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A MIXED NORM RESTORATION FOR MULTICHANNEL IMAGES

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Abstract

In this paper, we present a regularized mixed norm multichannel image restoration algorithm. The problem of multichannel restoration using both within- and between- channel deterministic information is considered. For each channel a functional which combines the least mean squares (LMS), the least mean fourth (LMF), and a smoothing functional is proposed. We introduce a mixed norm parameter that controls the relative contribution between the LMS and the LMF, and a regularization parameter that defines the degree of smoothness of the solution, both updated at each iteration according to the noise characteristics of each channel. The novelty of the proposed algorithm is that no knowledge of the noise distribution for each channel is required, and the parameters mentioned above are adjusted based on the partially restored image.

1. INTRODUCTION

The mean squared error (MSE) has been traditionally used in formulating the restoration problem, resulting in the least mean squared (LMS) approach. The reason for this is that it leads to mathematically tractable solutions and yields optimal results when the contaminating noise has Gaussian distribution [1, 2, 3]. In a number of applications, the contaminating noise may be non-Gaussian or a combination of several noise types. It has been shown that the LMF approach outperforms the LMS under certain noise distributions, such as sub-Gaussian distributions. [4]. The combination of the LMS and LMF approach was introduced in image restoration in [5, 6] for single channel images.

Since multichannel images may be degraded due to crosstalk or spectral blurs between channels, it is expected that the optimal restored multichannel image should show better performance than the restored image without the use of cross information. In this paper, a multichannel regularized mixed norm image restoration algorithm is proposed. The mixed norm smoothing functional for each channel is determined by incorporating not only within-channel information but also between-channel information. The two parameters (the mixed norm parameter controlling the relative importance between the LMS and the LMF, and the regularization parameter controlling the trade-off between the mixed norm functional and the smoothing functional) for each channel are adjusted from the partially restored image. No prior information about the noise distribution and the bound of high pass energy is therefore required. Also, it is shown that the appropriate choice of these two parameters yields to a convex regularized mixed norm smoothing functional and therefore a unique solution is obtained.

The paper is organized as follows. In section 2, the multichannel degradation model and the properties of the kurtosis are reviewed. A stabilizing multichannel smoothing mixed norm functional is proposed in section 3. In addition, the properties of the proposed functional and the convergence of the iterative solution used are analyzed. Experimental results and conclusions are discussed in section 4 and 5.

2. BACKGROUND

A linear multichannel degradation model is assumed, and the model is described by [2]

$$y = Hx + n, \tag{1}$$

where the vectors y, x, and n are the observed image, the original image, and the noise, respectively. We assume that the noise is uncorrelated zero mean. For an N channel image where each channel contains $M \times M$ pixels, we have $y = [y_1^T, y_2^T, \cdots, y_N^T]^T$, $x = [x_1^T, x_2^T, \cdots, x_N^T]^T$, and $n = [n_1^T, n_2^T, \cdots, n_N^T]^T$, where T denotes the transpose of a vector or matrix and each of the $M^2 \times 1$ vectors y_i, x_i, and n_i represents the lexicographic ordering of each channel image. The multichannel degradation matrix H of size $NM^2 \times NM^2$ is assumed to be known and is given by

$$H = \begin{bmatrix} H_{11} & H_{12} & \dots & H_{1N} \\ H_{21} & H_{22} & \dots & H_{2N} \\ \cdot & \cdot & \dots & \cdot \\ H_{m1} & H_{m2} & \dots & H_{NN} \end{bmatrix} \quad (2)$$

Submatrices H_{ii} and H_{ij}, for $i \neq j$ are of size $M^2 \times M^2$ and represent the within-channel and between-channel degradation, respectively. Clearly, if all elements of cross channel degradation are zero, the degradation model in (2) leads to N separated single channel models.

The ith channel degradation model providing a solution to Eq. (1) can be rewritten as

$$y_i = H_i x + n_i, \text{ for } i = 1, 2, \cdots, N, \quad (3)$$

where

$$H_i = [H_{i1}, H_{i2}, \cdots, H_{iN}]$$

is the ith block row matrix of size $M^2 \times NM^2$.

