Implication and Rational Application of Equivalent Load Method in Prestressed Concrete Structures

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ABSTRACT

The equivalent load method has been widely used in the design and analysis of prestressed concrete structures. The purpose of this paper is to explore several important methods of obtaining equivalent loads, and to clarify the advantages and limitations of each method. The methods devised in this study include the use of curvature of tendon, characteristics of primary moment, self-equilibrium condition, and linear segments approximation of tendon. It is shown that equivalent loading system is not uniquely determined in some cases and careful engineering judgement is required from the view point of accuracy and practical convenience.

1. INTRODUCTION

With the progress of analysis technique for prestressed concrete structures, many studies to investigate the effect of prestressing tendon on the structural behavior have been conducted.

Concerning the modeling of tendon in prestressed concrete structures, two alternative approaches are generally used. One is to deal with tendon as elements in the finite element analysis of prestressed concrete structures. The other is to evaluate only the forces exerted to structure by prestressing tendon, neglecting stiffness of tendon. The second approach can be classified into several categories, the representative of which is the equivalent load method that considers prestressing force exerted to the structure as external force. By employing this concept, one can obtain a clear picture of the forces acting in the prestressed concrete structures and it allows the designer to use familiar beam and frame analysis. That is, the analysis of prestressed concrete member is converted to that of nonprestressed member.

The main purpose of the present study is to devise several important methods of calculating equivalent loads, and to clarify the advantages and limitations of various methods studied.

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2. GENERAL PROCEDURES TO CALCULATE EQUIVALENT LOADS AND APPROXIMATIONS USED

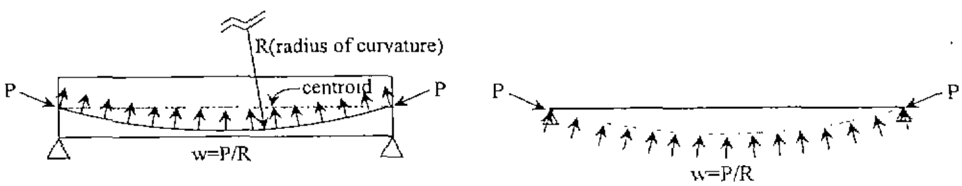
2.1 Equivalent Load Calculated by Curvature of Tendon

The equivalent load depends *only on the geometry of tendon, precisely speaking, curvature of* tendon irrespective of shape of the member and support condition. The distributed load exerted by the curvature of tendon at the contact face between tendon and surrounding concrete can be written as, $w = P/R = \kappa P$, with the direction normal to the tendon axis. Where, P is the prestressing force, R is the radius of curvature, and κ is the curvature.

This method is convenient to use for the tendon with constant curvature. Fig. 1(a) shows the exact equivalent loads in the simple beam with circular tendon profile. However, the problem is *now how one can accommodate these equivalent loads in the beam analysis.* The best answer to this is to deal with equivalent loads as it is without any modification in magnitude and direction already calculated. Fig. 1(b) shows the idealized simple beam in question with exact equivalent loads applied.

However, in many cases including above example, the equivalent load method is not used in a strict sense due to the practical considerations for the convenience of structural analysis. Fig. 2 is a generally accepted analysis model of Fig. 1(b) which transferred original equivalent distributed load to the location of centroidal axis with its magnitude maintained but direction slightly changed. Moreover, the direction of anchorage force P is often assumed to be horizontal. These approximations result in the following problems.

First, the equivalent loads do not satisfy self-equilibrium condition, which must be met from the consideration of statics. As a result, some reactions of small magnitude at the supports occur, which do not exist in the simple prestressed beam in reality Secondly, the section forces produced by approximate equivalent loads are also approximate ones.



(a) Equivalent loads in a strict sense (b) Idealized beam with exact equivalent loads

Fig. 1-Simple beam with circular tendon profile

Fig. 2-Approximate equivalent loading system for the simple beam of Fig. 1(a)

2.2 Equivalent Load Calculated by Primary Moment

When the curvature of tendon varies from point to point, the method 1 is not practical and the following method, especially for the polynomial type tendon, can be used instead. The primary moment can be defined as, $M_p = (P\cos\theta)e \approx Pe$, for small θ value.

This is very effective assumption in the sense that the bending moment diagram of M_p has the same shape as eccentricity with only difference being magnitude. One can obtain the equivalent loading diagram by differentiating the primary moment twice. Note that according to this procedure, the parabolic eccentric corresponds to uniform equivalent load, and the cubic eccentricity to linearly varying one, and so on.

Consider the simple beam with parabolic tendon profile. With the coordinate definition shown in Fig. 3, the eccentricity can be written as $e = 4hx(1-x/L)/L$, which results in $w = 8Ph/L^2$. This produces exact bending moment at the mid-span, but approximate one at other locations. This is due to the fact that $M_p = Pe$ is true expression at the mid-span and approximate one at other locations.

Next alternative procedure can also be used for the parabolic tendon, which is based on the above discussions. It is assumed that the equivalent load is applied at the centroidal axis of a beam with constant magnitude in the vertical direction. Then, the magnitude is determined so that it produces exact bending moment at the mid-span.

