuzzy set, Rule $i$ denotes the $i$th fuzzy inference rule. $\Delta A_i, \Delta B_i$ are time varying matrices with appropriate dimension, which represent uncertainties in the TS fuzzy system. The defuzzified output of this TS fuzzy system (2) is represented as follows:

$$x(t+1) = \sum_{i=1}^{q} \mu_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)). \tag{3}$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_j^i(x_j(t)), \qquad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q} \omega_i(x(t))}.$$

in which $\Gamma_i^j(x_j(t))$ is the grade of membership of $x_j(t)$ in $\Gamma_i^j$. Hereforth we assume, as usual, that the uncertain matrices $\Delta A_i$ and $\Delta B_i$ are admissibly norm-bounded and structured.

**Assumption 1** *The parameter uncertainties considered here are norm-bounded, in the form*

$$\begin{bmatrix}\Delta A_i & \Delta B_i\end{bmatrix} = D_i F_i(t) \begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix},$$

*where $D_i, E_{1i}$, and $E_{2i}$ are known real constant matrices of appropriate dimensions, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i(t)^T F_i(t) \le I$, in which $I$ is the identity matrix of appropriate dimension.*

In $i$th subspace divided by the fuzzy membership functions, the TS fuzzy system has much highly nonlinear interaction among the fuzzy rules, which complicates the analysis and control of the TS fuzzy system [4]. In order to get rid of these theoretical difficulties, we represent the uncertain TS fuzzy system as an uncertain multiple linear system with the following subspace [2].

$$\Theta_i = \{x(t) | \mu_i(x(t)) \ge \mu_j(x(t)), \quad j = 1, 2, \ldots, q, \quad i \ne j\} \quad i = 1, 2, \ldots, r. \tag{4}$$

The characteristic function of $\Theta_i$ is defined by

$$\eta_i = \begin{cases} 1, & x(t) \in \Theta_i \\ 0, & x(t) \notin \Theta_i \end{cases}, \qquad \sum_{i=1}^{r} \eta_i = 1. \tag{5}$$

Then, on every subspace the fuzzy system (2) can be represented with an uncertain multiple linear system as follows:

$$x(t+1) = \sum_{i=1}^{r} \eta_i(x(t)) \left( (A_i + \Delta A_i + \Delta \mathbf{A}_i) x(t) + (B_i + \Delta B_i + \Delta \mathbf{B}_i) \right) u(t). \tag{6}$$

where

$$\Delta \mathbf{A}_i = \sum_{j=1, j\ne i}^{r} \mu_j(x(t)) \Delta \mathbf{A}_{ij},$$
$$\Delta \mathbf{B}_i = \sum_{j=1, j\ne i}^{r} \mu_j(x(t)) \Delta \mathbf{B}_{ij},$$
$$\Delta \mathbf{A}_{ij} = A_j + \Delta A_j - A_i - \Delta A_j,$$
$$\Delta \mathbf{B}_{ij} = B_j + \Delta B_j - B_i - \Delta B_j. \quad i = 1, 2, \ldots, q.$$

## 3 Problem Statement

This section deals with the controller design problem for the discrete-time TS fuzzy system with parametric uncertainties. The state-space representation of the fuzzy system can be described as follows:

$$x(t+1) = \sum_{i=1}^{q} \mu_i(x(t)) (A_i + \Delta A_i) x(t) + (B_i + \Delta B_i) u(t)), \tag{7}$$

This TS fuzzy system can be represented by the uncertain multiple linear system of the form:

$$x(t+1) = \sum_{i=1}^{r} \eta_i(x(t)) (A_i + \Delta A_i + \Delta \mathbf{A}_i + (B_i + \Delta B_i + \Delta \mathbf{B}_i) u(t)) \tag{8}$$

The objective in this section is to design a state feedback controller of the following form:

$$u(t) = \sum_{i=1}^{r} \eta_i(x(t)) K_i x(t) \tag{9}$$

The main result on the robust control of the discrete-time TS fuzzy system in the presence of the parametric uncertainties is summarized in the following theorem.

**Theorem 1** *If there exist a symmetric positive definite matrices, $P_i$, a symmetric positive definite matrix, $Q$, and matrices, $K_i$ such that the following LMIs are satisfied, then the TS fuzzy system (7) is asymptotically stabilizable via TS fuzzy-model-based controller (9) in the presence of admissible parametric uncertainties with guaranteed-cost.*

$$\begin{bmatrix} -W_i & * & * & * & * & * & * \\ A_i W_i + B_i M_i & -W_i & * & * & * & * & * \\ \hat{E}_{1i} W_i + \hat{E}_{2i} M_j & 0 & -\epsilon_i I & * & * & * & * \\ \tilde{E}_{1i} W_i + \tilde{E}_{2i} M_i & 0 & 0 & -\epsilon_i I & * & * & * \\ 0 & \hat{D}_i^T & 0 & 0 & -\epsilon_i^{-1} I & * & * \\ 0 & \tilde{D}_i^T & 0 & 0 & 0 & -\epsilon_i^{-1} I & * \\ W_i & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0, \tag{10}$$
$$i = 1, 2, \ldots, r.$$

