Performance Bound of the OFDM Transmission with the Frequency Offset

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Abstract: It is well known that the OFDM system is weak against the frequency offset. The frequency offset attenuates the received signal power and makes the performance bound in the OFDM transmission. We show the performance bound of the OFDM transmission with the frequency offset under the additive white Gaussian noise channel by the numerical method. According to the results, increasing the transmitting power is not useful to improve the performance, when the frequency offset exits. The performance degradation is very severe as the frequency offset increases..

1. Introduction

Recently, the requirement for the high-speed transmission increases under the multipath channel environments. However, it is very difficult to achieve the high-speed transmission under multipath channels, because multipath channels make an Inter-Symbol Interference (ISI), which is the main source of the performance degradation. In the view of the frequency domain, the multipath channel can be represented as the frequency selective fading channel. One of solutions to overcome multipath channels is the Orthogonal Frequency Division Multiplexing (OFDM) transmission, because it changes the frequency fading into the frequency flat fading by using the divided subchannel with the small bandwidth comparing with the channel coherent bandwidth.. In addition, it has large bandwidth efficiency and con be easily implemented by using Fast Fourier Transform (FFT) and Inverse-FFT [1]. Practically, the OFDM transmission is used in digital terrestrial TV broadcasting [2] and in the wireless LAN standard [3].

However the OFDM is strong against multipath channels, it has some disadvantages. One of these disadvantages is that the OFDM weak against the frequency offset. For the single carrier system, the frequency offset results in the distortion of the desired signal. However, in the OFDM, the frequency offset makes not only the distortion of the desired signal but also Inter-Channel Interference (ICI) [4], because the frequency offset breaks down the orthogonality among the subcarriers. Thus, the performance degradation is very severe in the OFDM transmission, when the frequency offset exists. In addition, the performance of the OFDM system is bounded at the high Signal-to-Noise power Ratio (SNR) due to the ICI [5].

Thus, we derive the performance bound of the OFDM transmission in the sense of the SNR performance and the BER performance, when the frequency offset exists. To find the pure effect of the frequency offset in the OFDM, we use the Additive White Gaussian Noise (AWGN) channel.

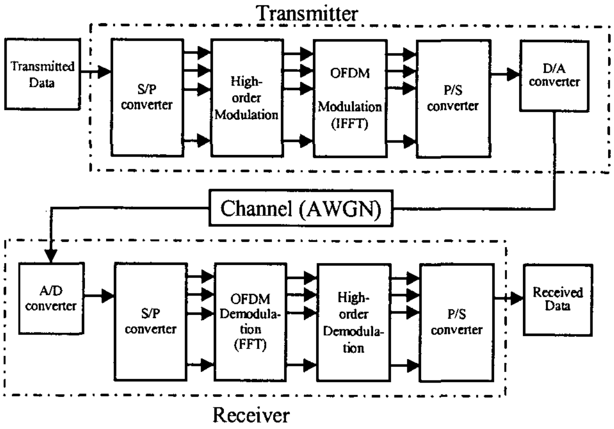

Figure 1. OFDM system block diagram

This paper is organized as follows. Section 2 shows signal representations in the OFDM system with the frequency offset. In Section 3, we derive the performance bound of the OFDM transmission with the frequency offset in the sense of the SNR performance and the BER performance. In Section 4, we show some numerical results. Finally, we conclude this paper in Section 5.

2. OFDM System

Figure 1 shows the block diagram of the OFDM system. The transmitted signal, $s(t)$, is given as follows :

$$s(t) = \frac{1}{\sqrt{N}} \sum_{i=-\infty}^{\infty} \sum_{k=0}^{N-1} X_{i,k} e^{j\frac{2\pi kt}{NT_s}} e^{j2\pi f_c t} p(t - iT) \quad (1)$$

where N is the number of the subcarriers, $X_{i,k}$ is the transmitted symbol at the i-th OFDM block through the k-th subcarrier, T_s is the effective OFDM symbol duration f_c is the carrier frequency and $p(t-iT)$ is the pulse shape function. This signal is transmitted through the wireless channel. In this paper, we use the AWGN channel to find the pure effect of the frequency offset and consider the frequency offset at the receiver side. Then, the received baseband signal, $r(t)$, is given as follows :

