

# Formation of Interdependence in Selecting Game

Hiroyuki Iizuka, Masahito Yamamoto, Hidenori Kawamura, Keiji Suzuki and Azuma Ohuchi

Research Group of Complex Systems Engineering,  
Graduate School of Engineering, Hokkaido University, Sapporo, Japan  
Kita 13, Nishi 8, Kita-ku, Sapporo, 060-8628 Japan  
Phone +81-11-716-2111(Ext.6498), Fax +81-11-706-7834  
E-mail: {ezca, masahito, kawamura, suzuki, ohuchi}@complex.eng.hokudai.ac.jp

**Abstract** Decision-makers in ecological system and social system have complex interactions and relations. Such interactions and relations are not predefined but dynamically constructed. We consider what relations are constructed or destroyed and how the relations change. Therefore, we focus on the formation and collapse of relations as one of the emergent phenomena of social or ecological complex phenomena. Game theory is the best way of analyzing phenomena in terms of interactions. However, it is difficult to analyze the dynamical system by game theory. Consequently, we propose Selecting Game with agents as players based on game theory. In this model, the relations among agents are not predefined but constructed by selecting subgames. As a result, we confirmed that the entire relation among agents is constructed by the agents' changing partial relations and that the relations dynamically change.

## 1 Introduction

Agent-based simulations in which multiple adaptive agents interact with each other are expected to enable analysis to social and ecological complex phenomena. In such simulations, one of the important behaviors is interacting among agents. It is studied well that the interactions among agents will cause emergent phenomena. However, it is difficult to analyze the universal relation between the interaction and the emergent phenomenon because the interaction is described independently in each system. Therefore, we think game theory is the best way of analyzing the emergent phenomena in terms of the interactions. Game theory is a way of analyzing situations in which each player's optimal decisions depend on his beliefs or expectations about the play of his opponents [1].

In this paper, we focus on the emergent phenomena of social or ecological complex phenomena and analyze them based on game theory. As one of the emergent phenomena, a formation of interdependence is adopted. Human or animals can not live independently without keeping relational mutually. Even if human or animals can live without having relation to the others, it means that the relation of no relation is formed. Such a rela-

tion is not predefined in ecosystem or social system but is constructed dynamically by decision-makers constituting the system. Namely, a relation of agents' having accepted would be remained and that of not having accepted would die out. Even if the relation of agents' having accepted is remained, the relation can not maintain stable among agents because a new agent move from another relation to there or an agent having accepted the relation move to another one, namely the number of agents constituting the relation is not fixed. Therefore, it occurred that the relation is formed or destroyed. It is important to analyze this dynamical interdependence in order to understand the ecosystem and society.

Therefore, we apply game theory to analyze this interdependence. However, there is a problem of game theory. Although game theory is suited for stationary situations and the case that the player can perceive complete information about the game, it is unnatural that the decision-makers make decisions on such conditions. Furthermore, the payoff matrix, which represents a relation between players, is predefined.

In order to resolve these problems and consider the dynamical interdependence, we deal with agent-based simulation based on game theory in which the agents are designed with bounded abilities to perceive the environments. The rationality of the players is realized by the mind that the player want to gain better rewards. In order to consider that the payoff matrix between players is not predefined, we propose Selecting Game as a new game. In Selecting Game, the various payoff matrices and many players exist. The players must select a game of the payoff matrices. If another player who want to play the game exists, both players can play the game. The payoff for the players is determined based on the result of the game. The match can be regarded as a formation of a relation. We focus on what kind of payoff matrix which represents the relation is selected by analyzing this game and show the dynamics of formation or collapse of interdependence.

## 2 Selecting Game

In order to treat the dynamical interdependence, we

propose Selecting Game. This game is represented as follows.

$$\text{SelectingGame} = (N, \mathbf{SG}), \quad (1)$$

where  $N$  denotes players who play this Selecting Game, and  $\mathbf{SG}$  represents a set of subgames.

