

# A Comparison of FFH/SSMA and DS/CDMA Communications in a Rician Fading Channel \*

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## Abstract

In this paper, we compare the bit-error-rate performance of the fast-frequency-hopped spread-spectrum multiple-access (FFH/SSMA) and direct-sequence code-division multiple-access (DS/CDMA) systems in a Rician fading channel. Each system has a same data rate, bandwidth and transmits over a Rician fading channel. The results illustrate tradeoffs in performance between the FFH/SSMA and DS/CDMA systems as a function of the parameters such as average signal to noise ratio and processing gain. The performance of the FFH/SSMA system is shown to be less sensitive to the change of fading environments, while the change of processing gain and average signal to noise ratio gives considerable affect to the FFH/SSMA system compared with the DS/CDMA system. Without respect to the change of system parameters, for most of Rician fading channels (except non-fading channel), FFH/SSMA system gives better performance than DS/CDMA system in  $BER < 10^{-3}$ .

## I. INTRODUCTION

The fast-frequency-hopped spread-spectrum multiple-access (FFH/SSMA) and direct-sequence code-division multiple-access (DS/CDMA) are often considered as alternatives for a given circumstance. The choice of the DS/CDMA system as a multiple access technique is attractive because of its potential capacity increase and other factors such as privacy and multipath rejection capabilities [1]. On the other hands, the FFH/SSMA technique is utilized widely in military communications, owing to its low probability of intercept (LPI) and antijamming capability [2]. The reason is why the FFH/SSMA and DS/CDMA systems use basically different methods to combat a fading channel and provide multiple access. The DS/CDMA system spreads narrowband signals over a much large bandwidth by using a unique code sequence, while the FFH/SSMA system covers a large bandwidth by generating a large number of carrier frequencies in a short period time, although only one carrier occupies a small bandwidth.

\*This work is supported in part by BK-21 Project and ADD, Korea.

Although such differences of the FFH/SSMA and DS/CDMA systems gives different performance characteristics, a comparison of the FFH/SSMA and DS/CDMA systems has been investigated only for very specific circumstances (e.g., [3]). Especially, in a comparison of the two systems, both the systems have used the same modulation scheme, or the same M-ary signaling with different modulation schemes to apply the equal complexity. However, such comparisons may be unfair to one of the systems. The binary phase-shift-keyed (BPSK) modulation, or quadrature phase-shift-keyed modulation is suitable to the DS/CDMA system. However, in the FFH/SSMA system, due to the rapid shifting of the transmitted frequency, it is difficult to maintain carrier phases synchronization. Hence, it is proper to choose the non-coherent frequency-shift-keyed (FSK) modulation for the FFH/SSMA system. In addition, if the FFH/SSMA system uses a M-ary signaling, the number of hops and bits per code word can be optimized to give a better performance for a given bandwidth and information rate. Therefore, we use the FFH/SSMA system with M-ary non-coherent FSK. The complexity of the FFH/SSMA system may relatively be higher than that of the DS/CDMA system. For the comparison of the FFH/SSMA and DS/CDMA systems, in this paper, we focus on the average probability of bit error rather than the complexity of the two systems for a single cell, and both two systems uses pure modulation schemes which do not include coding strategy and power control.

This paper is organized as follows: In Section II and III, the system models and an expression for the average probability of bit error for a Rician fading channel that are used in the comparison are described. In Section IV, numerical results are shown as changing various system parameters such as average received SNR and PG for a Rician fading channel. Finally, conclusions are drawn in Section V.

## II. FFH/SSMA SYSTEM DESCRIPTION

The FH system uses MFSK as the modulation scheme. At the transmitter, each user is assigned a unique address,

which is a sequence of  $L$   $K$ -bit code words. During a symbol duration  $T$ , each code word of the address is added modulo  $-2^K$  to the buffered  $K$ -bit message of a user to produce a modulated sequence of length  $L$ . The modulated sequence is then used by the tone generator to select the corresponding signal tones from the available  $2^K$  orthogonal frequencies. At the receiver, each one of the  $2^K$  frequency bins is determined every  $\tau$  (sec) whether or not a tone is present, by comparing the energy level in the frequency bin against a threshold level  $b$ . After the tones are detected, they are transformed back into the corresponding code words and subtracted from the address identical to that used in transmitter. In practice, AWGN and multipath fading can cause a transmitted tone to be omitted, i.e., deletion, or a false tone to be incorrectly detected, i.e., false alarm.

