Improvement of Tomographic Imaging in Coded Aperture System based on Simulated annealing

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Abstruct: In this paper, we propose a new method based on SA(simulated annealing) with a fast algorithm for 3D image reconstructrion from the coded apereture images[1]. The reconstructed images can be significantly improved by SA and to large computation cost of SA can be significantly reduced by the fast algorithm.

1.Introduction

A number of coded aperture imaging (CAI) techniques have been used or proposed for x-ray imaging[2]. The advantage offered by the CAI lay in its large photon collection efficiency due to its large open area. Uniformly redudant arrays (URA)[3] is one of CAI techniques. In URA coded aperture imaging, the pinhole is replaced by multi-pinholes arrays arranged in m-sequences[4]. So the URA camera can provide a two-dimensional image with a high resolution and high SN ratio. Furthermore, since the URA camera can view the object with a large solid angle, it can also provide some tomographic resolution for three-dimensional object[5][6]. Since the reconstructed tomographic images contain serious defocus artifacts, we propose a new method based on SA to reconstruct the tomographic images from the coded image. Since the SA is an iterative algorithm, the limitation of the SA is its large computation cost, we also propose a fast algorithm to reduce the large computation cost.

2.Tomographic imaging[7]

2.1 The basic concept of tomographic imaging

The basic concept of tomographic imaging using URA coded aperture is shown in Fig.1.

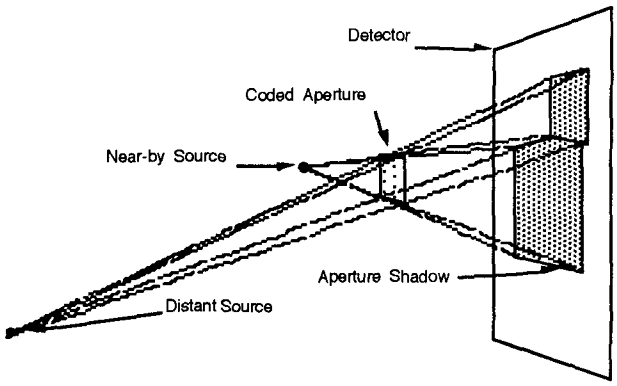


Fig.1. The basic concept of tomographic imaging Using URA coded aperture imaging.

Sources distant to the detector cast smaller aperture shadows than closer sources. The size of the shadow depend on the lateral displacement of the point. By correlating the recorded image with decoding patterns of different sizes, images of the source distribution at different depths can be retrieved.

The concept of the tomographic imaging can be expressed in simple mathematical terms. $P(x,y)$ is encoded image at position (x,y) on the detector and $O_z(x,y)$ is the distribution in a plane parallel to the aperture with a distance of z. If $A_z(x,y)$ is the appropriately magnified version of the aperture for the distance z, $P(x,y)$ can be expressed as

$$P(x,y) = \sum_{z=1}^{N}\left[O_z(x,y) * A_z(x,y)\right] \qquad (1)$$

That is, the coded image is the sum of the correlation of each object plane with an aperture pattern of $G_z(x,y)$, resulting in the estimate:

$$\hat{O}_z(x,y) = \sum_{z'=1}^{N}\left[O_{z'}(x,y) * A_{z'}(x,y)\right] * G_z(x,y)$$
$$= O_z(x,y) + \Delta\hat{O}_z \qquad (2)$$

where

$$\Delta\hat{O}_z = \sum_{z'=1}^{z-1} O_{z'}(x,y)*\left[A_{z'}(x,y)*G_z(x,y)\right] + \sum_{z'=z+1}^{N} O_{z'}(x,y)*\left[A_{z'}(x,y)*G_z(x,y)\right] \qquad (3)$$

It can be seen that only the z-th plane will be in focus, while all the others will be out of focus. Therefore, it is clear that if second term of Eq(2) becomes zero, we can obtain a perfect reconstruction of z-th plane. Since usually the $A_{z'} * G_z$ is not a delta function, the second term appears on the z-th plane as a defocus artifact (Eq(3)). The larger difference is the z and z', the smaller is the defocus artifact.

2.2 computer simulation

We have carried out computer simulations to study the performance of tomographic imaging using URA camera. The simulated objects consist three planar objects O_1, O_2, O_3, which are parallel to the aperture, as shown in Fig.2.