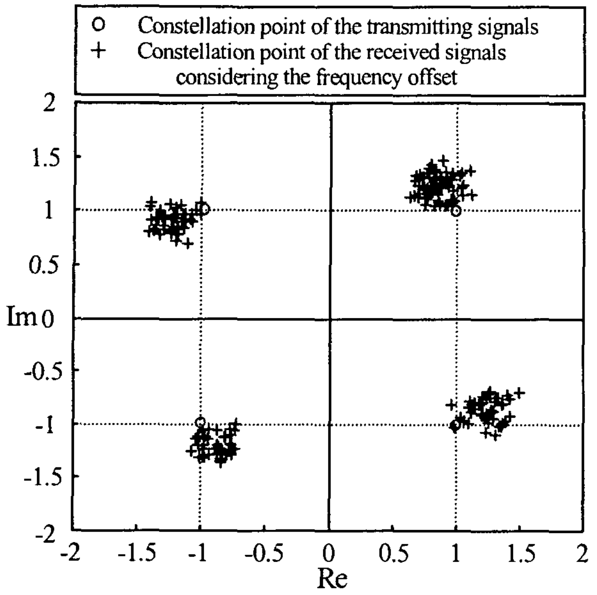

Figure 2. Constellation point of the transmitted and received signal, when the frequency offset exists and the AWGN does not exists

$$r(t) = \frac{1}{\sqrt{N}} \sum_{i=-\infty}^{\infty} \sum_{k=0}^{N-1} X_{i,k} e^{j\frac{2\pi k t}{NT_s}} e^{j2\pi\Delta f} p(t-iT) + n(t) \tag{2}$$

where Δf is the frequency offset due to the difference between transmitter and receiver frequencies and $n(t)$ is the AWGN which has the two-side power spectral density, $N_0/2$. The equivalent received baseband signal is enters FFT and the FFT output signal of the m-th subcarrier at the i-th OFDM symbol, $z_{m,i}$,is given as follows [6]:

$$z_{m,i} = e^{j\theta} \sum_{l=0}^{N-1} c_{l-m} X_{l,i} + N_{m,l} \tag{3}$$

where θ is the carrier phase error at the start of the received symbol period, $N_{m,l}$ is the output noise of the FFT due to the AWGN and c_{l-m} is the weighting coefficient, which gives the contribution of the l-th input X_l to the m-th output z_m and is given as follows:

$$c_{l-m} = \frac{1}{N} \sum_{k=0}^{N-1} e^{j\frac{2\pi k(l-m+\Delta f T_s)}{N}} \tag{4}$$

In eq. (4), the weighting coefficient depends on the normalized frequency offset, $\Delta f T_s$, which is normalized about the OFDM subcarrier tone spacing. We assume that the coherent modulation is employed, which means the carrier phase is known and compensated perfectly. Then, the FFT output signal of the m-th subcarrier at the i-th OFDM symbol, $z_{m,i}$,can be rewritten as follows:

$$z_{m,i} = \sum_{l=0}^{N-1} c_{l-m} X_{l,i} + N_{m,l} \tag{5}$$

In eq. (5), the ICI due to the frequency offset plays roll of the noise, although the AWGN does not exist. Figure 2 show the effect of ICI due to the frequency offset. The constellation point of the received signal is scattered due to the ICI's.