The received signal of  $i$ th user can be written as

$$r(t) = d_i(t) + \sum_{z=1}^{Z-1} I_z(t) + n(t) \quad (1)$$

where  $d_i(t)$  is the desired signal,  $I_z(t)$  is the interference caused by the  $z$ th interference,  $Z$  is the total number of users, and  $n(t)$  represents AWGN with double-sided power spectrum density of  $N_0/2$ . The desired signal can be represented as

$$d_i(t) = \sum_{l=0}^{L-1} \beta_l \sqrt{2P} p_\tau(t - l\tau) \cos[2\pi(f_0 + \frac{y_{il}}{\tau})t + \phi_l] \quad (2)$$

where  $P$  is the average signal power,  $f_0$  is the lowest frequency,  $\beta_l$  is a Rician distributed path gain, and  $y_{il}$  is the modulated value of  $l$ th hop for  $i$ th user.  $p_\tau(t)$  is the rectangular pulse defined during interval  $(0, \tau)$ . The signal of  $z$ th interference is modeled as

$$I_z(t) = \sum_{n=0}^{M-1} \sum_{l=0}^{L-1} \beta_{znl} \sqrt{2P} p_\tau(t - l\tau) \times \cos[2\pi(f_0 + \frac{n}{\tau})t + \phi_{znl}] \quad (3)$$

where  $M$  is the number of symbols.

### A. False Alarms and Deletions

By considering the transmission of each tone as noncoherent on-off keying, we can define the probability of false alarm  $P_F$  as the probability that the threshold will be exceeded by an energy level in a frequency bin containing no signal. This energy basically comes from the background noise. The expression for false alarm probability is represented as follows [5]:

$$P_F = \exp(-b_0^2/2) \quad (4)$$

where  $b_0 = b/\sqrt{N}$  denotes the actual threshold level  $b$  normalized by the average noise power  $N$ .

Unlike false alarms, deletions are due largely to both multipath fading and AWGN. Therefore, different types of fading will lead to different deletion probabilities. In

Rician channels, the probability distribution of the envelope  $r$  of the received signal can be expressed as

$$P(r) = \frac{r}{N + \alpha} \exp\left[-\frac{r^2 + u^2}{2(N + \alpha)}\right] I_0\left(\frac{ru}{N + \alpha}\right) \quad (5)$$

where  $\alpha$  is the average power of the multipath portion,  $u$  is the amplitude of the specular component, and  $I_0$  represent the modified Bessel function of the first kind and zeroth order. The probability of deletion may be defined as the probability that the envelope of a tone is smaller in magnitude than a threshold. For Rician fading, we define a parameter called Rician factor as

$$\rho = \frac{\text{power in specular component}}{\text{power in multipath component}} = \frac{u^2}{2\alpha} \quad (6)$$

The deletion probability is represented as follows [6]:

$$P_D = 1 - Q\left(\sqrt{\frac{2\rho\gamma_0}{1+\rho+\gamma_0}}, b_1\right) \quad (7)$$

where

$$b_1 = b_0/\sqrt{1 + (\gamma_0/(1 + \rho))} \quad (8)$$

and  $\gamma_0$  is average received SNR and  $Q(a, b)$  is Marcum's  $Q$  function

### B. BER Performance

In analyzing BER performance of FFH/SSMA, we employ the majority decision rule. By using the probability of entries existing in a spurious row (or correct row) of the decision matrix, the upper bound on the BER is represented as follow [6]:

$$P_B < \frac{2^{K-1}}{2^K - 1} \left(1 - \sum_{i=0}^L P_C(i) \left[ P(i, 0) + \frac{1}{2} P(i, 1) \right] \right) \quad (9)$$

where  $P_C(i)$  is the probability of  $i$  entries in the correct row, and  $P(n, k)$  is the probability of exactly  $k$  rows containing  $n$  entries when  $n$  is the maximum number of entries over the  $2^K - 1$  incorrect rows.

## III. DS/CDMA SYSTEM DESCRIPTION

We consider DS/CDMA system using BPSK modulation over Rician fading channels. In this system, each active users is assigned a unique code or signature sequence which identifies the user to the base station.

The received signal for a given user can be written as

$$r(t) = \sum_{k=1}^K \sum_{l=1}^L \sqrt{2P_k} \beta_{lk} b_k(t - t_{lk}) a_{lk}(t - t_{lk}) \times \cos(\omega_c t + \phi_{lk}) + n(t) \quad (10)$$

where  $P_k$  is the transmitted signal power of user  $k$ ,  $b_k(t)$  is a binary data signal, and  $a_k(t)$  is a signature sequence

Table 1: The optimum parameters of FFH/SSMA system.

Processing Gain (PG)	No. of bits per code word (K)	No. of hops per code word (L)
32	4	8
64	5	10
128	6	12
255	7	13
625	8	19

signal.  $\phi_k$  and  $t_k$  are the carrier phase and transmitter time delay of the  $k$ th user, respectively.  $\beta_k$  is Rician distributed path gain,  $L$  is the number of path, and  $K$  is total number of users.

