

## Study on $\alpha$ -LTS Hausdorff distance applying $\alpha$ -trimmed

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**Abstract:** It is effectively removed noise in the image using FCNN(Fuzzy Cellular Neural Network) applying fuzzy theory to CNN(Cellular Neural Network) structure and HD(Hausdorff Distance) commonly used measures for object matching. HD calculates the distance between two point set of pixels in two-dimensional binary images without establishing correspondence. Also, this method is proposed in order to improve the operation speed. In this paper,  $\alpha$ -LTSHD(Least Trimmed Square HD) operator applying  $\alpha$ -Trimmed to LTSHD, one field of HD, is applied to FCNN structure, and it is proposed as the modified method in order to remove noise in the image. Also, it is made a comparison with the other filters by using MSE and SNR after removing noise using the FCNNs which are applied  $\alpha$ -LTSHD operator through the computer simulation. In a result, FCNN performance which is applied the proposed  $\alpha$ -LTSHD demonstrated the superiority to the other filters in the noise removal.

### 1. Introduction

An important property of CNN(Cellular Neural Network) is that cell(processing unit) is able to connect locally with neighborhood cell, is very simple to the structure of interconnections elements between cell and cell, and is suitable to VLSI implementations with immutability of the space for the real processing[1]. This concept of CNN, using many applications of image processing, pattern recognition, etc, was introduced in 1987 in Leon O. chua's laboratory in Berkely[2]. CNNs consist of many parallel analogue processors computing in the real time. In classified to CNNs structure, FCNN(Fuzzy Cellular Neural Network) is introduced with a new structure applied the fuzzy concept in order to obtain the digital weight, input, state, output to this CNN. But FCNN occurs in case to sink into the local value when it is applied template, and then it needs very much time in order to obtain the global winner value. And in this paper, it

is effectively removed noise in the image using FCNN(Fuzzy Cellular Neural Network) applying fuzzy theory to CNN(Cellular Neural Network) structure and HD(Hausdorff Distance) commonly used measures for object matching. And this method is proposed in order to improve the operation speed. So, this paper use the FCNN structure operation as well as matching method in order to solve the above fault. Image matching, which measures the degree of similarity between two image sets that are superposed upon one another, plays a key role in many areas such as pattern recognition, image analysis and computer vision. And matching two images requires the images to be matching go through a number of operations before the similarity is determined. These operations include feature extraction, distance transformation, matching measurement and searching for the best match. In the past, many matching algorithms had been proposed, such as hierarchical Chamfer matching algorithm (HCMA), Borgefors matching algorithm, HD(Hausdorff Distance), etc. Where, a hausdorff distance(HD) is one of commonly used measures for object matching, it calculates the distance between two point sets of edges in two-dimensional binary images without establishing correspondences, and this method is using max-min distance, and HD operation construction has bidirection, this HD measure computes a distance value between two sets of edge points extracted from the object model and a test image[3][4][5].

$\alpha$ -LTSHD(Least Trimmed Square HD) operator applying  $\alpha$ -Trimmed to LTSHD, one field of HD, is applied to FCNN structure, and it is proposed as the modified method in order to remove noise in the image in this paper.

### 2. The basic structure FCNN

The  $M \times N$  FCNN structure applying the fuzzy operation and the fuzzy template is that  $u$ ,  $x$  and  $y$  are represented as every input, state, output. It is likely as equation(1). Voltage  $V_{uij}$ ,  $V_{xij}$ ,  $V_{yij}$  give expression of

input, state, output voltage of Cell  $C_{ij}$ . Equation is presented as below[2][6].

$C_{ij}$  state equation :

$$\begin{aligned}
C \frac{dv_{x_{ij}(t)}}{dt} &= \frac{-1}{R_x} v_{x_{ij}(t)} + \sum_{\langle k, l \rangle \in N_{A(i,j)}} A(i, j, k, l) v_{y_{kl}(t)} \\
&+ \sum_{\langle k, l \rangle \in N_{A(i,j)}} B(i, j, k, l) v_{u_{kl}} + I \\
&+ \bigwedge_{\langle k, l \rangle \in N_{A(i,j)}} A_{fmin}(i, j, k, l) y_{kl} \\
&+ \bigvee_{\langle k, l \rangle \in N_{A(i,j)}} A_{fmax}(i, j, k, l) y_{kl} \\
&+ \bigwedge_{\langle k, l \rangle \in N_{A(i,j)}} B_{fmin}(i, j, k, l) u_{kl} \\
&+ \bigvee_{\langle k, l \rangle \in N_{A(i,j)}} B_{fmax}(i, j, k, l) u_{kl}
\end{aligned} \quad (1)$$