In most applications the ith noise term n_i is assumed to be uncorrelated zero-mean Gaussian. There are application, however, for which the additive noise is characterized by other distributions, such as, Uniform, Laplacian, or a combination of them. In the Gaussian noise case, the LMS approach is used for solving the problem described in Eq. (3). However, for other noise distributions norms of higher order need to be used. It was shown that under certain conditions such as sub-Gaussian noise, the LMF and other higher criteria exhibit improved performance compared to the LMS. The converse is true for Gaussian and super-Gaussian noise signals.

The kurtosis is utilized to determine the degree of Gaussianity of a random signal. For a zero mean random variable n_i, the kurtosis is defined by

$$\chi(n_i) = E[n_i^4] - 3E^2[n_i^2]. \quad (4)$$

The kurtosis is zero for Gaussian signals, it is positive for super-Gaussian or leptokurtic signals and negative for sub-Gaussian or platykurtic signals.

According to the properties of the kurtosis analyzed in Ref. [6], the kurtosis of the combined noise is determined by the dominating noise term.

In analogy to Eq. (4), we estimate the kurtosis of an M^2 sample random signal when only one realization is available by

$$\chi(n_i) = \frac{1}{M^2}||n_i||_4^4 - 3(\frac{1}{M^2}||n_i||_2^2)^2, \quad (5)$$

where $||\cdot||_p$ denotes the l_p norm.

3. A MIXED NORM RESTORATION FOR MULTICHANNEL IMAGES

As pointed out, under sub-Gaussian noise conditions the LMF outperforms the LMS. When the LMF and the LMS are combined to utilize the advantages of both of them, and the smoothing functional is incorporated to avoid some of the difficulties caused by the ill-posed nature of the degradation operator, the ith channel mixed norm smoothing functional can be written as

$$\begin{aligned} M_i(x) &= J_i(x) + \alpha_i(x)||C_i x||_2^2 \\ &= (1-\gamma_i(x))||y_i - H_i x||_2^2 \\ &+ \gamma_i(x)||y_i - H_i x||_4^4 + \alpha_i(x)||C_i x||_2^2, \quad (6) \end{aligned}$$

where

$$C_i = [C_{i1}, C_{i2}, \cdots, C_{iN}],$$

where $J_i(x)$ represents the ith channel mixed norm functional. C_{ij} represents the multichannel high pass filter of size $M^2 \times M^2$, $\alpha_i(x)$ the ith channel regularization parameter, and $\gamma_i(x)$ the ith channel mixed norm parameter, respectively. With C_{ii}, smoothness within each channel is imposed, while with C_{ij} $(i \neq j)$, smoothness between channels is imposed. In Eq. (6), it is desired that the LMF term dominates the mixed norm functional ($\gamma_i(x) \approx 1$) if the ith channel noise is sub-Gaussian, while the LMS term dominates the mixed norm functional ($\gamma_i(x) \approx 0$) if the ith channel noise is Gaussian or super-Gaussian. The regularization parameter $\alpha_i(x)$, represents the trade-off between the mixed norm functional and the smoothing functional of the ith channel image. Then a solution is obtained by minimizing the multichannel mixed norm smoothing functional

$$M(x) = \sum_{i=1}^{N} M_i(x) = \sum_{i=1}^{N}(J_i(x) + \alpha_i(x)||C_i x||_2^2). \quad (7)$$

The two parameters can be expressed as functions of the original image. However, since the original image is not available, the information about the original image can be obtained from the partially restored image when an iterative approach is implemented. Following the steps as in Ref. [5, 6], each functional becomes convex when $\gamma_i(x)$ and $\alpha_i(x)$ are chosen as

$$\gamma_i(x) = \frac{\exp(-c\chi(n_i))}{A + \exp(-c\chi(n_i))}, \quad (8)$$

and

$$\alpha_i(x) = \frac{(1-\gamma_i(x))||y_i - H_i x||_2^2 + \gamma_i(x)||y_i - H_i x||_4^4}{\frac{1}{\tau} - ||C_i x||_2^2}, \quad (9)$$

where A and c are positive scalars, $\frac{1}{\tau} \geq ||C_i x||_2^2$ for convexity, and $||x||_2^2 \approx ||y||_2^2 \geq ||C_i x||_2^2$ when C is normalized such that $\sigma_{max}(C_i^T C_i) = 1$.