### A. Decision Statistic

Without loss of generality, we shall restrict our consideration to the  $i$ th user and calculate the average probability of error of the data bit  $b_i^0$  in the signaling interval. Further, we will assume that the desired receiver can coherently recover the carrier phase and delay lock to the first arriving desired signal. Hence, we can set  $t_{1i} = \phi_{1i} = 0$ .

Next let  $Z$  denote the output of the correlator receiver matched to user  $i$  at  $t = T_b$ . Then

$$\begin{aligned} Z &= \int_0^{T_b} r(t) a_i(t) \cos(\omega_c t) dt \\ &= \eta + \sqrt{\frac{P_i}{2}} b_0^i T_b g_{1i} + T_b \sqrt{\frac{P_i}{2}} \sum_{l=2}^L I_l^i \\ &\quad + T_b \sqrt{\frac{P_k}{2}} \sum_{k=1, k \neq i}^K \sum_{l=1}^L I_l^k \end{aligned} \quad (11)$$

where

$$\begin{aligned} \eta &= \int_0^{T_b} n(t) a_i(t) \cos(\omega_c t) dt \\ I_l^k &= \frac{1}{T_b} g_{lk} \cos(\phi_{lk}) [ b_{-1}^{(k)} \chi_{ik}(t_{lk}) + b_0^{(k)} \hat{\chi}_{ik}(t_{lk}) ] \end{aligned} \quad (12)$$

$\chi_{ik}(\zeta)$  and  $\hat{\chi}_{ik}(\zeta)$  are the crosscorrelation function defined during interval  $(0, t_{lk})$  and  $(t_{lk}, T_b)$ , respectively.

### B. BER Performance

We present one of the simplest methods for estimating the BER of DS/CDMA systems; namely, the Gaussian approximation [7].

Let  $Z$  be defined as above and suppose that the signature sequences are random and uniform on  $(-1, 1)$ . Then it can be easily shown that

$$E[(b_{-1}^k \chi_{ik}(t_{lk}) + b_0^k \hat{\chi}_{ik}(t_{lk}))^2] = \frac{2}{3\Lambda} T_b^2 \quad (13)$$

where  $\Lambda$  is the minimum period of the spreading sequence. By using above equation, we can easily show that the variance of  $Z$  is given by

$$\text{Var}[Z] = \frac{P}{2} T_b^2 \left( \frac{2\rho(L-1)}{3\Lambda} + \frac{2\rho L(K-1)}{3\Lambda} \right) + \frac{N_o T_b}{4} \quad (14)$$

where  $\rho = (1/2)E[g_{lk}^2]$ . The Gaussian approximation can now be obtained by assuming that the total interference

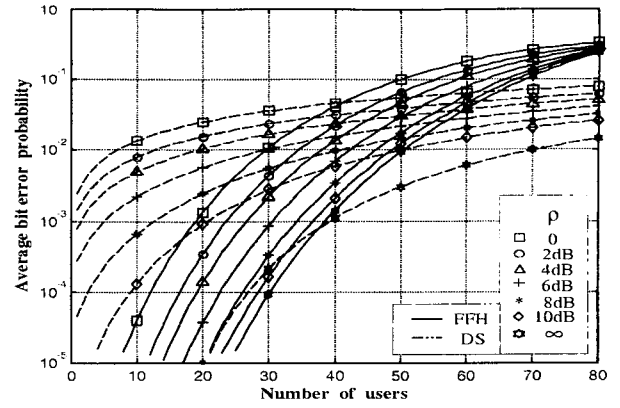


Figure 1: The comparison of FFH/SSMA and DS/CDMA systems for Rician fading (received SNR = 20dB, PG = 128).

has Gaussian distribution. We consider  $L = 1$ . Then BER of DS/CDMA in Rician fading is represented like this

$$\begin{aligned} P_B &= \int_0^\infty Q \left( \sqrt{\frac{\frac{1}{2}\gamma}{\frac{1}{2}(K-1)\frac{1}{3\Lambda}\gamma_0 + \frac{1}{4}}} \right) \times \\ &\quad \frac{K+1}{\gamma_0} \exp \left[ -\frac{K+1}{\gamma_0} \gamma - K \right] I_0 \left[ 2\sqrt{\frac{K^2+K}{\gamma_0} \gamma} \right] d\gamma \end{aligned} \quad (15)$$

where  $\gamma$  is instantaneous received SNR which changes as the fading environment, and  $\gamma_0$  is average received SNR.