$1 \leq i \leq M; 1 \leq j \leq N$

Where,  $A_{fmin}(i, j, k, l)$ ,  $A_{fmax}(i, j, k, l)$ ,  $B_{fmin}(i, j, k, l)$ ,  $B_{fmax}(i, j, k, l)$  is an original element of the fuzzy feedback MIN template, the fuzzy feedback MAX template, the fuzzy feed-forward MIN template, The fuzzy feed-forward MAX template.  $\bigwedge$  and  $\bigvee$  show the fuzzy AND and Fuzzy OR, respectively[2].

Output equation of  $C_{ij}$  :

$$\begin{aligned}
v_{y_{ij}(t)} = f(x_{ij}(t)) &= \frac{1}{2} (|v_{x_{ij}(t)} + 1| - |v_{x_{ij}(t)} - 1|) \\
&1 \leq i \leq M; 1 \leq j \leq N
\end{aligned} \quad (2)$$

Constraint conditions :

$$|x_{ij}| \leq 1, |u_{ij}| \leq 1, 1 \leq i \leq M; 1 \leq j \leq N \quad (3)$$

Parameter assumption :

$$\begin{aligned}
C > 0, R_x > 0 \\
A(i, j, k, l) &= A(k, l; i, j) \\
A_{fmin}(i, j, k, l) &= A_{fmin}(k, l; i, j), \\
A_{fmax}(i, j, k, l) &= A_{fmax}(k, l; i, j) \\
1 \leq i, k \leq M; 1 \leq j, l \leq N
\end{aligned} \quad (4)$$

### 3. The proposed $\alpha$ -LTSHD algorithm

FCNN occurs in case to sink into the local value when it is applied template, and then it needs very much time in order to obtain the global winner value. So, this paper use the FCNN structure operation and matching method in order to solve the above fault. Where the matching method is Hausdorff Distance, HD operation structure has the bidirection, the HD measure computes a distance value between two set,  $A = \{a_1, \dots, a_{N_A}\}$  and  $B = \{b_1, \dots, b_{N_B}\}$  of sizes  $N_A$  and  $N_B$ , respectively, it is defined as[3][7]

$$h(A, B) = \max(h(A, B), h(B, A)) \quad (5)$$

Where  $h(A, B)$  and  $h(B, A)$  represent the directed distance between two sets A and B. The distance for

the point set B from a point a represent as  $d_B(a) = \min_{b \in B} \|a - b\|$ . The distance for the point set A

from a point b defined as  $d_A(b) = \min_{a \in A} \|b - a\|$ .

And  $h(A, B)$  and  $h(B, A)$ , the direction HD, is defined as

$$h(A, B) = \max_{a \in A} d_B(a) \quad (6)$$

$$h(B, A) = \max_{b \in B} d_A(b) \quad (7)$$

$\|\cdot\|$  is defined as the Euclidean distance norm, also restrict to being the  $L_2$  norm. The modified HD matching for the object matching is proposed because the conventional HD matching is sensitivity to the image injury of the noise or the interference. So, Huttenlocher et al, proposed the partial HD measure in comparing partial portions of images containing severe occlusions or degradation. The directed distance of the partial HD is defined as[7]

$$h_K(A, B) = K \frac{th}{a \in A} d_B(a) \quad (8)$$

Where  $d_B(a)$  represents the minimum distance value at point a to the point set B, and  $K \frac{th}{a \in A}$  denotes the Kth ranked value of  $d_B(a)$ . This HD measure needs one parameter,  $K = fN_A$ , whose range is from 0.0 to 1.0. Depending on the fractional value of f, its performance widely varies. By modifying the HD based on the ranked order statistics, Azencott et al, proposed the CHD measure in comparing binary images. The directed distance of the CHD  $h_{CHD}(A, B)$  is defined as[4][5]

$$h_{CHD}(A, B) = K \frac{th}{a \in A} L \frac{th}{b \in B} \|a - b\| \quad (9)$$