We propose to use a steepest descent algorithm for obtaining a solution to the minimization problem of Eq. (7). The gradient of $M(x)$ with respect to x is equal to

$$\begin{aligned} \nabla_x M(x) &= \nabla_x \sum_{i=1}^{N} M_i(x) = \sum_{i=1}^{N} \nabla_x M_i(x) \\ &= \sum_{i=1}^{N} [(1-\gamma_i(x)) H_i^T (y_i - H_i x) \\ &\quad +2\gamma_i(x) P_i(x)(y_i - H_i x) \\ &\quad -\alpha_i(x) C_i^T C_i x] = 0, \end{aligned} \tag{10}$$

where $P_i(x)$ represents a diagonal matrix with diagonal elements $P_i(x)(j,j) = (y_i(j) - (H_i x)(j))^2$.

With the error residual in Eq. (10), an iteration results in

$$\begin{aligned} x_{k+1} &= x_k + \beta \sum_{i=1}^{N} [(1-\gamma_i(x_k)) H_i^T (y_i - H_i x_k) \\ &\quad +2\gamma_i(x_k) H_i^T P_i(x_k)(y_i - H_i x_k) \\ &\quad -\alpha_i(x_k) C_i^T C_i x_k)], \end{aligned} \tag{11}$$

where β is the relaxation parameter to control the convergence as well as the convergence rate.

For two consecutive iteration steps, Eq. (11) can be rewritten as

$$\begin{aligned} x_{k+1} - x_k &= (x_k - x_{k-1}) \\ &\quad +\beta \sum_{i=1}^{N} [-H_i^T H_i (x_k - x_{k-1}) \\ &\quad -H_i^T (F_i(x_k) - F_{i-1}(x_{k-1})) \\ &\quad +H_i^T H_i (G_i(x_k) - G_i(x_{k-1})) \\ &\quad +2H_i^T (K_i(x_k) - K_i(x_{k-1})) \\ &\quad -2H_i^T (L_i(x_k) - L_i(x_{k-1})) \\ &\quad -C_i^T C_i (Q_i(x_k) - Q_i(x_{k-1}))], \end{aligned} \tag{12}$$

where $F_i(x_k) = \gamma_i(x_k)\, y_i$, $G_i(x_k) = \gamma_i(x_k)\, x_k$, $K_i(x_k) = \gamma_i(x_k) P_i(x_k) y_i$, $L_i(x_k) = \gamma_i(x_k) P_i(x_k) H_i x_k$, and $Q_i(x_k) = \alpha_i(x_k) x_k$. These nonlinear factors are linearized as

$$\begin{aligned} F_i(x_k) - F_i(x_{k-1}) &\approx J_{F_i}(x_k - x_{k-1}), \\ G_i(x_k) - G_i(x_{k-1}) &\approx J_{G_i}(x_k - x_{k-1}), \\ K_i(x_k) - K_i(x_{k-1}) &\approx J_{K_i}(x_k - x_{k-1}), \\ L_i(x_k) - L_i(x_{k-1}) &\approx J_{L_i}(x_k - x_{k-1}), \\ Q_i(x_k) - Q_i(x_{k-1}) &\approx J_{Q_i}(x_k - x_{k-1}), \end{aligned} \tag{13}$$

where $J_{F_i}(x_k)$, $J_{G_i}(x_k)$, $J_{K_i}(x_k)$, $J_{L_i}(x_k)$, and $J_{Q_i}(x_k)$ are the Jacobian matrices of the ith channel. Following similar steps as in [6] and by using Eq. (13), Eq. (12) can be rewritten as

$$x_{k+1} - x_k = [I - \beta \sum_{i=1}^{N} A_i(x_k)](x_k - x_{k-1}), \tag{14}$$

$A_i(x_k) = (1 - \gamma_i(x_k)) H_i^T H_i + 6\gamma_i(x_k) H_i^T P_i(x_k) H_i + \alpha_i(x_k) C_i^T C_i$. Since $A_i(x_k)$ is positive definite matrix, the sufficient condition for convergence becomes

$$0 < \beta < \frac{2}{N \cdot \sigma_{max}(A_i(x_k))}, \tag{15}$$

where $\sigma_{max}(Z)$ represents the maximum singular value of the matrix Z.