3. Performance Bound

3.1 SNR performance bound

In eq. (5), the FFT output can be divided according to the signal components. Then, we can rewrite the FFT output of the m-th subcarrier at the i-th OFDM symbol can be rewritten as follows:

$$\begin{aligned} z_{m,i} &= c_{m-m} X_{l,m} \\ &+ \sum_{l=0,\neq m}^{N-1} c_{l-m} X_{l,i} \\ &+ N_{m,l} \end{aligned} \tag{6}$$

The first term is wanted signal, the second term is ICI's and the other is AWGN. From eq. (6), we can find powers of three terms as follows:

$$\begin{aligned} E_{want} &= \left|c_{m-m}\right|^2 \left|X_{l,m}\right|^2 = \left|c_{m-m}\right|^2 E_s \\ E_{ICI} &= \sum_{l=0,\neq m}^{N-1} \left|c_{l-m}\right|^2 \left|X_{l,i}\right|^2 = \sum_{l=0,\neq m}^{N-1} \left|c_{l-m}\right|^2 E_s \\ P_{AWGN} &= \frac{N_0}{2} \end{aligned} \tag{7}$$

where E_{want} is the power of the wanted signal, E_{ICI} is the power of the ICI's, P_{AWGN} is the power of the AWGN and E_s is the average power of the symbol.

In this paper, we consider the E_s/N_0, because it relates with the BER performance of the systems. The effective E_s/N_0 is defined as follows:

$$\begin{aligned} \frac{E_s}{N_{0\,eff}} &= \frac{E_{want}}{E_{ICI} + P_{AWGN}} \\ &= \frac{\left|c_{m-m}\right|^2 E_s}{\sum_{l=0,\neq m}^{N-1} \left|c_{l-m}\right|^2 E_s + \frac{N_0}{2}} \\ &= \frac{\left|c_{m-m}\right|^2}{\sum_{l=0,\neq m}^{N-1} \left|c_{l-m}\right|^2 + \frac{N_0}{2E_s}} \end{aligned} \tag{8}$$

We can get the best performance of the OFDM system, when the transmitting power is infinite. Thus, we can get the effective E_s/N_0 bound of the OFDM system by replacing E_s/N_0 into infinite value. The effective E_s/N_0 bound is given as follows:

$$\frac{E_s}{N_{0\,bound}} = \frac{\left|c_{m-m}\right|^2}{\sum_{l=0,\neq m}^{N-1} \left|c_{l-m}\right|^2} \tag{9}$$

From eq. (9), we can confirm that the bound of the effective E_s/N_0 is not dependent to the transmitting E_s/N_0.

3.2 BER performance bound

In this subsection, we evaluate the performance in the sense of the BER performance, because the BER is immediate factor of digital communications. To find the

System parameter values	
Modulation	QPSK,8-PSK,16-PSK 16-QAM, 64QAM
Number of subcarriers	64
OFDM symbol duration	4us
Guard interval	800ns
Carrier frequency	5Ghz

Table 1. Used system parameter values

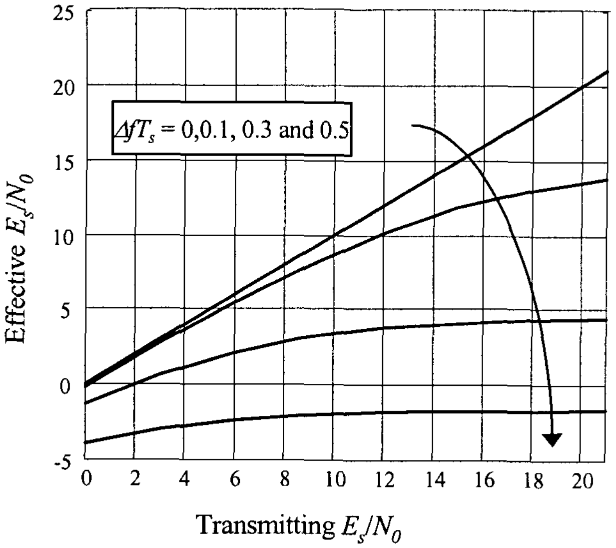

Figure 3. The effective E_s/N_0 of the OFDM system with 128 subcarriers according to the variation of the normalized frequency offset

