

A Deterministic Channel Simulation Model Generating Spatiotemporally Correlated Fading Waveforms

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ABSTRACT

We propose a deterministic vector channel simulation model satisfying not only rigorous temporal correlation but also arbitrary spatial correlation using the method of Doppler phase difference sampling. The model is more efficient than the conventional PN filtered Gaussian model with coloring process in evaluating the laboratory performance of mobile communication systems employing adaptive array antennas or space diversity.

I. INTRODUCTION

Antenna array structures such as adaptive array antennas and space diversity technique have emerged to enhance the capacity of mobile communication network. In order to analyze the performance of antenna array systems, the fading channel simulator generating spatiotemporally correlated fading waveforms must be employed, since the multiple fading waveforms on each element have spatial correlation as well as temporal correlation. Recently, *spatiotemporally correlated fading model*, which implements temporally correlated processes by means of Doppler filter and spatially correlated processes by means of coloring matrix obtained from a desired spatial correlation matrix, has been proposed [1]. The model is based on the conventional PN filtered Gaussian model. Though the fading waveforms generated by the model in [1] satisfy the rigorous spatiotemporal correlation characteristics, the high order Doppler filters and the interpolators, which are essential to enhance the temporal correlation characteristics, make it difficult to apply to the digital simulation. Deterministic models such as Jakes' model [2] and Patzold's models [3] have been proposed to solve such problems that occur in the PN filtered Gaussian model. Moreover, a deterministic model, which generates spatiotemporally correlated fading waveforms by the vector sum of the complex sinusoids with Doppler

frequencies experienced by each scatterers, is proposed [4].

This paper proposes a novel deterministic channel model satisfying not only rigorous temporal correlation but also arbitrary spatial correlation by Doppler phase difference sampling. No use of the Doppler filters and interpolators gives the model computing efficiency in digital simulation

II. DETERMINISTIC CHANNEL MODEL

Assuming narrow-band signals whose signal bandwidth is very smaller than the carrier frequency, the received signal for an unmodulated carrier can be represented as $\alpha(t) = \alpha_r(t) + j\alpha_i(t)$, which is known as fading waveform, by baseband equivalent model. And the following sum of sinusoids approximates $\alpha(t)$ by the deterministic channel model [3].

$$\begin{aligned}\tilde{\alpha}_r(t) &= \sum_{k=1}^N C_{r,k} \cos(2\pi f_{r,k} t + \phi_{r,k}) \\ \tilde{\alpha}_i(t) &= \sum_{k=1}^N C_{i,k} \cos(2\pi f_{i,k} t + \phi_{i,k}).\end{aligned}\quad (1)$$

where N designates the number of sinusoids and the parameters such as Doppler frequencies $f_{r,k}$, $f_{i,k}$, Doppler coefficients $C_{r,k}$, $C_{i,k}$, and Doppler phases $\phi_{r,k}$, $\phi_{i,k}$, are determined to satisfy statistical properties

and given criteria. If $\phi_{l,k}$ and $\phi_{Q,k}$ are uniformly distributed over $(-\pi, \pi)$ and $f_{l,k} \neq f_{l,l}$, $f_{Q,k} \neq f_{Q,l}$ for $k \neq l$. $\tilde{\alpha}_l$ and $\tilde{\alpha}_Q$ tend to the Gaussian processes in the limit $N \rightarrow \infty$ by central limit theorem. Therefore, the envelope of $\tilde{\alpha} = \tilde{\alpha}_l + j\tilde{\alpha}_Q$ becomes Rayleigh distributed process, which is the important stochastic characteristics of fading waveforms in the typical land mobile channel environment. The deterministic models are classified into several models, such as Jakes' model [4], equal distance method, equal gain method, and Monte-Carlo method [3], by how to select Doppler frequencies and coefficients. While Doppler coefficients of Jakes' model and equal distance vary with k , those of equal gain method and Monte-Carlo method are constant. To sample Doppler phase differences easily, the Doppler frequencies are determined by equal area model in this paper. The cross-correlation between distinct processes using the same Doppler frequencies is dependent on Doppler coefficients and Doppler phases. Constant Doppler coefficients make the cross-correlation changed by Doppler coefficients be ignored.