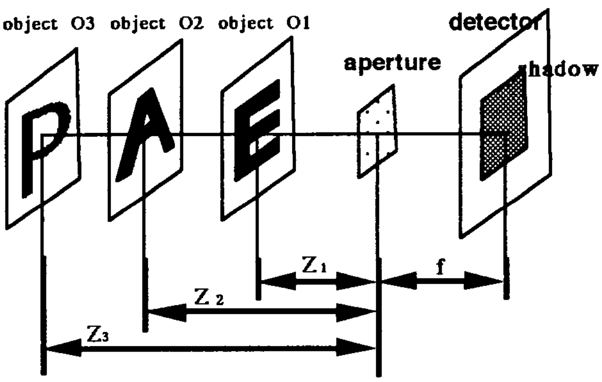


Fig.2. An arrangement of URA camera for tomographic imaging

The magnifications for O_1, O_2 and O_3 are $M_1 = f/z_1 = 5$ and $M_2 = f/z_2 = 4$, $M_3 = f/z_3 = 3$, respectively. The encoded image on the detector plane is the sum of the correlation of each object with an aperture pattern of appropriate magnification. The coded images with corresponding magnification are shown in Fig.3(a), and the reconstructed tomographic images with the corresponding coded image are shown in Fig.3(b), (c), (d), respectively. It can be seen that reconstructed images are significantly blurred by defocus artifacts.

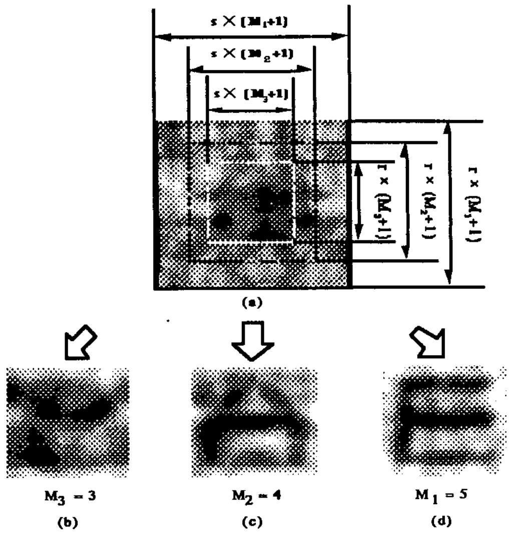


Fig.3. Reconstructed result of tomographic imaging

3.SA-based image reconstruction

In this paper, we propose to use simulated annealing (SA) to reconstruct the tomographic images from the coded image. The SA is an optimization technique based on calculation of state function in statistical mechanics.The flowchart of the SA-based image reconstruction is shown in Fig.4.

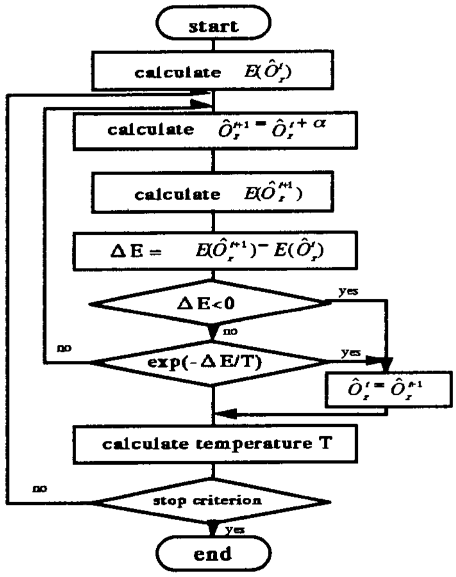


Fig.4. Flowchart of a SA for image reconstruction

The cost function E is shown in Eq(4).

$$E(\hat{O}_z^t) = \|P - \hat{P}^t\|^2$$
$$\hat{P}^t = \sum_{z=1}^{N}\left(\hat{O}_z^t * A_z\right) \qquad (4)$$

where $\hat{O}_z^t$ is the estimate of O_z, t is the iteration number. We want to find an estimate of O_z which minimize the cost function Eq(4). As shown in Fig.(4), we add a small random perturebation α on each element $\hat{O}_z^t$ and make a new estimation $\hat{O}_z^{t+1}$. Next caluculate the difference ΔE between $E(\hat{O}_z^{t+1})$ and $E(\hat{O}_z^t)$ after and before the perturebation and the image become a new one. On the other hand, if $\Delta E > 0$, the new estimate is accepted or rejected (make no change to image) with the acceptance probability $\exp(-\Delta E / T)$, where T is the control parameter known as a temprature.