Subgame is a two-person non-zero-sum game in which players have pure strategies. Namely, subgame is represented with following expression:

$$\text{SubGame} = (N', \mathbf{S}, F), \quad (2)$$

where  $\mathbf{S}$  denotes a set of strategies, and  $F$  means a payoff function.

### 2.1 Game Rule

At first, all players select one game out of subgames. In each subgame, the players who have selected each subgame randomly form teams of two players each. When the number of the players who have selected the subgame is odd, one player is left. These selected two players play this game  $\alpha$  times.  $\alpha$  is a constant value. In each times, the players are given the payoff based on payoff function. The remained player is given no payoff. This evaluation of the players is performed in all subgames. This process is regarded one cycle of Selecting Game. This cycle is performed repeatedly.

### 2.2 Matrices of Subgame

In order that the player can freely select the relation (subgame), many kinds of matrices are needed. We treat 2x2 games as subgames because all 2x2 two-person games are well discussed and classified in [2]. It is expected that this paper is very useful for the analysis of our experiment.

There are four kinds of payoff values of matrix for one player in 2x2 game. Those values are represented by  $a, b, c, d$ . There are 24 kinds of permutations and combinations according to height and, for both players, there are 576 kinds. But in this 576 games, equivalent games to the other games exist. Those games are removed and 78 kinds of games are left. 12 games of 78 are symmetrical games and 66 games of 78 are unsymmetrical. We create 78 game matrices by parameter settings as  $a = 1, b = 2, c = 3, d = 4$ .

## 3 Decision Making of Agents as Players

Agents as players must make decisions at two stages. First, the agent need to determine which subgame he plays. Second, in the game, the agent need to determine which strategy he chooses.

### 3.1 Decision Making for Selection of Subgame

It is hard for us to think that human or animals realize all the relations and it is difficult for agents to select one subgame (relation) out of all the relations. However, it is natural to think that an agent, at first, randomly select a subgame and if the agent gets a good payoff, the agent stays in the subgame and if not, the agent moves to another subgame. Therefore, we designed the agent with a probability which represents the extent of his desire to move to another subgame. The probability,  $MP$ , which each agent possesses is defined as follows.

$$MP(i) = \beta^i, \quad i = 1 \dots, L \quad (3)$$

where  $\beta$  represents a constant value in  $[0, 1]$ . The agent adaptively changes this  $MP$  in response to the environment. When the agent selects a subgame, the subgame the agent selected last time is normally selected, but the agent randomly changes another game with the  $MP$  probability. After the agents play subgame and get the payoff, each agent compares his payoff with randomly selected agent's payoff. When the payoff of the agent is larger than that of randomly selected agent or the subgame the agent played is the same as the subgame randomly selected agent played,  $MP$  level  $i$ , increases and the agent make the probability low because the agent expects better payoff than the others in this subgame and wishes that the agent stays here. On the contrary, when the payoff of the agent is smaller than that of randomly selected agent, the agent expects worse payoff and thinks that there may be better subgame. Namely,  $i$  decreases. By using this updated  $MP$ , selection of subgame for the agent is performed.

### 3.2 Decision Making in Subgame

The agents must choose one strategy of two because of 2x2 game. Here, two strategies are represented by "0" and "1". The strategy is determined by the history of interactions between players. Robust and unexploitable strategies for playing the Iterated Prisoner's Dilemma (2x2 game) require a memory of the outcomes of the last two games played. Therefore, we designed the agent with a strategy string as a memory of the outcomes of the last two games. The strategy string provides a response for every possible situation that could arise from a memory of outcomes of the last two games, i.e. 16 possible situations. For example, if the point on the strategy string labelled "0000" holds a "1", this is interpreted to mean that if the last two strategies of both players is "00", then the next strategy is "1". The strategy string must also provide appropriate responses for when an agent first meets a new player, i.e. either "always 0" or "always 1" (1 bit), and for the second game (4 bit). In total the strategy string must be 21 bits long to cover all possible situations.