## IV. NUMERICAL RESULTS

In this section, the capacity of the FFH/SSMA and DS/CDMA systems is evaluated. For the comparison of the two systems, we focus on the average probability of bit error for a single cell system. Each system has the same available bandwidth and transmits at the same information rate. We optimize the parameters of FFH/SSMA system, which is shown in table.1 The optimum values of  $K$  and  $L$  can be obtained using the search method which maximizes the number of users for a specified values of  $P_B$ ,  $P_F$ ,  $P_D$ , and the bandwidth  $W_{ss}$ . In Fig.1, the performance of the FFH/SSMA and DS/CDMA systems with SNR = 20dB and PG = 128 is compared in a Rician fading channel. This figure shows that DS/CDMA system is more sensitive to the change of fading factor than the FFH/SSMA system, and the FFH/SSMA system performs better at low BER for all Rician factors. However, in case of no fading, where the Rician factor is infinite, DS/CDMA system can accommodate more users at BER < 10<sup>-3</sup>. Therefore, the FFH/SSMA system is relatively more tolerant in the fading channel. However, as the number of user increases the performance of the FFH/SSMA system decreases faster, whereas that of DS/CDMA system is relatively slow. Fig.2 shows the BER performance of the FFH/SSMA and DS/CDMA systems with PG = 128 and Rician factor = 8dB for various SNRs. This figure shows that FFH/SSMA system is more sensitive to the change

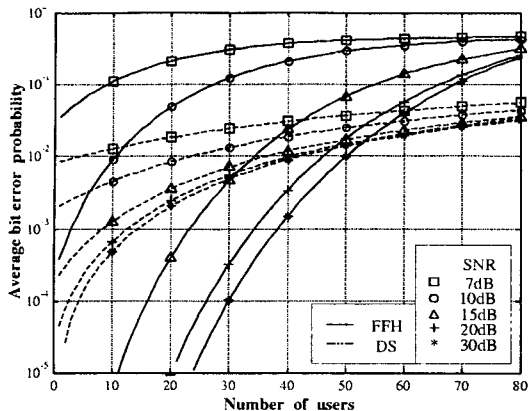


Figure 2: The comparison of FFH/SSMA and DS/CDMA systems contaminated by Rician fading for various SNRs (Rician factor = 8dB, PG = 128).

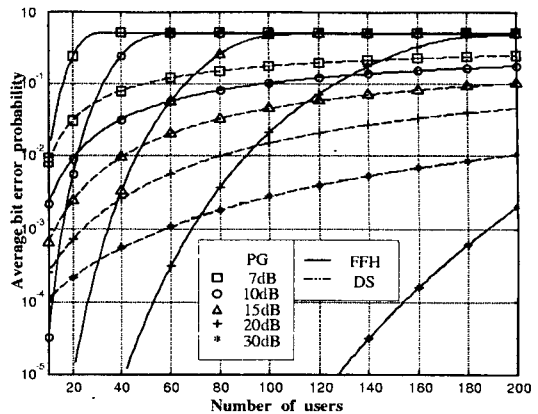


Figure 3: The comparison of FFH/SSMA and DS/CDMA systems contaminated by Rician fading for various PGs (average SNR = 20dB, Rician factor = 8dB).

of average received SNR. In practice, because FFH/SSMA system uses energy detection, SNR is one of the important parameters which determine the system performance. For example, when the number of user is about 20, the change of SNR from 15dB to 10dB reduces the BER performance about from  $6 \times 10^{-4}$  up to  $6 \times 10^{-2}$  in FFH/SSMA system, whereas the BER performance of DS/CDMA system is reduced by a small amount.

In a wideband transmission, a large processing gain is available. Therefore, we need to evaluate and compare the performance of two systems contaminated by a fading channel with various PGs. In Fig.3, the performance of the FFH/SSMA and DS/CDMA systems with average SNR = 20dB and Rician factor = 8dB is compared for various PGs. This figure shows that as the PG increases, FFH/SSMA system can accommodate more users with low BER in the same fading environment and same SNR. For example, when BER is  $10^{-3}$ , the change of PG from 255 to 625 increases the number of allowable users to about 190% in FFH/SSMA system, whereas the number of users in DS/CDMA system increases about 132%.

## V. CONCLUSION

In this paper, the performance of the FFH/SSMA and DS/CDMA systems, represented by the average probability of bit error, is investigated for a variety of Rician factors, SNRs, and PGs. The FFH/SSMA system is relatively insensitive to the change of Rician fading factors compared with DS/CDMA system. Numerical results show the performance of FFH/SSMA system due to the change of Rician fading changes gradually, whereas DS/CDMA system may be severely contaminated. However, in a given fading channel, FFH/SSMA system is more sensitive to the change of average SNR and PG. In addition, as the number of users increases the performance of FFH/SSMA system severely degrades, whereas the performance of DS/CDMA system gradually degrades. FFH/SSMA system can ac-

commodate more users with low BER in the same fading environment and same SNR as the processing gain increases.

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