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내부고장요인과 외부고장요인이 있는 제품에 대한 가속수명시험의 분석

Analysis of Accelerated Life Tests with Intrinsic and Extrinsic Failure Modes

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Abstract

This paper proposes a method of estimating the lifetime distribution at use condition for constant stress accelerated life tests when extrinsic failure mode as well as intrinsic one exists. A mixture of two log-normal distributions is introduced to describe these failure modes and it is assumed that a linear relation exists between the location parameter and stress. An estimation procedure using the expectation and maximization algorithm is proposed and a numerical example is given.

1. Introduction

Accelerated life tests (ALTs) of units under higher-than-usual levels of stress involving high temperature, voltage, pressure, vibration, cycle rate, load, etc., are commonly used to quickly obtain information on the lifetime distribution of durable products at use condition stress. The inference on ALTs usually assumes that the lifetime distribution at each stress comes from a prespecified parametric family of distributions such as exponential, Weibull, lognormal etc.; In lognormal distributions, Nelson and Kielpinski(1976) have considered maximum likelihood method for constant stress ALTs. Chung and Bai (1998) suggested estimation method and optimal design for step-stress ALTs. See Nelson(1990) for detailed treatment of ALTs.

Most of previous works assume that the lifetime distribution has only intrinsic failure which occurs due to wearout. However, this assumption may not be appropriate in some populations of electronic devices or other system components, since extrinsic failure caused by defects also exists. Mori et al.(1991) showed that the lifetime data of capacitors include extrinsic failures. Prendergast et al. (1997) predicted the reliability of dielectrics when both intrinsic and extrinsic failures exist. Martin et al.(1997) monitored intrinsic and extrinsic breakdown properties of capacitors and Croes et al.(1998) fitted resistors failure data on log-normal probability paper and found that they are subject to two failure modes.

This paper proposes a method of estimating the lifetime distribution at use condition for constant stress ALTs when extrinsic failure mode as well as intrinsic one exists. A mixed distribution is introduced to describe the two failure modes. It is commonly used for modeling the lives of electrical and mechanical components when the failure is caused by more than one failure mode. Assuming that each failure mode follows a log-normal distribution and its location parameter is a linear function of stress, the maximum likelihood estimates (MLEs) of the distribution parameters and mixing proportion are obtained by expectation and maximization(EM) algorithm. Section 2 describes an ALT model with intrinsic and extrinsic failure modes. EM algorithm and estimators of the lifetime distribution are presented and a numerical example is given in Section 3. The following notations will be used in this paper.

Notation

h	Number of stress levels.
k	Failure mode number; 1(intrinsic), 2(extrinsic).
s_j	j th stress level , $j = 1, L, h$.
n_j	Number of test units at stress s_j, $j = 1, \Lambda, h$.
ξ_j	Standardized stress, $\xi_j = \dfrac{s_j - s_0}{s_h - s_0}$, $j = 1, \Lambda, h$.
α_{0k}, α_{1k}	Parameters of linear relation, $k = 1,2$.
β_{0k}, β_{1k}	Parameters of standardized linear relation, $k = 1,2$.
μ_{jk}	Location parameter at stress s_j, $j = 1, \Lambda, h$, $k = 1,2$.
σ_k	Scale parameter, $k = 1,2$.
η_j	Censoring time at at stress s_j, $j = 1, \Lambda, h$.
$F_k(\cdot)$	Normal cumulative distribution function(*cdf*),

$k = 1,2$.

$F(\cdot)$ Mixture of normal *cdf*s.

Θ_1, Θ_2 Parameter sets of intrinsic and extrinsic failure distributions

Y_{ij} Log-lifetime of unit i under stress s_j , $i = 1,2,\Lambda,n_j$, $j = 1,2,\Lambda,h$.

π_1, π_2 Mixing proportions of intrinsic and extrinsic failure modes.

