Steady-State Analysis of Reactance Oscillators having Multiple Oscillations

K.MATSUO, T.MATSUDA, Y.NISHIO, Y.YAMAGAMI and A.USHIDA
Department of Electrical and Electronic Engineering,
Tokushima University, Tokushima, 770-8056 JAPAN
Tel: +81-88-656-7469, Fax.: +81-88-656-7471
E-mail: ushida@ee.tokushima-u.ac.jp

Abstract: In this paper, we discuss an efficient steady-state analysis of reactance oscillators having multiple oscillations. Our oscillator is consisted of the Cauer or Foster reactance sub-circuit and a negative resistor such as tunnel diode. The reactance circuit has many resonance and anti-resonance points on the frequency response curve. Such a circuit having the specified resonance and anti-resonance points can be easily synthesized with the fundamental circuit theory [6]. In this case, the multiple oscillations may occur near at the anti-resonance points. We have developed a user friendly simulator for getting the exact steady-state responses using the SPICE.

1. Introduction

Analysis of oscillator circuits is very important to design the modulators and mixers which have two signals of the signal input and a local oscillator. There have been published many papers for the analysis of steady-state responses of communication circuits driven by multiple inputs [1-3,7], where they have many frequency components because of the intermodulations. On the other hand, some papers discuss the steady-state analysis of oscillator circuits [4,5,7]. There are two fundamental techniques of the time-domain and the frequency-domain relaxation methods. Of course, if the transient term does not continue for a long period, we can easily calculate the steady-state response using the transient analysis of SPICE. However, there are many types of high Q oscillators such as crystal oscillators and Hartley, which are widely used because of the frequency stability. Unfortunately, the attenuation in the transient response is usually very small, which causes very difficult to calculate the steady-state response by the transient analysis.

In this paper, we propose an efficient method for calculating the steady-state multiple oscillations, which is based on the circuit partitioning in the frequency domain techniques. We have developed a simulator combining both C-program and SPICE fundamental tools.

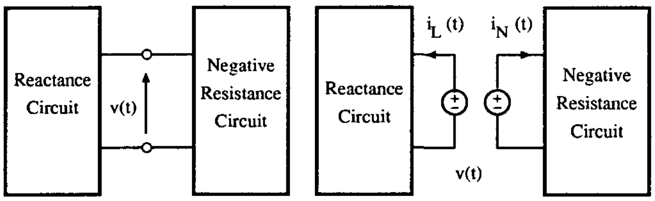
The multiple reactance oscillators are consisted of a ladder reactance sub-circuit such as the Cauer or Foster circuit and a negative resistor element. The reactance sub-circuit may have many resonance and anti-resonance points on the frequency response curve. We found that the periodic oscillations may be happened near at the anti-resonance points, where the energy consumption of the reactance circuit becomes smallest [9-10]. It is sometimes happened the quasi-periodic oscillation at one of the multiple oscillations, where the steady-state oscillation seems to become unstable depending on the Hopf bifurcation [11].

2. Basic Algorithm

To focus on the main ideas of our algorithm, consider a circuit shown in Fig.1. Assume that the reactance circuit consists of the Cauer or Foster circuit, where it may contain small parasitic resistances in series to the inductors, and the nonlinear circuit has a negative resistance characteristic. Suppose that the driving point impedance has p resonance and the same number of anti-resonance frequencies excluding zero and infinity frequencies. Then, the oscillator may happen to oscillate near at the p anti-resonance frequency points. The fundamental frequencies ω_k, $k = 1, 2, \ldots, p$ are a function of the nonlinear characteristic, whose frequencies can be decided in our iteration algorithm. Since the oscillator is autonomous system, we can arbitrarily choose time axis in the analysis [8]. Thus, the waveform can be described by

$$v_k(t) = V_{k,0} + V_{k,1} \cos \omega_k t + \sum_{n=2}^{N} \{V_{k,2n-1} \cos n\omega_k t$$

