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An Algorithm for Portfolio Selection Model

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Abstract

The problem of selecting a portfolio is to find an investment plan that achieves a desired return while minimizing the risk involved. One stream of algorithms are based upon mixed integer linear programming models and guarantee an integer optimal solution. But these algorithms require too much time to apply to real problems. Another stream of algorithms are for a near optimal solution and are fast enough. But, these also have a weakness in that the solution generated can't be guaranteed to be integer values. Since it is not a trivial job to tansform the soultion into integer valued one simutaneously maintaining the quality of the solution, they are not easy to apply to real world portfolio selection.

To tackle the problem more efficiently, we propose an algorithm which generates a very good integer solution in reasonable amount of time. The algorithm is tested using Korean stock market data to verify its accuracy and efficiency.

1. Introduction

The classical portfolio optimization problem is to find an investment plan for stock market with a reasonable trade-off between the rate of return and risk involved. The classical quadratic programming model by Markowiz[1] has a disadvantage in terms of computational complexity and is difficult to apply to real world problems. The complexity problem is alleviated partly by the recent works. More specifically, Konno and Yamazaki[2] proposed a linear programming model, which is called mean-absolute deviation(MAD) model, as an alternative to Markowiz model. The linear form of the model is made possible by a risk function they introduced. The model has been analyzed further by Zenios and Kang[3] with asymmetric distributions of the rate of return. It is shown that it may be a good alternative to the Morkowiz model.

Speranza[4] generalized Konno and Yamazaki's model with a weighted risk function. He showed that suitable coefficients in the linear combination can make his model more compact and can generate the same solution as Konno and Yamazaki's. Lastly, Mansini and Speranza[5,6,7,8] introduced more flexible model with fixed transaction costs and minimum transaction lots and proposed three different algorithms. In this paper, we propose a revised form of Mansini and Speranza's algorithm. The algorithm we are introducing has similar accuracy but requires less time to obtain a solution.

In the remaining part of this paper, we describe our mathematical model in section 2 followed by an explanation of the algorithm we are introducing. Results of computational experiments and final comments are in sections 4 and 5.

2. The relaxed model

In this section we present the mathematical programming formulation of the problem. We define the required notations as follows :

Indices

J : set of candidate securities,

j : index for security, $j=1, \cdots, J$,

t : index for time period, $t=1, \cdots, T$,

i : index for security, $i=1, \cdots, I$.

Parameters

r_{jt} : observed rate of return of security j at time period t,

r_j : expected value of security j,

C_0 : lower bound of investment asset,

C_1 : upper bound of investment asset,

c_j : purchase price of trading unit of security j,

d_j : transaction cost rate of security j,

l_j : lower bound of number of security j that must be included in the portfolio (in transaction unit)

u_j : upper bound of number of security j that must be included in the portfolio (in transaction unit)

ρ : rate of the expected return.

Variables

y_t : summation of the rate of risk at time period t

x_j : number of security j to be purchased in transaction unit,

$\widetilde{X_L}$: optimal value of the relaxed LP,

$\tilde{x}_j$: optimal number of security j to purchase based on the optimal solution of relaxed LP model,

$\tilde{s}_t$: slack variable from the relaxed LP model for time period t.

The mixed integer linear program for the portfolio selection problem with minimum lot constraints is as follows:

$$\min \frac{\sum_{t=1}^{T} y_t}{T} \quad (1)$$

s.t.

$$y_t \geq -\sum_{j\in J}(r_{jt}-r_j)x_j, \quad t=1,\cdots,T \quad (2)$$

$$y_t \geq 0, \quad t=1,\cdots,T \quad (3)$$

$$\sum_{j\in J}(1+d_j)c_jx_j \geq C_0 \quad (4)$$

$$\sum_{j\in J}(1+d_j)c_jx_j \leq C_1 \quad (5)$$

$$\sum_{j\in J}[(r_j-\rho-\rho d_j)c_jx_j] \geq 0 \quad (6)$$

$$l_j \leq x_j \leq u_j, \quad j\in J \quad (7)$$

$$x_j \ integer, \quad j\in J \quad (8)$$

Constraints (2), (3) is needed to transform objective function to be linear. Constraints (4) and (5) force investment amount to be between C_0 and C_1, that are the minimum and maximum amount of money available for the investment. The constraint on the expected return (6) implies that the selected portfolio has a combined rate of return greater than ρ.

Finally constraints (7) and (8) enable number of each stock to be within allowable range.

3. The proposed algorithm

Three heuristic methods developed by Mansini and Speranza[8] are all based on the solution of an LP-relaxation model and on a subsequent adjustment of the solution to obtain integer feasibility with respect to the original constraints.