Where  $K \frac{th}{a \in A}$  denotes the Kth ranked value of  $L \frac{th}{b \in B} \|a - b\|$ , with  $L \frac{th}{b \in B}$  representing the Lth ranked value of the Euclidean distance set  $\|a - b\|$ . Where, the parameters  $\beta, \gamma$  is defined in order to decide the similarity as the image impair measure,  $\beta$  has the values between 0 and 1,  $\gamma$  has the values between 0 and 1. And the CHD measure requires two parameters :  $K = \beta N_A$  and  $L = \gamma N_B$ . Dubuisson and Jain are proposed to MHD(Modified HD) of the distance values mean type of  $D_B(A, B)$ , and they are simulated for the impaired composition images by four type noise. The direction HD  $h_{MHD}(A, B)$  of MHD is defined as equation(10).

$$h_{MHD}(A, B) = \frac{1}{N_A} \sum_{a \in A} d_B(a) \quad (10)$$

The MHD measure dose not require a parameter, however its matching performance is not good, compared with that of the partial HD and the CHD, because it employs the summation operator over all distance some of which might be computed form outliers. And MHD does not have the parameter, while the partial HD has one parameter, CHD has two parameters. Partial HD and MHD values except the CHD obtain as the minimum distance for the model image obtained by the pixel operator and the point sets from a point of the structure image, it is need to next computing process,

$$\begin{aligned} d_B(a) &= \min_{b \in B} \| a - b \|, \\ d_A(b) &= \min_{a \in A} \| b - a \| \end{aligned} \quad (11)$$

This process is very simple operator, but it is need a lot of computing as the bidirection characteristic. Also the modified  $\alpha$ -Least trimmed square(LTS) of LMS(Least median square) is proposed in order to improve breakdown point produced in the HD operation process, This object function is likely to equation(12).

$$\sum_{i=1}^n (r^2_i)_{j,n} \quad (12)$$

Where,  $(r^2_i)_{1:n} \leq (r^2_i)_{2:n} \leq \dots \leq (r^2_i)_{n:n}$  array as rank for smalling this function. Firstly, LTS array as the rank, and it is exclude the large error from arrayed error values, and then reducing the remainder error. The proposed  $\alpha$ -LTS is excluded as  $\alpha$  rate from the large part in arrayed error values as order, and used as the reduction at the remainder error method. It is minimum of the error by modifying the element size. Where,  $\alpha$  has value between 0 and 1. And the direction distance  $h_{\alpha-LTS}(A, B)$  of  $\alpha$ -LTSHD is likely to equation (13) as the order state linear combination.

$$h_{\alpha-LTS}(A, B) = \frac{1}{\alpha K} \sum_{i=1}^n d_B(a)_i \quad (13)$$

Where  $\alpha$  is the removed error rate, K is represented to  $f \times N_A$  as the partial HD matching, and  $d_B(x)_i$  is represented ith distance value among the arrayed distance values  $d_B(x)_1, d_B(x)_2, \dots, d_B(x)_n$  as order. The parameter f has value between 0 and 1, decide as impairing or noise level. E is the normalized  $[0, 1]$  in order to apply HD operation to

FCNN, and FCNN in order to apply  $\alpha$ -LTSHD has the variant value as equation (14) and (15).

$$\begin{aligned} A=0, B=0, I=0, \\ A_{fmin} = \text{undefined}, A_{fmax} = \text{undefined}, \\ B_{fmax} = \text{undefined}, B_{fmin} = -S_{\alpha-LTS} \end{aligned} \quad (14)$$

$$\begin{aligned} f_y(x) = f(v_{xy}(t)) = \frac{1}{2} (|v_{xy}(t)+1| - |v_{xy}(t)-1|), \\ t \in [0, 1] \end{aligned} \quad (15)$$

Where,  $S_{\alpha-LTS} = \{-t : t \in S\}$ .

#### 4. Simulation analysis

The total gray level value is very low in Fig. 1, and Fig. 1 apply the proposed  $\alpha$ -LTSHD in order to conquest a fault which has the local winner to FCNN structure. Also, it consists of  $n(n+1)/2$   $\alpha$ -LTSHD and  $n-1$  MIN filter as the flow chart for removing the image noise. And the structure elements are increased as the linear, the total structure without being connected with the input image size consist of the parallel structure and the pipeline type, and each windows use  $3 \times 3$  window image. Fig. 2 is the original lena image,  $256 \times 256$  image. Fig. 3 is the result image applying the conventional FCNN, Fig. 4 is the result image applying the proposed  $\alpha$ -LTSHD to FCNN in the paper. Also, Fig. 5 is the graph for MSE and SNR applying the proposed algorithm and the conventional algorithm. Where MSE of the proposed  $\alpha$ -LTSHD algorithm is less about  $30.3424 \sim 30.8276$  than the conventional FCNN algorithm, and then the proposed algorithm demonstrated the superiority to the conventional algorithm in the computer simulation. FCNN algorithm applying the proposed the proposed  $\alpha$ -LTSHD demonstrated the superiority to the conventional FCNN in the filter capacity through table 1 and Fig. 3, 4, 5.