$$+ V_{k,2n} \sin n\omega_k t\}, \quad k = 1, 2, \ldots, p \qquad (1)$$



(a) Reactance oscillator (b) Circuit partition

Fig.1 Circuit partitioning technique

The original circuit shown in Fig.1(a) may have many types of the steady-state oscillations, and describe one of them by (1). Then, the *substitution theorem* says that the steady-state solution $v_k(t)$ satisfies the following *determining equation*:

$$F_k(v_k(t)) = i_{k,L}(t) + i_{k,N}(t) = 0, \quad k = 1, 2, \ldots, p \qquad (2)$$

at the partitioning point, where $i_{k,L}(t)$ and $i_{k,N}(t)$ are responses of the linear reactance sub-circuit and the nonlinear resistive circuit, respectively.

Let us solve it by an iteration method, and assume the waveform at the jth iteration by

$$v_k^j(t) = V_{k,0}^j + V_{k,1}^j \cos \omega_k^j t + \sum_{n=2}^{N} \{V_{k,2n-1}^j \cos n\omega_k^j t$$

$$+ V_{k,2n}^j \sin n\omega_k^j t\} \qquad (3)$$

To get the solution at $(j+1)$st iteration, set

$$v_k^{j+1}(t) = v_k^j(t) + \Delta v_k(t) \qquad (4)$$

where

$$\Delta v_k(t) = \Delta V_{k,0} + \Delta V_{k,1} \cos \omega_k^{j+1} t$$

$$+ \sum_{n=2}^{N} \{\Delta V_{k,2n-1} \cos n\omega_k^{j+1} t + \Delta V_{k,2n} \sin n\omega_k^{j+1} t\} \qquad (5.1)$$

$$\omega_k^{j+1} = \omega_k^j + \Delta\omega_k \qquad (5.2)$$

Substituting (4) into (2), we have

$$\frac{\partial i_{k,L}}{\partial v_k} \Delta v_k + \sum_{k=1}^{N} \frac{\partial i_{k,L}}{\partial \omega_k} \Delta\omega_k + \left.\frac{\partial i_{k,N}}{\partial v_k}\right|_{v_k = v_k^j} \Delta v_k$$

$$= -\varepsilon_k^j(t) \qquad (6.1)$$

for the residual error

$$\varepsilon_k^j(t) = i_{k,L}^j(t) + i_{k,N}^j(t) \qquad (6.2)$$

For simplicity, we assume that

$$\left.\frac{\partial i_{k,N}}{\partial v_k}\right|_{v_k = v_k^j} \approx \left.\frac{\partial i_{k,N}}{\partial v_k}\right|_{v_k = v_k^0} \qquad (7)$$

and set

$$D_0 = \left.\frac{\partial i_{k,N}}{\partial v_k}\right|_{v_k = v_k^0}$$

where v_k^0 is the dc operating point. Now, applying the *harmonic balance method* to (6), we have

$$\mathrm{Re}\{Y(0)\}\Delta V_{k,0} + D_0 \Delta V_{k,0} = -\varepsilon_{k,0}^j \qquad (8.1)$$

$$\mathrm{Re}\{Y(\omega_k^j)\}\Delta V_{k,1} + D_0 \Delta V_{k,1} = -\varepsilon_{k,1}^j \qquad (8.2)$$

$$-\mathrm{Im}\{Y(\omega_k^j)\}\Delta V_{k,1} - \frac{\partial \mathrm{Im}\{Y(\omega_k)\}}{\partial \omega_k} V_{k,1} \Delta\omega_k = -\varepsilon_{k,2}^j \qquad (8.3)$$

$$\{\bar{Y}(n\omega_k^j) + D_0\}(\Delta V_{k,2n-1} + j\Delta V_{k,2n}) = -\varepsilon_{k,2n-1}^j$$

$$-j\varepsilon_{k,2n} + \frac{\partial \mathrm{Im}\{Y(n\omega_k)\}}{\partial \omega_k}(V_{k,2n-1} + jV_{k,2n})\Delta\omega_k \qquad (8.4)$$