2. The Model

Assumptions

1) At any stress s , the log-lifetime of a test unit follows a mixed normal distribution with location and scale parameters, $\mu_k(s)$ and σ_k , $k =$ 1(intrinsic), 2(extrinsic).
2) μ_k is a linear function of a (possibly transformed) stress s ; that is,

$$\mu_k(s) = \alpha_{0k} + \alpha_{1k}s, \quad k = 1,2$$

3) The scale parameter σ_k, $k = 1,2$, is constant and is independent of the stress.
4) The lifetimes of test units are independent and identically distributed.

Lifetime distribution

Let Y_{ij} be the random variable representing the log-lifetime of unit i under stress s_j. From the assumption of the mixture of 2 normal distributions, the *pdf* of Y_{ij} , $i = 1,2,\Lambda,n_j$, $j = 1,2,\Lambda,h$, is:

$$f(y_{ij};\Theta) = \pi_1 f_1(y_{ij};\Theta_1) + \pi_2 f_2(y_{ij};\Theta_2)$$

where $\Theta = \{\pi_1,\Theta_1,\Theta_2\}$, $\Theta_k = \{\beta_{0k},\beta_{1k},\sigma_k\}$, $k = 1,2$, and $f_k(\cdot)$, $k = 1,2$, is the *pdf* of normal distribution with location parameter $\mu_{jk} = \beta_{0k} + \beta_{1k}\xi_j$ and scale parameter σ_k at stress ξ_j .

3. Estimation with EM algorithm

When more than one failure mode exist, the *pdf* of the time to failure can be multimodal, and a mixed distribution is a good candidate to describe the times to failure. See, for instance, Mann et al.(1974, CH.4), Lawless(1982, CH.5), Jiang and Kececioglu(1992), Moosa et al.(1996) and Gerhold(1998). In the mixed distribution, however, one can not obtain an explicit solution to the maximum likelihood equation for the mixing proportion since the number of intrinsic failures is unknown and thus it can be regarded as a missing variable, and the EM algorithm can be utilized.

The EM algorithm can obtain iterative solutions to the maximum likelihood equations in a wide class of missing data problems. On each iteration of the EM algorithm there are two steps; the expectation step and the maximization step. In the expectation step, log-likelihood including missing data are replaced by their conditional expectations given the observed data. In the maximization step, MLEs of the parameters are computed which maximize the conditional expectations calculated in the expectation step. See Dempster et al.(1977) for details.

In this section, an estimation procedure using the EM algorithm is proposed for the data from ALTs without censoring.

When all failure times are observed, the log-likelihood is;

$$\log L = \sum_{j=1}^{h}\sum_{i=1}^{n_j}\log f(y_{ij};\Theta)$$

$$= \sum_{j=1}^{h}\sum_{i=1}^{n_j}\log\left(\pi_1 f_1(y_{ij};\Theta_1) + \pi_2 f_2(y_{ij};\Theta_2)\right)$$

To estimate π_1, we need to know the number of intrinsic failures. It is, however, unobservable or missing, since we only get to observe the sum of intrinsic and extrinsic failures. Thus the log-likelihood of a complete data set should be obtained before expectation step.

Let I_{ij1} and $I_{ij2}(= 1 - I_{ij1})$ be the indicator variable denoting whether i th unit at j th stress follows intrinsic or extrinsic failure mode, respectively. If these I_{ijk} were observable, then the log-likelihood of a complete data set would become

$$\log L_c = \sum_{j=1}^{h}\sum_{i=1}^{n_j}\sum_{k=1}^{2} I_{ijk}\left[\log f_k(y_{ij};\Theta_k) + \log\pi_k\right]$$

In the above formula, however, I_{ijk} 's are missing variables.