BER performance, we consider the specific modulation: M-ary Phase Shift Keying (M-PSK) modulation and M-ary Quadrature Amplitude Modulation (QAM), which are very popular modulation schemes for the high-speed transmission. From the effective E_s/N_0, the BER performance can be founded easily. For the M-PSK, the SER performance is given as follow [7]:

$$P_{E(M-PSK)} = 2Q(\sqrt{\frac{2}{\log_2 M}\frac{E_b}{N_0}_{eff}}\sin\frac{\pi}{M}) \quad (10)$$

where M is the number of the modulated symbols, $(E_b/N_0)_{eff}$ is the effective (E_b/N_0), which is equal to $(E_s/N_0)_{eff}/\log_2 M$ and $Q(x)$ is the Gaussian Q-function and defined as follows :

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{u^2}{2}} du \quad (11)$$

Then, we can get the BER performance of the gray-coded M-PSK system by using the relation between SER and BER given as follows [7]:

$$P_B \approx \frac{P_E}{\log_2 M} \quad (\text{for } P_E << 1) \quad (12)$$

For M-QAM, the BER performance is given as follows [7]:

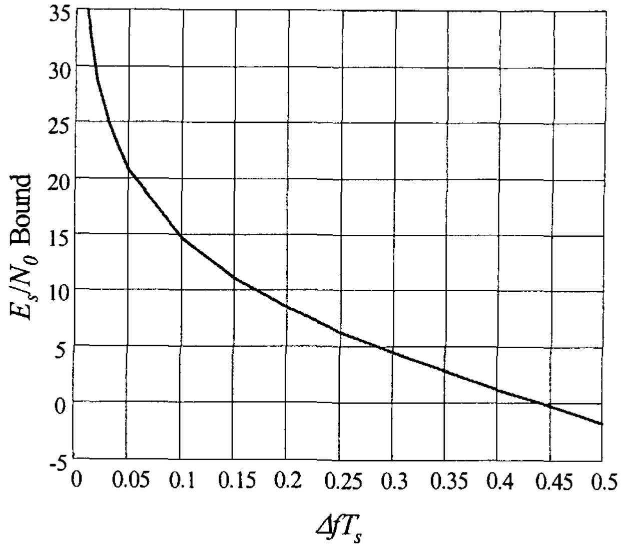

Figure 4. The bound of the effective E_s/N_0 performance in the OFDM system with 128 subcarriers

$$P_{B(M-QAM)} \cong \frac{2(1-L^{-1})}{\log_2 L} Q(\sqrt{(\frac{3\log_2 L}{L^2-1})\frac{2E_b}{N_0}_{eff}}) \quad (13)$$

where L is the number of amplitude levels, which is equal to $\sqrt{M}$.

From the BER performance eq. (10) and (13), we can get the BER performance bound by replacing $(E_b/N_0)_{eff}$ into $(E_b/N_0)_{bound}$,.

4. Numerical Results

The used system parameter values are shown in Table 1. We consider the OFDM system with 128 subcarriers with the frequency offset under the AWGN channel. To evaluate the BER performance bound, the M-PSK modulations, which has M equal to 4, 8 and 16, and the M-QAM modulation, which has M equal to 16 and 64, are employed.

Figure 3 shows the effective E_s/N_0 of the OFDM system according to the variation of the normalized frequency offset, ΔfT_s, which is normalized about the OFDM subcarrier tone spacing. When the normalized frequency offset is 0.1 and the transmitting E_s/N_0 is 20dB the loss in effective E_s/N_0 is about 6dB comparing with E_s/N_0 of no frequency offset case. However, the transmitted E_s/N_0 increases, the loss in E_s/N_0 becomes larger and the effective E_s/N_0 does not improved. When the normalized frequency offset is equal to 0.3 and 0.5, the effective E_s/N_0 is bounded about 5dB and -2dB, respectively. That means that the attainable effective E_s/N_0 is bounded according to the normalized frequency offset. It is because ICI is large, as the normalized frequency offset increases.