In order to get better payoff, the agent changes strategies in response to the environment. In the same way as the probability,  $MP$ , the agent has a probability which represents the extent of his desire to change his strategy. The probability,  $CP$ , is defined as follows:

$$CP(j) = \beta^j, \quad j = 1 \dots, L \quad (4)$$

After a match in subgame, if the payoff an agent gets is larger than the payoff another agent in same subgame gets,  $j$  increases and the agent make the probability of changing the strategy low. On the contrary, if the payoff an agent gets is smaller,  $j$  decrease.

## 4 Simulation

### 4.1 Experimental Settings

A simulation investigates dynamics of selected subgame among the agents in Selecting Game. We regard this selection of subgames as that of relation. Namely, the only relation which most of the agents accepted is left and the relation among a mass of the agents is constructed.

The parameter settings for these experiments were as follows:

$$\begin{aligned} \alpha &= 100, & \beta &= \frac{1}{1.5} \\ L &= 20 \end{aligned} \quad (5)$$

In this paper, for simplicity and analyses, we used the symmetrical 12 games as the subgames. Future research will include an analysis of Selecting Game using all 78 games as subgames.

### 4.2 Experimental Results

At first, we experimented with fixing all the outputs of the strategy string providing the next strategy of each agent on "1" in order to investigate influence of only the selection of subgame on dynamics of formation of relation. Namely, when the agent selects a subgame, the payoff only depends on whether or not there is a opponent that he plays game with. In this experiment, the number of population of the agents is set to 1000. Figure 1 shows the transition of the number of the agents that selected each game. Figure 2 shows the average of  $MP$  of all the agents at that time. According to these graphs, at the beginning of the iterations, some subgames which can provide both players with good payoff 4 are regarded as the relations among the agents. This agrees with our expectation. After that, by chance, agents that selected one subgame increase compared with the other subgame. This causes that one subgame made a monopoly of the agents step by step. In response to the monopoly, the average of  $MP$  decreases and the relation become stable. To investigate the influence of the number of population, we performed the same experiment with 500 agents ( Fig. 3 ). Consequently, these

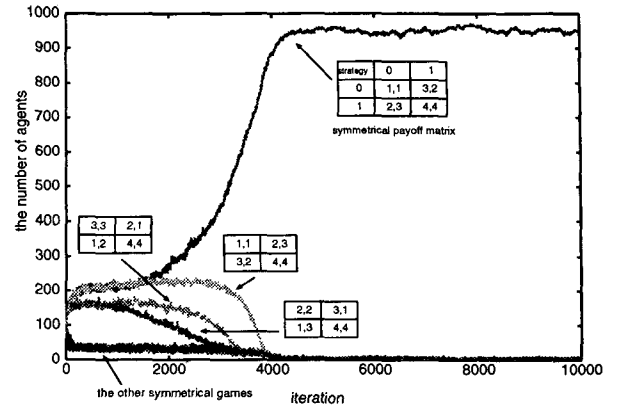


Fig. 1: The number of agents in each subgame. The number of population is 1000.

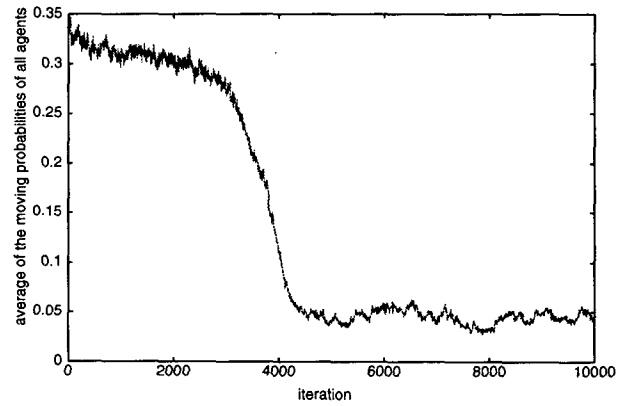


Fig. 2: The average of the probabilities,  $MP$ , of all agents which represent a extent of desire to move to another game.

two graphs show that there is no influence of the number of the population on the monopoly of one subgame.