Expectation step

As the log-likelihood of a complete data set is linear in the unobservable data I_{ijk} , the expectation step simply requires the calculation of the conditional expectation of I_{ijk} given the observed data y_{ij} . Given the y_{ij} , the probability of $I_{ijk} = 1$ is

$$\Pr\{I_{ijk} = 1 \mid y_{ij}\} = \frac{\pi_k f_k(y_{ij};\Theta_k)}{f(y_{ij};\Theta)}$$

$$= \tau_k(y_{ij};\Theta),$$

for $i = 1,2,\Lambda\ ,n_j$ and $j = 1,2,\Lambda\ ,h$. The quantity $\tau_1(y_{ij};\Theta)$ and $\tau_2(y_{ij};\Theta)$ $\left(= 1-\tau_1(y_{ij};\Theta)\right)$ are, respectively, the posterior probabilities that the failure is intrinsic and extrinsic given y_{ij} . Thus the expectation of I_{ijk} given y_{ij} on the $(p+1)$th iteration is

$$E_{\Theta^{(p)}}\left(I_{ijk} \mid y_{ij}\right) = \tau_k\left(y_{ij};\Theta^{(p)}\right),$$

where $\Theta^{(p)}$ is the parameter set obtained on the pth iteration. Thus the conditional expectation of log-likelihood is

$$Q\left(\Theta;\Theta^{(p)}\right) = \sum_{j=1}^{h}\sum_{i=1}^{n_j}\sum_{k=1}^{2}\tau_k\left(y_{ij};\Theta^{(p)}\right)\left[\log f_k\left(y_{ij};\Theta_k\right) + \log \pi_k\right]$$

Maximization step

At the $(p+1)$th stage of the maximization step, the intent is to maximize $Q\left(\Theta;\Theta^{(p)}\right)$ with respect to Θ to produce a new estimate $\Theta^{(p+1)}$ of Θ . First partial derivatives of the conditional expectation of log-likelihood are given in Appendix A.

The esitmates of π_k , β_{0k} , β_{1k} and σ_k , $k = 1,2$, obtained at the $(p+1)$th stage of the M-step from formulas in Appendix satisfy

$$\hat{\pi}_1^{(p+1)} = \frac{A_{0,1}^{(p)}}{n},$$

$$\hat{\beta}_{1k}^{(p+1)} = \frac{A_{1,k}^{(p)}\cdot B_{0,k}^{(p)} - A_{0,k}^{(p)}\cdot B_{1,k}^{(p)}}{\left(A_{1,k}^{(p)}\right)^2 - A_{0,k}^{(p)}\cdot A_{2,k}^{(p)}},\quad k = 1,2,$$

$$\hat{\beta}_{0k}^{(p+1)} = \frac{B_{0,k}^{(p)} - A_{1,k}^{(p)}\cdot \hat{\beta}_{1k}^{(p+1)}}{A_{0,k}^{(p)}},\quad k = 1,2,$$

$$\hat{\sigma}_k^{(p+1)} = \left[\frac{A_{3,k}^{(p)}}{A_{0,k}^{(p)}}\right]^{1/2},\quad k = 1,2,$$

where $n = \sum_{j=1}^{h} n_j$,

$$A_{l,k}^{(p)} = \sum_{j=1}^{h}\sum_{i=1}^{n_j}\tau_k\left(y_{ij};\hat{\Theta}^{(p)}\right)\xi_j^l,\quad l = 0,1,2,$$

$$A_{3,k}^{(p)} = \sum_{j=1}^{h}\sum_{i=1}^{n_j}\tau_k\left(y_{ij};\hat{\Theta}^{(p)}\right)\left(y_{ij} - \hat{\mu}_{jk}^{(p+1)}\right)^2,$$

and $B_{m,k}^{(p)} = \sum_{j=1}^{h}\sum_{i=1}^{n_j}\tau_k\left(y_{ij};\hat{\Theta}^{(p)}\right)y_{ij}\xi_j^m$, $m = 0,1$.

As the iteration of the expectation and the maximization steps progresses, $\hat{\Theta}$ converges to the stationary solution. If the likelihood function is unimodal, the stationary solution of the algorithm is the unique MLE(Wu (1983)).

Example 1

The proposed method is illustrated with failure data generated from a mixed normal distribution with parameters;

$\pi_1 = 0.8$;

$\beta_{01} = 16$, $\beta_{11} = -6$, $\sigma_1 = 0.5$;

$\beta_{02} = 14$, $\beta_{12} = -8$, $\sigma_2 = 1.0$;

Given $\xi_1 = 0.5$, $\xi_2 = 1.0$ and $n_1 = n_2 = 30$, Table 1 contains the log of failure times(minutes).