Figure 4 shows the bound of the effective E_s/N_0 performance in the OFDM system with 128 subcarriers according to the variation of the normalized frequency offset. When the normalized frequency offset is 0.1, the maximum attainable effective E_s/N_0 is about 15dB. When

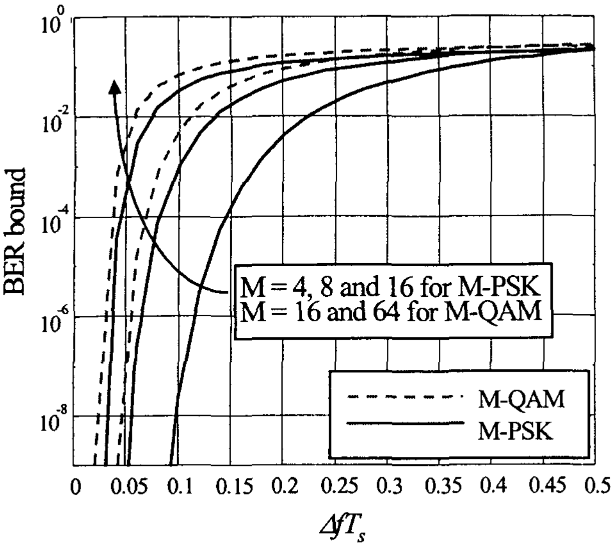

Figure 5. The bound of the BER performance in the OFDM system with128 subcarriers and various modulation types (M-PSK and M-QAM)

the normalized frequency offset is 0.4, the maximum attainable effective E_s/N_0 is about 2dB similarly. As the normalized frequency offset increases in the OFDM system, the attainable maximum E_s/N_0 is reduced significantly. If we assume that the required E_s/N_0 is about 10dB under the AWGN channel to support the some required service, the OFDM system must have smaller normalized frequency offset than 0.16 to support the required E_s/N_0. Thus it is more important to employ technique to reduce frequency offset itself, for example frequency offset estimator [8,9], than the increasing transmitting power when the normalized frequency offset is greater than 0.16.

Figure 5 shows the bound of the BER performance of the OFDM system with various modulation types: M-PSK or M-QAM. To find the effect of the frequency offset in terms of the Quality of Services (QoS), we assume that the required BER is 10^{-5}. To support the required BER, 10^{-5}, the allowable normalized frequency offset are limited about 0.125, 0.07, and 0.035 for M-PSK with M equal to 4, 8 and 16, respectively. As M increases, the allowable normalized frequency offset is more reduced. It means that the complex modulation scheme is sensitive to the frequency offset than simple modulation schemes.

For M-QAM, the allowable frequency offset also reduces, as M increases. For example, to support the required BER equal to 10^{-5}, the maximum allowable normalized frequency offset is 0.05 and 0.025 for the 16-QAM and 64-QAM, respectively. However, M-QAM is stronger than M-PSK for the same M value. For examples, when M is equal to 16, the allowable normalized frequency offset of 16-PSK and 16-QAM are 0.035 and 0.06, respectively. To combat frequency offset, M-QAM is more suitable for the same data rate.

5. Conclusions

In this paper, we evaluated the performance bound of the OFDM system with the frequency offset in the sense of the effective E_s/N_0 and the BER under the AWGN channel. When the frequency offset existed, the increasing transmitting power did not help to improve the performance and the bound of the performance occurred. When the normalized frequency offset, which was normalized about the OFDM tone spacing, was 0.1, the maximum attainable effective E_s/N_0 was about 15dB and the BER performance bound was about 5×10^{-8} for the OFDM system with the QPSK modulation. When the normalized frequency offset was 0.4, the maximum attainable effective E_s/N_0 was about 2dB and the best BER performance is 0.1, respectively. To overcome this loss, other technique must be considered, for example, frequency offset diversity.

In addition, we evaluated the BER performance according to the variation of the modulation (M-PSK and M-QAM). According to results, M-QAM was stronger against the frequency offset than M-PSK for the same value of M.

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