As we already mentioned, our algorithm is a modification of the Mansini and Speranza's algorithm rather than a brand new one. (Mansini and Speranza's algorithm is referred to as M&S algorithm from now on.) The major enhancement lies in that we utilize solution and slack information together while the M&S algorithm use the information from solution only. In other words, M&S algorithm starts investigation of the integer optimal solution from a random vertex and our method starts from the vertex corresponding the LP optimal. Since the integer optimal solution usually lies near the vertex corresponding to the optimal solution of the relaxed model, utilizing slack information speeds up the subsequent phase of search for integer optimal.

Now, we introduce our method.

Step 1. solve the LP relaxation model of the original problem. Let x_R denote the solutions vector and n be the number of components in x_R with value greater than zero.

Step 2. construct an IP model as followings:

$$\min \frac{\sum_{t=1}^{T} y_t}{T} \quad (9)$$

s.t.

$$-\sum_{j\in J}(r_{jt}-r_j)(x_j-\widehat{x_j})-\widehat{s_t} \leq y_t,$$

$$t=1,\cdots,T \quad (10)$$

$$\sum_{j\in J}(1+d_j)c_j(x_j-\widehat{x_j}) \leq 0 \quad (11)$$

$$\sum_{j\in J}(1+d_j)c_j(x_j-\widehat{x_j}) \geq 0 \quad (12)$$

$$\sum_{j\in J}[(r_j-\rho-\rho d_j)c_jx_j] \geq 0 \quad (13)$$

$$x_j \ integer, \quad y_t \geq 0 \quad (14)$$

Let $\widehat{s_t}$ denote the slack of equation (2). First, in equation (2), we have to note that RHS value can be increased without affecting y_t as long as the change is less than current slack. If the change is over the slack, then y_t starts to have positive value and value of objective function (1) is increased. Constraint (10) is formulated to reflect this situation. $\widehat{x_j}$, integer amount of security j included in the portfolio, is determined so as to minimize the increase in the objective function value. Constraints (11), (12) define the limitation in the investment capital remaining. With constraint (13), the $\widehat{x_j}$ is determined to satisfy the return rate requirement.

Step 3. Calculate the objective value : The objective value from IP model is added to it from relaxed LP model. Because y_t is calculated by difference x_j and $\widetilde{x_j}$. However x_j from this IP model is used to final solution without any conversion.

4. Computational experience

Our algorithm is tested using the data from Korean stock market and compared to the original M&S algorithm. The algorithm was coded in DELPHI on a personnel computer with Intel Pentium(333MHz) and with 96 Mb memory. Inside LP solution is obtained using LINDO.

Experiments are performed with the rate of return, ρ set to 0.01. We try to compare the solutions with those of the procedure A, C of M&S algorithm. The performance criteria is percent error from the optimal solution

The experiments have been carried out on a set of data from 1998 with a total of 80 securities.

A summary of results in Table 1 shows that the percent errors from the optimal solutions of each algorithm. When the size is less than 40, the solution of the proposed algorithm is not better than others. But as the size is increased, the solution of the proposed shows better performance than others.

Table 1 Percent errors from the optimal solution

Size	10	20	30	40
Optimum	1.4800	0.9629	0.8108	0.7777
Proposed	1.4858 (0.3)	0.9662 (0.3)	0.8224 (1.4)	0.7991 (2.8)
Procedure A	1.4800 (0)	0.9636 (0.07)	0.8163 (0.7)	0.7813 (0.75)
Procedure C	1.4800 (0)	0.9632 (0.03)	0.8162 (0.7)	0.7780 (0.03)
	50	60	70	80
Optimum	0.7052	0.3200	0.0020	0.0053
Proposed	0.7055 (0.04)	0.3200 (0)	0.0048 (1.4)	0.0130 (1.4)
Procedure A	0.7055 (0.04)	0.3200 (0)	0.0048 (1.4)	0.012 (1.3)
Procedure C	0.7102 (0.7)	0.3200 (0)	0.7728 (385.4)	0.0053 (0)

*( ) : percent error from the optimal solution
Bold face represents the best result

Table 2 The computational times(in seconds) for the proposed and procedures

Size	10	20	30	40
Proposed	0.77	0.77	1.19	1.50
Procedure A	0.94	0.88	1.34	1.66
Procedure C	1.26	3.26	6.55	9.61
	50	60	70	80
Proposed	1.36	1.03	27.10	4.12
Procedure A	3.20	6.11	54.33	5.94
Procedure C	10.66	136.33	61.41	190.02

As shown in Table 2, the computational time of the proposed algorithm is shorter than the others. The procedure C is inferior to others in all problem sizes. The procedure A takes a little longer to obtain a solution for the smaller problems. But, as the problem size is increased, our algorithm shows a dominant supremacy.

5. Conclusions

In this paper we propose a algorithm that is a modified version of Mansini and Speranza's algorithm. The proposed algorithm shows superior performance both in solution accuracy and computational time during the computational experiments using real data of Korea stock market. Therefore, it is safe to conclude that the algorithm is a good candidate for a real world application and a good addition to the existing solution methodologies.

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