<Table 1. Failure times>

Low Stress			High Stress		
8.1825	12.5160	13.2323	4.1623	9.4378	10.0166
8.4499	12.6121	13.2434	4.3560	9.5391	10.0903
9.6558	12.7010	13.2639	4.5516	9.6015	10.2457
10.0676	12.7776	13.3425	4.8514	9.7468	10.2995
10.7134	12.9200	13.3474	5.9849	9.7648	10.3698
11.7885	12.9481	13.5994	6.0113	9.8527	10.5003
12.1650	12.9721	13.6342	6.3306	9.8932	10.5014
12.2501	13.0243	13.7395	6.6691	9.9554	10.5097
12.2720	13.0373	13.7558	7.2190	9.9597	10.6152
12.4404	13.1980	14.0548	9.3319	9.9981	10.6823

Initial step:

Initial estimates of $\pi_1^{(0)}$, $\Theta_1^{(0)}$ and $\Theta_2^{(0)}$ are chosen. Let $\pi_1^{(0)} = 0.5$, $\beta_{01}^{(0)} = 20$, $\beta_{11}^{(0)} = -4$, $\sigma_1^{(0)} = 1.0$, $\beta_{02}^{(0)} = 10$, $\beta_{12}^{(0)} = -10$, $\sigma_2^{(0)} = 2.0$.

Expectation step:

With the initial estimates, the $\tau_k\left(y_{ij};\Theta^{(0)}\right)$ can be computed for $i = 1,2,\Lambda\ ,n_j$, $j = 1,2,\Lambda\ ,h$ and $k = 1,2$. For example,

$$\tau_1\left(y_{11};\Theta^{(0)}\right) = \frac{\pi_1^{(0)} f_1\left(y_{11};\Theta_1^{(0)}\right)}{f\left(y_{11};\Theta^{(0)}\right)} = 0.0004 .$$

Maximization step:

With $\tau_1\left(y_{ij};\Theta^{(0)}\right)$ and $\tau_2\left(y_{ij};\Theta^{(0)}\right)$, $\pi_1^{(0)}$, $\Theta_1^{(0)}$ and $\Theta_2^{(0)}$ are updated.

$$\hat{\pi}_1^{(1)} = 0.1209,$$

$$\hat{\beta}_{01}^{(1)} = 16.8183, \hat{\beta}_{11}^{(1)} = -6.2751, \quad \hat{\sigma}_1^{(1)} = 0.2279,$$

$$\hat{\beta}_{02}^{(1)} = 15.8089, \hat{\beta}_{12}^{(1)} = -7.2874, \quad \hat{\sigma}_2^{(1)} = 1.8748.$$

Computations are iterated until all parameters satisfy inequality

$$\left|\theta^{(p+1)} - \theta^{(p)}\right| < 10^{-5}.$$

The stationary solution is obtained, after 31 iterations, as

$$\hat{\pi}_1 = 0.7570,$$

$$\hat{\beta}_{01} = 15.9919, \quad \hat{\beta}_{11} = -5.9484, \quad \hat{\sigma}_1 = 0.4697,$$

$$\hat{\beta}_{02} = 13.7814, \quad \hat{\beta}_{12} = -8.2091, \quad \hat{\sigma}_2 = 1.1102.$$

4. Conclusion

We have proposed a ML estimation method estimating the lifetime distribution containing intrinsic and extrinsic subsets for constant stress ALTs. EM algorithm is used to estimate the lifetime distributions of each failure mode and the mixing proportion. The results can be extended to the censored data case. Designing ALTs with intrinsic and extrinsic failure mode is still under study.