Self-equilibrium condition and section forces produced by the method 2 are only approximate expressions, because the above procedure has the same effect as slightly modifying the magnitude and direction of exact equivalent load.

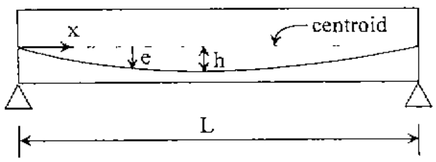


Fig. 3-Simple beam with parabolic tendon profile

2.3 Equivalent Load Calculated by Self-Equilibrium Condition

This approach evaluates the equivalent load by equilibrium condition with anchorage forces given. Therefore, the self-equilibrium condition, which is satisfied in an approximate sense in the methods 1 and 2, is exactly satisfied, although section forces are still approximate values.

Two equilibrium equations can be used to find equivalent transverse load in the plane beam, i.e., vertical force equilibrium and moment equilibrium. One should try to find out the equivalent loads that satisfy these two equilibrium equations with the anchorage forces given at both ends of the beam. This may raise a question, because there exist many load cases that satisfy equilibrium condition with anchorage forces. Therefore, it is necessary that one should obtain some information of equivalent loads from the geometry of tendon with two unknowns left before detailed calculations.

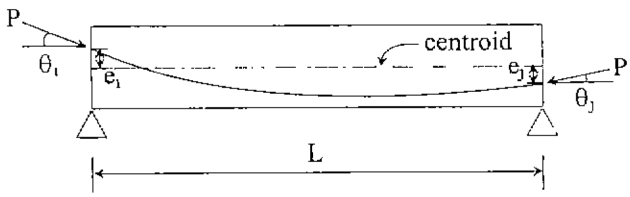
Consider the member with cubic tendon profile and anchorage forces given in Fig. 4(a). It is already known that assuming constant horizontal prestressing force, the cubic tendon profile corresponds to linearly varying equivalent load from the method 2. Therefore, it is reasonable to set the magnitudes of linearly varying load at both ends of the member as unknowns, and solve for these two unknowns by two equilibrium equations. Referring to Fig. 4(b) with positive sign conventions shown, the following results are derived from the equilibrium conditions.

$$w_i = -\frac{2}{L}(2P\sin\theta_i - P\sin\theta_j) - \frac{6}{L^2}(e_i P\cos\theta_i + e_j P\cos\theta_j) \quad (1)$$

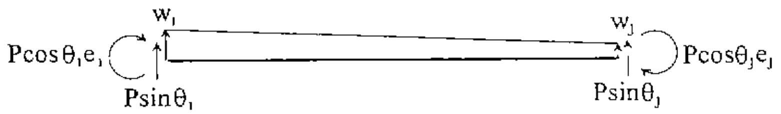
$$w_j = \frac{2}{L}(P\sin\theta_i - 2P\sin\theta_j) + \frac{6}{L^2}(e_i P\cos\theta_i + e_j P\cos\theta_j) \quad (2)$$

Note that cubic tendon profile also includes parabolic shape, which would result in equivalent uniform load. However, equivalent transverse load corresponding to tendon profile higher than cubic order cannot be uniquely determined by only two equilibrium equations, because the number of unknowns exceeds two

When the several kinds of profiles are used in one tendon as in the continuous beam, the beam should be divided into segments so that each segment has unique polynomial expression. After that, the method 3 can be applied to each segment to find corresponding equivalent load, respectively.



(a) Simple beam with tendon



(b) Forces applied to the beam in relation to the vertical equilibrium

Fig. 4-Simple beam with cubic tendon profile

2.4 Equivalent Load by Linear Segments Approximation of Tendon

According to the method 4, the member is divided into segments and the curved tendon is assumed to be straight tendon connecting two intersection points in each segment. Then, the equivalent load exists as the form of concentrated load only at the interface of two segments meeting each other and the equivalent distributed load arising from the curvature of tendon disappears. This method is also used when the tendon is implemented as elements in finite element method. As can be expected, to simulate curved geometry of tendon by this method to some extent, many segments are required depending on the problem.

3. CONCLUSIONS

The equivalent load method has wide applications in the design and analysis of prestressed concrete structures and is recognized as simple yet powerful one. The purpose of this paper is to explore several important methods of obtaining equivalent loads, and to clarify the advantages and limitations of each method. The methods devised in this study include the use of curvature of tendon, characteristics of primary moment, self-equilibrium condition, and linear segments approximation of tendon. The present study emphasizes that careful engineering judgement is required to obtain realistic equivalent loading system for the prestressed concrete members

REFERENCES

1. Aalami, B. O., "Structural Modeling of Posttensioned Members," *Journal of Structural Engineering*, V. 126, No. 2, Feb. 2000, pp. 157-162
2. Aalami, B. O., "Load Balancing: A Comprehensive Solution to Post-Tensioning," *ACI Structural Journal*, V. 87, No. 6, Nov.-Dec. 1990, pp. 662-670
3. Naaman, A. E., *Prestressed Concrete Analysis and Design: Fundamentals*, McGraw-Hill, New York, 1982, pp. 360-364
4. Lin, T. Y., and Burns, Ned H., *Design of Prestressed Concrete Structures*, 3rd Edition, John Wiley & Sons, New York, 1981, pp. 392-427