4. EXPERIMENTAL RESULTS

The proposed multichannel regularized mixed norm restora tion algorithm was tested with a noisy blurred three channel true color image. The point spread function of the blurring system within- and between- channel is Gaussian with support region 7 and variance 5. For the within-channel PSF $h_{i,i}(m,n)$, $\sum_m \sum_n h_{i,i}(m,n) = 0.8$, while for between-channel PSF $h_{i,j}(m,n)$, $\sum_m \sum_n h_{i,j}(m,n) = 0.1$. The three dimensional Laplacian high pass filter was used for C, and the criterion $||x_{k+1} - x_k||^2/||x_k||^2 \leq 10^{-7}$ was used for terminating the iteration.

The contaminating noises are 20 dB uniform noise for the red channel, 20 dB Laplacian noise for the green channel, and 20 dB Gaussian noise for the blue channel. The noisy blurred R channel image is shown in Fig. 1. The proposed algorithm is compared with the LMS multichannel restoration algorithm, as shown in Table 1.

Δ_{SNR}	R Channel	G channel	B channel
proposed algorithm	2.91 (dB)	2.41 (dB)	1.95 (dB)
LMS algorithm	2.52 (dB)	2.35 (dB)	1.96 (dB)

Table 1: Performance comparison

The corresponding restored R channel image by the proposed algorithm is shown in Fig. 2 with $c = 1$ and $A = 1$ in Eq. (8), and $\tau^{-1} = 2||y||_2^2$.

As mentioned, the proposed multichannel mixed norm approach does not require any information about the noise distribution, the power of the noise, and the original image. The proposed algorithm is capable of controlling the relative importance of the LMS and LMF, and estimating the smoothing functional from the partially restored image. Fig. 3 shows the values of the mixed norm parameter as function of iteration number. The parameter $\gamma_R(x_k)$ for the red channel contaminated by uniform noise (sub-Gaussian) is close to 1, $\gamma_G(x_k)$ for the green channel contaminated by Laplacian noise (super-Gaussian) is close to 0, and $\gamma_B(x_k)$ for the blue channel contaminated by Gaussian noise is close to 0.5. In Fig. 4, the values of the regularization parameter of each channel as functions of the iteration number are shown.

Figure 1: Noisy blurred image ; R channel, 20 dB uniform noise

Figure 2: Restored image by proposed algorithm ; R channel

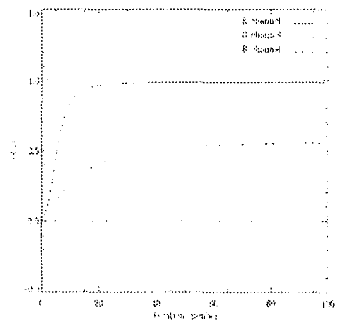

Figure 3: Values of $\gamma(x)$ as function of iteration number for each channels

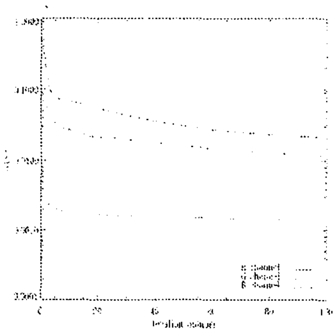

Figure 4: Values of $\alpha(x)$ as function of iteration number for each channels

5. CONCLUSIONS

In this paper, we propose a multichannel regularized mixed norm image restoration algorithm. The proposed multichannel mixed norm smoothing functional to be minimized is formulated to have a global minimizer with the proper choice of the multichannel mixed norm and the regularization parameters, resulting in better performance compared with the single restoration approach and the least mean squared approach.

The novelty of this paper is that with the proposed algorithm no knowledge of the noise distributions is required, and the relative contribution of the LMS and LMF approaches is adjusted based on the partially restored image. Experimental result shows the capability of the proposed approach.

6. REFERENCES

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