$$n = 2, 3, \ldots, N$$

where $Y(\cdot)$ and $\bar{Y}(\cdot)$ are the admittance of reactance circuit, and the complex conjugate, respectively. Observe that the variational value of the fundamental frequency component $\Delta V_{k,1}$ and $\Delta\omega_k$ are calculated from (8.2) and (8.3), respectively. The variational value at the high frequency components $\Delta V_{k,2n-1}$ and $\Delta V_{k,2n}$ are calculate from (8.4). The algorithm belongs to the *relaxation method*[5], and it can be applied weakly nonlinear circuits, efficiently.

Remark that *the reactance oscillator may happen to oscillate near at the anti-resonant frequency point.*

Proof In order that the circuit has a nonzero amplitude steady-state oscillation, it satisfies the relations (8.2) and (8.3) for nonzero $\Delta V_{k,1}$ and $\Delta\omega_k$. Since, from the assumption of (1), $\varepsilon^j_{k,2}$ in (8.3) is always setted to be zero, we can assume a nonzero amplitude $\Delta V_{k,1}$ at the anti-resonance frequency point($\Delta\omega_k = 0$) because $\mathrm{Im}\{Y(\omega^j_k)\}$ is always zero at the frequency.

On the other hand, since $\mathrm{Im}\{Y(\omega^j_k)\}$ is very large at the resonant point in (8.3), the amplitude must be $\Delta V_{k,1} \cong 0$. Therefore, the circuit never oscillates at the resonant frequency points of the reactance sub-circuit. Thus, we can prove the important property. The result is exactly equal to the theorem [9,10] saying that the reactance oscillator may oscillate at the frequency, where the energy consumption of a reactance circuit being smallest.

Now, our algorithm is as follows: We first construct a reactance sub-circuit by specifying the resonant and anti-resonant frequencies. The nonlinear sub-circuit is constructed by a negative resistor such as tunnel diode. Suppose that the driving point characteristic of the nonlinear resistive circuit is as follows:

$$i_N = -c_1 v + c_3 v^3$$

Assuming $v_k = V^0_{k,0}\cos\omega_k t, \quad k = 1, 2, \ldots, p$ at the anti-resonant points, we can calculate the initial guess from (8.2) and (8.3) as follows:

$$G - c_1 + 1.5c_3 {V^0}^2 = 0 \Rightarrow V^0 = \sqrt{\frac{c_1 - G}{1.5c_3}}$$

where $G = Re\{Y(j\omega_k)\}$.

0. Set kth frequency of the oscillations, the maximum frequency component N and $j = 0$.

1. Calculate the response($i^j_{k,N}$) of nonlinear resistive circuit , and describe it in the Fourier series using FFT.

2. Calculate the response($i^j_{k,L}$) of linear reactance circuit by the phasor technique.

3. Calculate the residual error (6.2), and describe it in the form of the Fourier series. If it satisfies $||\varepsilon^j_k(t)|| < \delta$ for a small specified δ, then stop. Otherwise, go to 4.

4. Calculate the variational value $\Delta V_{k,0}$, $\Delta V_{k,1}$, $\Delta V_{k,3}, \ldots$, $\Delta V_{k,2N}$ and $\Delta\omega_k$ from (8.2), (8.3) and (8.4). Thus, we have

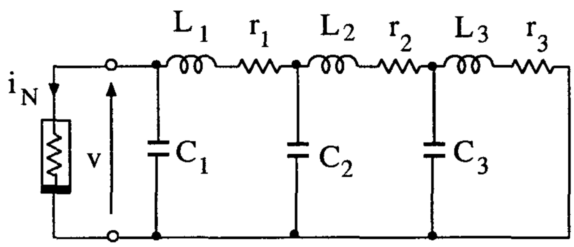
$$v^{j+1}_k(t) = v^j_k(t) + \Delta v_k(t), \quad \omega^{j+1}_k = \omega^j_k + \Delta\omega_k$$

5. Using the ac-sweep of SPICE, calculate $Y(n\omega^{j+1}_k)$ and $\frac{\partial \mathrm{Im}\{Y(n\omega^{j+1}_k)\}}{\partial\omega^{j+1}_k}$ in (8.2)-(8.4). Set $j = j + 1$ and go to 1.