III. PROPOSED CHANNEL MODEL

Extending single fading waveform by equal gain method to M fading waveforms on M array elements, the m th fading waveform is described as

$$\tilde{\alpha}_m(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^N \left\{ \cos(2\pi f_k t + \phi_{m,k}^{(I)}) + j \cos(2\pi f_k t + \phi_{m,k}^{(Q)}) \right\} \quad (2)$$

$$f_k = f_d \sin\left(\frac{k\pi}{2N}\right)$$

where $\phi_{n,k}^{(I)} - \phi_{n,k}^{(Q)} = \pi/2$ to suppress the cross-correlation between inphase and quadratic components of resultant waveforms. Then the cross-correlation between m th and n th fading waveforms is calculated in terms of Doppler phase difference $\phi_{m,k}^{(I)} - \phi_{n,k}^{(I)} = \Delta\phi_{m,n,k}$ as

$$\tilde{\rho}_{m,n} = E\{\tilde{\alpha}_m(t)\tilde{\alpha}_n^*(t)\} = \frac{1}{N} \sum_{k=1}^N \exp(-j\Delta\phi_{m,n,k}). \quad (3)$$

The spatial correlation between fading waveforms is determined by the electrical distance between array

elements d/λ and the normalized power distribution $f_{\Theta}(\theta)$ as

$$R(d/\lambda) = \int_{-\pi}^{\pi} e^{-j2\pi d \sin\theta/\lambda} f_{\Theta}(\theta) d\theta \quad (4)$$

$$\int_{-\pi}^{\pi} f_{\Theta}(\theta) d\theta = 1,$$

where λ denotes the wavelength of carrier signal and $f_{\Theta}(\theta)$ is a function of incident angle $\theta \in (-\pi, \pi)$. Considering d the distance $d_{m,n}$ between m th and n th array element, integration of (4) can be approximated into summation as (5) in such a manner that θ_k s are sampled more densely in the incident angle region where $f_{\Theta}(\theta)$ increases.

$$R(d_{m,n}/\lambda) \cong \frac{1}{N} \sum_{k=1}^N \exp(-j2\pi d_{m,n} \sin\theta_k/\lambda) \quad (5)$$

By applying equality between (3) and (5), the relationship between discrete incident angle sample and Doppler phase difference is described as

$$2\pi d_{m,n} \sin\theta_k/\lambda = \Delta\phi_{m,n,k}. \quad (6)$$

Prior to selecting proper θ_k , equal power distribution regions over incident angle are chosen as the rule of (7a). The distributed powers in the region of (d_{k-1}, d_k) are equal for all $k=1,2,\dots,N$. Since incident angle θ is defined in $(-\pi, \pi)$, d_0 and d_N are $-\pi$ and π respectively. Then discrete incident angle samples are selected in such a manner that θ_k becomes the average incident angle in the k th region (d_{k-1}, d_k) as (7b).

$$d_k = F_{\Theta}^{-1} \left\{ F_{\Theta}(d_{k-1}) + \frac{1}{N} \right\} \quad (7a)$$

where $F_{\Theta}(x) = \int_{-\pi}^x f_{\Theta}(\theta) d\theta$ is the cumulative density function (cdf) for $f_{\Theta}(\theta)$.

$$\theta_k = N \int_{d_{k-1}}^{d_k} \theta f_{\Theta}(\theta) d\theta \quad (7b)$$

Our proposed model can be easily expanded into channel simulation model for frequency-selective fading channel by T-spaced model [5]. The Gaussian processes that are generated by the distinct Doppler frequencies are

mutually uncorrelated, and the distinct numbers of sinusoids make the distinct Doppler frequencies to be selected according to (2). Define $\tilde{\alpha}_{m,l}$ as the fading waveform for the l th resolvable path and m th element and N_l as the number of sinusoids for $\tilde{\alpha}_{m,l}$, then $N_l = N_1 + l - 1$ guarantees selection of the distinct Doppler frequencies for different path if N_1 is sufficiently large.