We have carried out computer simulations to validate the applicability of the proposed method to image reconstruction. The parameters for the simulated annealing, which are obtained through several testing run, are shown below:

$$T_0 = 200$$

$$T_{n+1} = \frac{T_0}{1+n}$$

$$\alpha = \frac{1}{\log(n)+1}$$

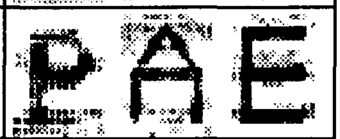
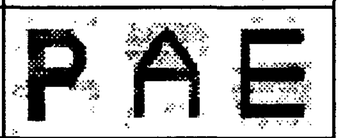
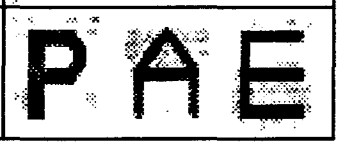
Iteration (n)	Reconstructed image
0 (initial image)	
25	
50	
75	
100	

Fig.5. Reconstructed images based on SA

The image error Ie is defined as Eq(5),

$$Ie_{(z)} = \frac{\| O_z(x,y) - \hat{O}_z(x,y) \|^2}{\| O_z(x,y) \|^2} \qquad (5)$$

where $O_z(x,y)$ is pixel value of the original image and $\hat{O}_z(x,y)$ is the pixel value of the reconstructed image. The reconstructed images based on SA are shown in Fig.5. The reconstructed images without SA are used as initial estimates. It can be seen that the reconstructed images are improved an increasing the iteration number. The image errors are shown in Fig.6. The image error are significantly improved.

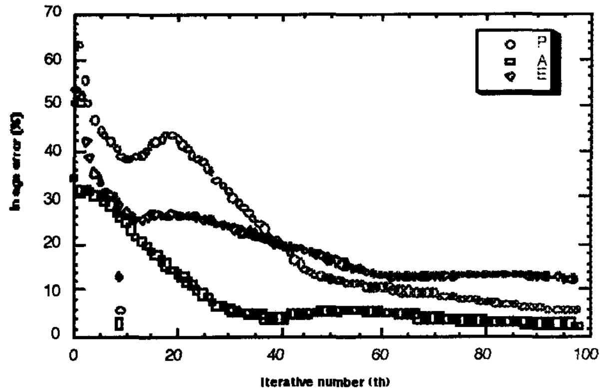


Fig.6. Image error are improved as increasing the iteration number

4.Fast algorithm

The SA has been demonstrated that it can reconstruct image well. But as shown in section 3, the SA is an iterative algorithm. The 3D correlation calculation(Eq (4)) should be done for each pixel and each annealing. It takes a large conputation cost. Therefor, we also propose a fast algorithm to reduce the large computation cost. Since only one pixel (a,b) value are perturebated from $\hat{O}_z^t(a,b)$ to $\hat{O}_z^{t+1}(a,b)$, caluculation of $\hat{P}^t$ can be simplified as Eq.(6).

$$\hat{P}^{t+1}(k,l) = \hat{P}^t(k,l) + A(\mathrm{mod}(a+k,r), \mathrm{mod}(b+l,s)) \times (\hat{O}_z^{t+1}(a,b) - \hat{O}_z^t(a,b)) \quad (6)$$

So we just need to caluculate only the initial coded image with 3D correlation of Eq.(4), the others can be calculated by simplified Eq.(6).

The improvement of the computation time by the proposed fast algorithm is shown in Fig.7. It can be seen that the computation time was improved by a facter of 10.

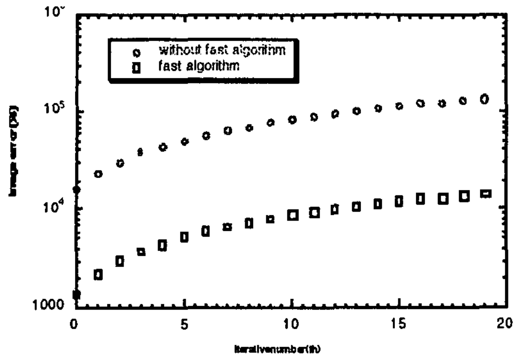


Fig.7. Comparison of computation time

5.Summary

We have proposed a new method based on SA with a fast algorithm for 3D image reconstruction from the coded aperture images. The reconstructed images can be significantly improved by SA and to large computation cost of SA can be significantly reduced by the fast algorithm.

References

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