Our algorithm is called the *frequency domain relaxation method*, and it can be efficiently applied any kind of weakly nonlinear circuits.

3. An Illustrative Example

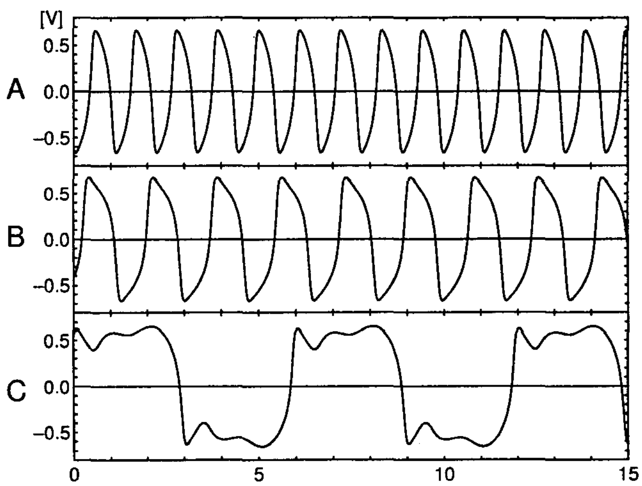
Consider a reactance oscillator shown in Fig .2(a). The linear reactance circuit is synthesized by the Cauer circuit, whose anti-resonance points are chosen at $\omega_1 = 1$, $\omega_2 = 4$ and $\omega_3 = 6$, respectively.



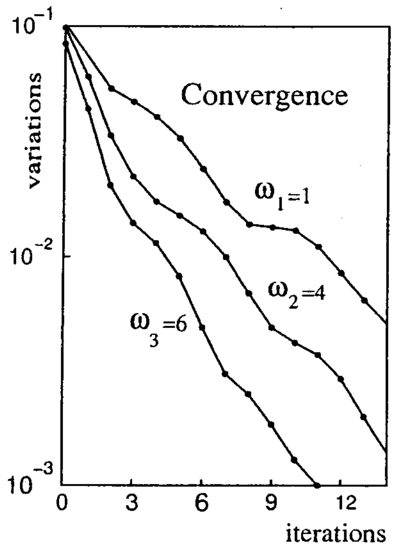
(a) Reactance oscillator

$C_1 = 0.100$, $C_2 = 0.343$, $C_3 = 0.439$, $L_1 = 0.417$, $L_2 = 0.262$, $L_3 = 1.05$

$r_1 = r_2 = r_3 = 0.01$



(b) The steady-state waveforms of 3 type oscillations.



(c) Convergence ratio

Fig.2 Reactance oscillator

The characteristic of nonlinear resistor is chosen by

$$i_N = -v + 3v^3$$

We found that the reactance oscillator has 3 types of oscillations as shown in Fig .2(b) depending on the initial conditions. The fundamental frequency components are as follows:

$$\omega_1 = 1.0531, \qquad \omega_2 = 3.6210, \qquad \omega_3 = 5.6947$$

which are something different from the original anti-resonant frequencies. The nonlinearity for the first case ω_1 is largest and the convergence ratio is smallest, where we consider 18 higher harmonic components, and 7, 5 components for second ω_2 and ω_3 cases, respectively.

4. Conclusions and Remarks

In this paper, we have proposed an analytical method of reactance oscillators whose linear sub-circuit is consisted by the Cauer or Foster circuit, and the nonlinear sub - circuit consisted by negative resistance circuit. We have efficiently applied the frequency domain relaxation method to solve it. We found that the circuit has multi - type oscillations around the vicinity of the anti-resonance frequency points.

As the future problem, we need to solve the quasi-periodic oscillations which will happen in the weakly nonlinear circuits.

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