IV. DERIVATION OF PARAMETERS FOR SPECIAL INCIDENT ANGLE DISTRIBUTION

Although any distribution can be applied for $f_{\theta}(\theta)$, the Gaussian and Laplacian distributions, which are described as (8a) and (8b), are applied in this paper. Recent measurement shows that Laplacian distribution approximates $f_{\theta}(\theta)$ in the realistic outdoor environment [6].

$$f_{\theta}(\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\theta-m)^2}{2\sigma^2}} \quad (8a)$$

$$\text{where } S = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\pi}^{\pi} e^{-\frac{(\theta-m)^2}{2\sigma^2}} d\theta \text{ and } m \in (-\pi, \pi)$$

$$f_{\theta}(\theta) = \frac{1}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}|x-m|}{\sigma}} \quad (8b)$$

$$\text{where } S = \frac{1}{\sqrt{2\sigma}} \int_{-\pi}^{\pi} e^{-\frac{\sqrt{2}|x-m|}{\sigma}} d\theta \text{ and } m \in (-\pi, \pi)$$

The inverse cdfs for Gaussian and Laplacian distributions are introduced as (9a) and (9b), respectively.

$$F_{\theta}^{-1}(x) = \sqrt{2\sigma} \operatorname{erf}^{-1}(2x-1) + m \quad (9a)$$

$$F_{\theta}^{-1}(x) = \begin{cases} m - \frac{\sigma}{\sqrt{2}} \ln(2(1-x)) & x \geq 0.5 \\ m + \frac{\sigma}{\sqrt{2}} \ln(2x) & x < 0.5 \end{cases} \quad (9b)$$

where erf is error function.

Finally, $\{d_i\}$ and $\{\theta_i\}$ deterministic channel model are derived using (9a) and (9b). The equations of (10) show how to determine the parameters for channel simulation for Gaussian incident angle distribution, and those of (11) are for Laplacian distribution.

$$d_i = \sqrt{2\sigma} \operatorname{erf}^{-1} \left(\operatorname{erf} \left(\frac{d_{i-1} - m}{\sqrt{2\sigma}} \right) + \frac{2S}{N} \right) + m. \quad (10a)$$

$$\text{where } S = \frac{1}{2} \left(\operatorname{erf} \left(\frac{\pi - m}{\sqrt{2\sigma}} \right) - \operatorname{erf} \left(\frac{-\pi - m}{\sqrt{2\sigma}} \right) \right)$$

$$\theta_i = \frac{N}{S} [g_{m\sigma}(d_{i-1}) - g_{m\sigma}(d_i)]. \quad (10b)$$

$$\text{where } g_{m\sigma}(x) = \frac{\sigma}{\sqrt{2\pi}} \exp \left(-\frac{(m-x)^2}{2\sigma^2} \right) + \frac{m}{2} \operatorname{erf} \left(\frac{m-x}{\sqrt{2\sigma}} \right)$$

$$d_i = F_{\theta}^{-1} \left\{ F_{\theta}(d_{i-1}) + \frac{S}{N} \right\}. \quad (11a)$$

$$\text{where } S = 1 - e^{-\frac{\sqrt{2}\pi}{\sigma}} \cosh \left(\frac{\sqrt{2}m}{\sigma} \right)$$

$$\theta_i = \begin{cases} \frac{N}{S} [g_{m\sigma}^{(1)}(d_{i-1}) - g_{m\sigma}^{(1)}(d_i)] & d_{i-1}, d_i \geq m \\ \frac{N}{S} [g_{m\sigma}^{(2)}(d_{i-1}) - g_{m\sigma}^{(1)}(d_i) + m] & d_{i-1} < m \leq d_i \\ \frac{N}{S} [g_{m\sigma}^{(2)}(d_{i-1}) - g_{m\sigma}^{(2)}(d_i)] & d_{i-1}, d_i < m \end{cases} \quad (11b)$$

$$\text{where } g_{m\sigma}^{(1)}(x) = \left(\frac{x}{2} + \frac{\sqrt{2}\sigma}{4} \right) \exp \left(\frac{\sqrt{2}(m-x)}{\sigma} \right).$$

$$g_{m\sigma}^{(2)}(x) = \left(-\frac{x}{2} + \frac{\sqrt{2}\sigma}{4} \right) \exp \left(-\frac{\sqrt{2}(m-x)}{\sigma} \right).$$

$$g_{m\sigma}^{(1)}(m) = \left(\frac{m}{2} + \frac{\sqrt{2}\sigma}{4} \right) \text{ and } g_{m\sigma}^{(2)}(m) = \left(-\frac{m}{2} + \frac{\sqrt{2}\sigma}{4} \right)$$

V. NUMERICAL RESULTS AND DISCUSSION

Fig. 1 and Fig. 2 show the envelope of the spatial correlation of fading waveforms generated by the proposed model in comparison with the theoretic results, which are calculated with (4) considering the Gaussian and Laplacian distribution respectively. These results confirm that the proposed model can be adapted for arbitrary spatial correlation, and the larger N makes the channel simulator generate the more rigorously correlated fading waveforms. Our model is based on equal area model, which adjusts the only Doppler frequencies to satisfy the temporal correlation, and Doppler phases are adjusted to satisfy given spatial correlation. Therefore, the model does not distort the temporal correlation. Like spatial correlation, the larger N guarantees temporal correlation of the generated fading waveforms to agree

with the intended correlation characteristics the more rigorously.