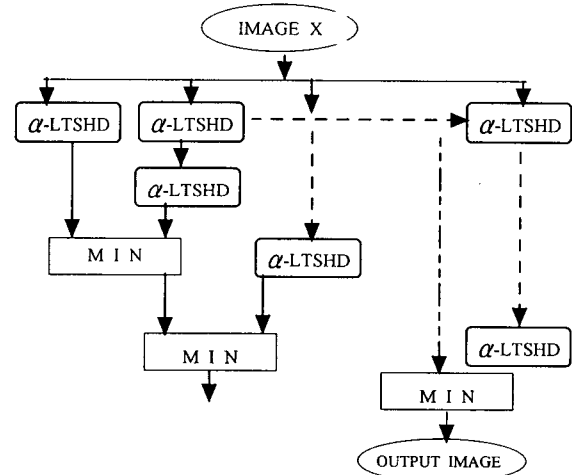


Fig. 1 FCNN Structure applying  $\alpha$ -LTSHD



Fig. 2 Lena image (a) (b)

Fig. 3 The conventional FCNN result image (a) Result image for 10% noise (b) 20%

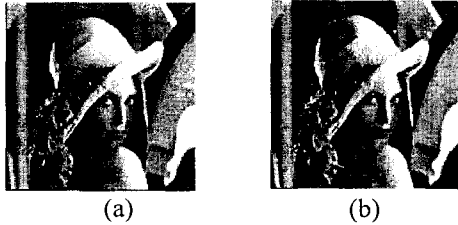


Fig. 4 The proposed  $\alpha$ -LTSHD operator result image (a) Result image for 10% noise (b) 20%

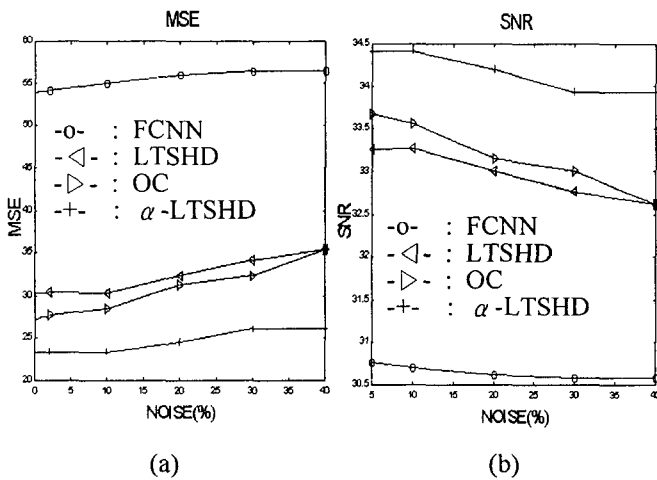


Fig. 5 (a) MSE (b) SNR

### 5. Conclusion

$\alpha$ -LTSHD (Least Trimmed Square HD) operator applying  $\alpha$ -Trimmed to LTSHD is applied to FCNN structure, and it is proposed as the modified method in order to remove noise in the image. Also, it is made a comparison with the other filters by using MSE and SNR after removing noise using the FCNNs which are applied  $\alpha$ -LTSHD operator through the computer simulation. In a result, MSE of the proposed  $\alpha$ -LTSHD algorithm is less about 30.3424 ~ 30.8276 than the conventional FCNN algorithm, and then the proposed algorithm performance demonstrated the superiority to the conventional algorithm in the noise removal.

In the future, we will apply the Hausdorff distance to a property detection of the real-time system, the

color image and pattern recognition, the edge detection, etc.

Table 1. MSE and SNR of each algorithm and noise

Algorithm \ Noise		5%	10%	20%	30%
FCNN	MSE	54.1732	54.9419	55.9569	56.4620
	SNR	30.7588	30.6976	30.6181	30.5791
LTSHD	MSE	30.4808	30.3568	32.3112	34.1448
	SNR	33.2564	33.2741	33.0031	32.7634
OC	MSE	27.7061	28.4302	31.2553	32.3228
	SNR	33.6709	33.5589	33.1474	33.0016
FCNN applying $\alpha$ -LTSHD	MSE	23.3456	23.3559	24.5420	26.1196
	SNR	34.4146	34.4127	34.1976	33.9270

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