*where*

$$\tilde{D}_i = \begin{bmatrix} \mu_{1max} D_1 & \mu_{2max} D_2 & \cdots & \mu_{qmax} D_q \end{bmatrix},$$
$$\tilde{E}_{1i} = \begin{bmatrix} E_{11} \\ E_{12} \\ \vdots \\ E_{1q} \end{bmatrix}, \quad \tilde{E}_{2i} = \begin{bmatrix} E_{21} \\ E_{22} \\ \vdots \\ E_{2q} \end{bmatrix},$$
$$\hat{D}_i = \begin{bmatrix} \mu_{1max} I & \cdots & \mu_{i-1max} I & \mu_{i+1max} I & \mu_{qmax} I \end{bmatrix},$$
$$\hat{E}_{1i} = \begin{bmatrix} E_{11} \\ E_{12} \vdots \\ E_{1q} \end{bmatrix}, \quad \hat{E}_{2i} = \begin{bmatrix} E_{21} \\ E_{22} \vdots \\ E_{2q} \end{bmatrix},$$

*and $W_i = P_i^{-1}$, $M_i = K_i P_i^{-1}$, and $*$ denotes the transposed elements in the symmetric positions.*

*proof* : The proof is omitted due to lack of space.

## 4 An Example

We now apply the above design technique to the control of a computer simulated truck trailer. We use the following truck trailer model formulated in [5]

$$x_1(t+1) = (1 - v \cdot t/L)x_1(t) + v \cdot t/l \cdot u(t)$$
$$x_2(t+1) = x_2(t) + v \cdot t/L \cdot x_1(t)$$
$$x_3(t+1) = v \cdot t \cdot \sin(x_2(t) + v \cdot t \cdot x_1(t)/2L) + x_3(t)$$

The following fuzzy model is used to design a fuzzy controller:

$$R^1 : \text{If } x_2(t) + v \cdot t/L \cdot x_1(t) \text{ is about } 0$$
$$\text{Then}, x(t+1) = A_1 x(t) + B_1 u(t)$$
$$R^2 : \text{If } x_2(t) + v \cdot t/L \cdot x_1(t) \text{ is about } \pi$$
$$\text{Then}, x(t+1) = A_2 x(t) + B_2 u(t)$$

where

$$A_1 = \begin{bmatrix} 1 - v \cdot t/L & 0 & 0 \\ v \cdot t/L & 0 & 0 \\ (v \cdot t)^2/2L & v\dot{t} & 1 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 1 - v \cdot t/L & 0 & 0 \\ v \cdot t/L & 0 & 0 \\ g \cdot (v \cdot t)^2/2L & g \cdot v\dot{t} & 1 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} v \cdot t/l \\ 0 \\ 0 \end{bmatrix}$$

and $g = 1/100\pi, l = 0.2, L = 0.32, v = -0.1, t = 0.5$, and $g = 10^{(} - 2)/pi;$. The membership functions are

$$\mu_1(x(t)) = \frac{\sin(x_2 + v \cdot t/2L \cdot x_1(t)) - g \cdot x_2 + v \cdot t/2L \cdot x_1(t)}{(x_2 + v \cdot t/2L \cdot x_1(t))(1-g)}$$
$$\mu_2(x(t)) = 1 - \mu_1(x(t))$$

Based on Theorem 1, we obtain

$$P_1 = 1.0\text{e} - 4 \begin{bmatrix} 0.1108 & -0.0229 & 0.0000 \\ -0.0229 & 0.0217 & 0.0000 \\ 0.0000 & 0.0000 & 0.0009 \end{bmatrix},$$
$$P_2 = 1.0\text{e} - 4 \begin{bmatrix} 0.1132 & -0.0234 & -0.0000 \\ -0.0234 & 0.0225 & 0.0000 \\ -0.0000 & 0.0000 & 0.0009 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 2.2806 & -0.4016 & -0.0002 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 2.3171 & -0.4090 & 0.0002 \end{bmatrix},$$
$$Q = 1.0\text{e} - 7 \begin{bmatrix} 0.5811 & -0.1099 & 0.0017 \\ -0.1099 & 0.3248 & 0.0037 \\ 0.0017 & 0.0037 & 0.4021 \end{bmatrix}.$$

We consider the uncertainties of the backward velocity $v$, that is, $v$ is assume to have 10% variation of its nominal value. Figure 1 shows the computer simulation results. From Fig. 1, the backward movement control based on the proposed technique in this paper is excellent.

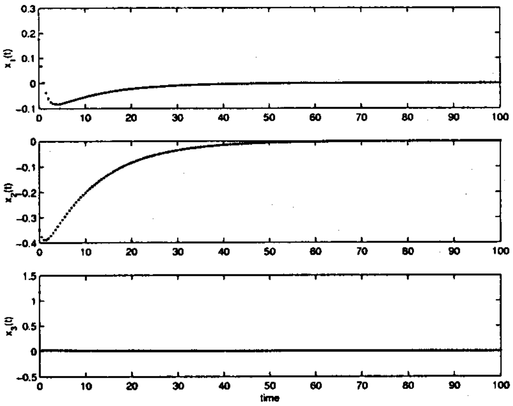

Figure 1: The controlled system response in the presence of parametric uncertainties

# 5 Conclusion

In this paper, the robust controller design technique for TS fuzzy system with parametric uncertainties is presented. The stabilization problem of uncertain TS fuzzy system was converted into the stabilization problem of the uncertain multiple linear system. The sufficient condition was formulated in LMI framework. The simulation example ensured us the feasibility of the developed design technique.

**Acknowledgement**

본 연구는 기초전력공학연구소의 지원(과제번호: 99-009)에 의하여 수행되었습.

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