Next, a simulation with the agents adaptively changing the strategy was performed. The initial outputs of the strategy string of each agent were randomly set to "0" or "1". Figure 4 shows the transition of the number of the agents in each subgame. Since the agents were absorbed in selecting subgames, a subgame try to make of monopoly early in the iterations like Fig 1. However, when the subgame was making of monopoly, the agents begin to regard the subgame as their fixed relation. The agents come to change their strategies because of desire of getting better payoff in the subgame. If combination of both agents' strategies is not well there, the agents can not get good payoff. Even if the combination between them is well, since the other agents come to participate in the relation, it is difficult to harmonize their strategies with those of the new agents. Namely, one subgame can not make a monopoly. The state that a

monopoly was destroyed is shown with A in Fig 4 and Fig 5. After that, the agents in the subgame keep the moving probability according to Fig 5. Namely, the entire relation keeps stable by the agents' changing the relations because the agents can not make monopoly of the relation. Because of this influence, a fixed relation does not exist, and formation and collapse of the relations occur repeatedly.

## 5 Conclusion

We focused on the formation of interdependence as one of the emergent phenomena and proposed Selecting Game for analyzing it. The results we confirmed are summarized as follows: (1) in the case of the relation in which the players always get the routine payoff, one relation was accepted among the agents and become stable; and (2) in the case of using agents with the abilities to adaptively change the strategies, the agents were absorbed in selecting a subgame early in the iterations. After that, the relations were dynamically changed like the relations in ecosystem or social system.

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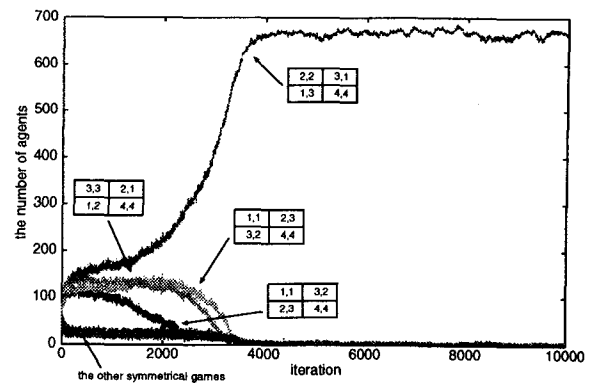


Fig. 3: The number of agents in each subgame. The number of population is 750.

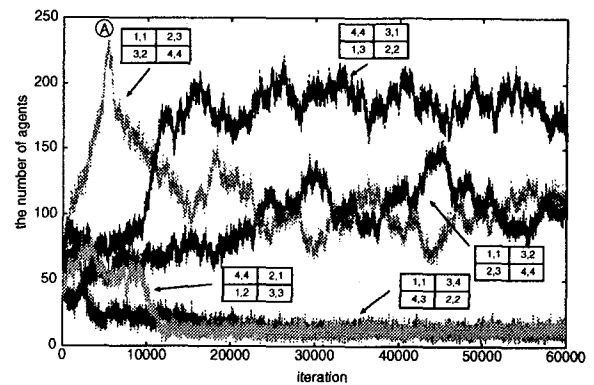


Fig. 4: The number of the agents in each subgame. In this case, the agent has the ability of changing the strategy adaptively. The number of population is 500.

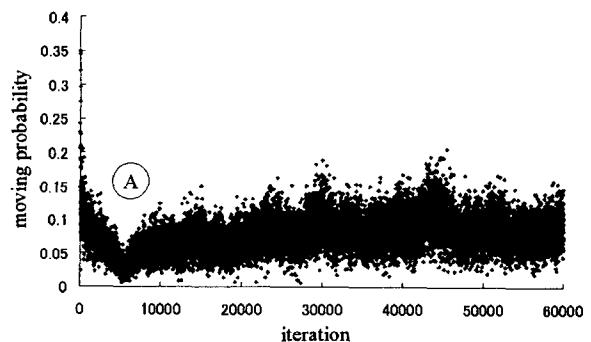


Fig. 5: The average of the probabilities,  $MP$ , of the agents in the subgame which could collect the most agents from 0 to 10000 iterations in Fig. 4