VI. CONCLUSION

This paper presents a deterministic vector channel simulation model satisfying arbitrary spatial correlation as well as rigorous temporal correlation in such a manner of selecting proper Doppler phase differences. It can be shown that the correlation characteristics of generated fading waveforms are in good agreement with theory in both time and space domain. Our proposed model can be applied to evaluate the laboratory performance of mobile communication systems employing adaptive array antennas or space diversity.

References

[1] S.T. Kim, J.H. Yoo, and H.K. Park, "A Spatially and Temporally Correlated Fading Model for Array Antenna Applications," *IEEE Trans. on Veh. Technol.*, Vol. 48, No. 6, pp. 1899-1905, 1999.

[2] W.C. Jakes, *Microwave Mobile Communications*, John Wiley & Sons, pp. 65-76, 1974.

[3] M. Patzold, U. Killat, and F. Laue, "On the Statistical Properties of Deterministic Simulation Models for Mobile Fading Channels," *IEEE Trans. on Veh. Technol.*, Vol. 47, No. 1, pp. 254-269, 1998.

[4] J.H. Yoo, C. Mun, J.K. Han, and H.K. Park, "Spatiotemporally Correlated Deterministic Rayleigh Fading Model for Smart Antenna Systems," *Proc. of Veh. Technol. Conf. 1999-Fall*, Amsterdam, Vol. 3, pp. 1397-1401, Sept. 1999.

[5] STUBER, G.L. : *Principles of Mobile Communication*, Kluwer Academic Publishers, pp. 83-87, 1996.

[6] K.I. Pedersen, P.E. Mogensen, and B.H. Fleury, "Spatial Channel Characteristics in Outdoor Environments and their Impact on BS Antenna System Performance," *Proc. of Veh. Technol. Conf. '98*, pp.719-723, 1998.

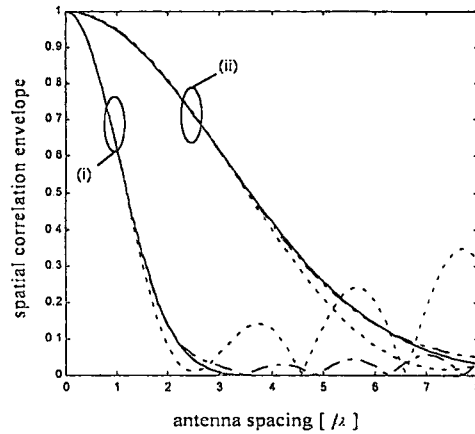


Fig 1. Spatial correlation characteristics in the Gaussian incident angle distribution.

—— theory, simulation ($N=8$),
 - - - - simulation ($N=32$)
 (i) $m=0^\circ, \sigma=9^\circ$, (ii) $m=0^\circ, \sigma=3^\circ$

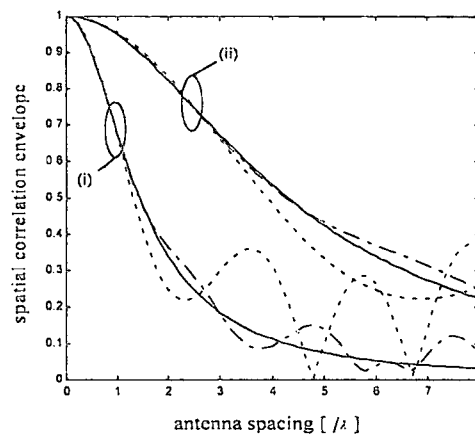


Fig 2. Spatial correlation characteristics in the Laplacian incident angle distribution.

—— theory, simulation ($N=8$),
 - - - - simulation ($N=32$)
 (i) $m=10^\circ, \sigma=9^\circ$, (ii) $m=10^\circ, \sigma=3^\circ$