Appendix A

Let $z_k\left(y_{ij}\right) = \dfrac{y_{ij} - \left(\beta_{0k} + \beta_{1k}\xi_j\right)}{\sigma_k}$, and the first partial derivatives of (3.4) with respect to Θ are

$$\frac{\partial Q\left(\Theta;\Theta^{(p)}\right)}{\partial \beta_{0k}} = \sum_{j=1}^{h}\sum_{i=1}^{n_j} \frac{\tau_k\left(y_{ij};\Theta^{(p)}\right) z_k\left(y_{ij}\right)}{\sigma_k}, \qquad k = 1, 2,$$

$$\frac{\partial Q\left(\Theta;\Theta^{(p)}\right)}{\partial \beta_{1k}} = \sum_{j=1}^{h}\sum_{i=1}^{n_j} \frac{\tau_k\left(y_{ij};\Theta^{(p)}\right) z_k\left(y_{ij}\right)\xi_j}{\sigma_k}, \qquad k = 1, 2,$$

$$\frac{\partial Q\left(\Theta;\Theta^{(p)}\right)}{\partial \sigma_k} = \sum_{j=1}^{h}\sum_{i=1}^{n_j} \frac{\tau_k\left(y_{ij};\Theta^{(p)}\right)\left\{-1 + \left(z_k\left(y_{ij}\right)\right)^2\right\}}{\sigma_k}$$

$k = 1, 2$.

References

1. S. W. Chung, D. S. Bai, "Optimal Designs of Simple Step-Stress Accelerated Life Tests for Lognomal Lifetime Distributions", *Int. J. of Rel., Qual. and Safety Eng.* Vol 5, 1998, pp 315-336.
2. K. Croes, W. De Ceuninck, L. De Schepper, L. Tielemans, "Bimodal Failure Behavior of Metal Film Resistors", *Qual. Reliab. Engng. Int.*, vol. 14, 1998, pp.87-90.
3. A. P. Dempster, N. M. Laird, D. R. Rubin, "Maximum Likelihood from Incomplete Data", *J. of Royal Statistical Society, Series B*, vol.39, 1977, pp1-38.
4. J. Gerhold, M. Hubmann, E. Telser, "Breakdown Probability and Size Effect in Liquid Helium", *IEEE trans. Dielectrics and Electrical Insulation*, vol. 5, 1998 Jun., pp321-333.
5. S. Jiang, D. Kececioglu, "Maximum Likelihood Estimates, from Censored Data, for Mixed-Weibull Distributions," *IEEE Trans. on Reliability*, 1992, R-41, pp248-255.
6. J. F. Lawless, *Statistical Models and Methods for Lifetime Data*, 1982, John and Wiley & Sons.
7. N. R. Mann, R. E. Schafer, N. D. Singpurwalla, *Methods for Statistical Analysis of Reliability and Life Data,* John Wiley and Sons, New York, 1974.
8. A. Martin, P. O'sullivan, A. Mathewson, "Study of Unipolar Pulsed Ramp and Combined Ramped/ Constant Voltage Stress on Mos Gate Oxides" *Microelectron. Reliab.* Vol. 37, 1997, pp.1045-1051.
9. M. S. Moosa, K. F. Poole, "EFSIM: An Integrated Circuit Early Failure Simulator", *Qual. Reliab. Engng. Int.*, vol. 12, 1996, pp.229-234
10. S. Mori, N. Arai, Y. Kaneko, K. Yoshikawa, "Polyoxide Thinning Limitation and Superior ONO Interpoly Dielectric fo Nonvolatile Memory Devices", *IEEE trans. Electron Devices*, vol. 38, 1991 Feb., pp270-276.
11. W. Nelson, *Accelerated Testing: Statistical Models, Test Plans, and Data Analyses*, 1990, John Wiley & Sons.
12. W. Nelson, T. J. Kielpinski, "Theory for Optimum Censored Accelerated Life Tests for Normal and Lognormal Distributions", *Technometrics*, vol 18, 1976 Feb., pp310-320.
13. J. G. Prendergast, E. Murphy, M. Stephenson, "Predicting Gate Oxide Reliability from Statistical Process Control Nodes in Integrated Circuit Manufacturing – a Case Study", *Qual. Reliab. Engng. Int.* vol.13, 1997, pp267-277.
14. C. Wu, "On the Convergence Property of the EM Algorithm", *Annals of Statistics*, Vol. 11, 1983, pp95-103.