MATHEMATICAL MODELING OF FERTILIZER IMPACTION USING RecurDyn®

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ABSTRACT

Fertilizer impaction mechanism was simulated using a commercial program RecurDyn® a dynamic program that could handle contact problems. Even if there had been numerous papers on modeling of fertilizer applicator, the performance predictions were not satisfactory due to simplification in modeling. The most significant simplification was assumption of fertilizer particles as a solid particle.

The assumption would eliminate rotation of fertilizer particles during the impaction mechanism. However, impaction of rotating body would be different from that of a solid particle. This paper introduced how the impaction was modeled using RecurDyn®. In order to simulate, restitution coefficient and contact time was measured. A stiffness coefficient and a damping coefficient of a fertilizer was theoretically estimated using the measured data. Validity of the simulation result was not proved yet, but judged to be promising.

Key Word : Fertilizer, Impaction, Broadcaster, Simulation, RecurDyn®

INTRODUCTION

Fertilizer application is one of the most important crop management. Recently new paradigm in agriculture such as Sustainable Agriculture, Least Input Sustainable, Precision Farming have been popular. One of the targets of the paradigms is reduction in amount of chemical fertilizer.

This research is done to develop a broadcaster with uniform application pattern as well as fixed relatively narrow swath, that could be attached to the head feeding type combines. The broadcaster attached to combines are useful for farmers who cultivates rice and barley in a year. In this research, the uniform swath was expected to be achieved using a rotor with vanes and deflectors.

Mathematical modeling of transport in the air was fairly reliable but spreading by the rotor involves complex processes caused by particle interactions and impaction on rotors as well as rotation of particles. For research convenience, RecurDyn® was selected and proper values in simulation parameters were found by trial and error. In this paper, a broadcaster was modeled as a rotating spinner with vanes and simulated under several condition.

Numerous studies has been done on fertilizer particle movement on a spinner of broadcaster and their application patterns. Pitt(1982) developed an proximate model on fertilizers using probability theory and theory on drag force. He concluded that spray pattern was mainly affected by the mean size of fertilizers. Size distribution was reportedly not dominant factor in application pattern.

Patterson et al.(1962) reported that departing velocity of fertilizers form a spinner had tangential component as well as radial component, of which magnitude depended on mode of fertilizer movement(rolling, sliding or impacting on a spinner) and friction coefficient between fertilizer particles and spinner surface. They also argued that variation of applicator performance were caused by variation of motion in group of fertilizer particles.

Olieslagers et al. (1996) developed a mathematical model which involved spreading factors, particle characteristics and environment factors. However their model did not included impaction of fertilizers against a spinner. They simulated the model under various conditions and evaluated the effect of changes in spreading factors. They also evaluated performance of a spreader for site-specific application and revealed difficulties in site-specific fertilizer application.

MATERIALS AND METHODS

The experimental material was a compound fertilizer(10-16-10) produced by D company in Korea. Fertilizers passed a standard sieve(mesh size : 6.3mm) was used as the experimental material.

Friction coefficient and restitution coefficient

Fertilizer particles were in order to prevent rolling when they were located on slope. The group of fertilizer particles was located on a plane and one edge of the plane was elevated to change slope. The slope angle(θ) of the surface was the same as the static friction coefficient($\mu = \tan\theta$) of the fertilizer particles with the surface. The slope angle was measured by a digital level with 0.1° accuracy.

Restitution coefficient (ε) was defined as a ratio between restituting velocity(V_{Ai}) to impacting velocity(V_{Af}). The coefficient was known as interaction properties between an object to the other. A test surface was located in a horizontal plane and a fertilizer particles was released from a set height(h_{in}) with zero initial velocity as shown in Fig. 1. After impaction, the restituted height and the range of projectile were measured using a video camera. A equation for the restitution coefficient could be represented as Eq.(1)

$$\varepsilon = -\frac{V_{Af}}{V_{Ai}} = \sqrt{\frac{h_{out}}{h_{in}}} \times \frac{1}{\sin(\tan^{-1}(4h_{out}/d))} \qquad \text{--------(1)}$$

Determination of contact time

Contact time between a fertilizer particle and a surface can not be measured directly. To determine the contact time, a high-speed camera and a machine vision program were employed. Focal length of the camera(Kodak EktaPro, EM Motion Analyzer Model 1000-E) was 50 mm, and distance between a lens and the plane where a fertilizer particle impacted on surface was set to be 400 mm. Image sampling rate was 1000 frame/sec and saved in a hard diskette as a MAW format file.

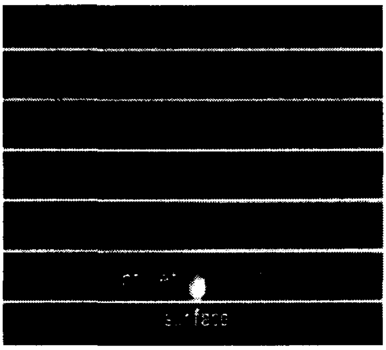

Fig. 1 View of the pellet
(just before the impact)

The image processing program for the high-speed camera was MAW1.2. Velocity of a particle before and after impaction, V_{in} and V_{out} could be measured using time interval and distance between particle positions from different frames before and after impaction. Also, remaining distance, h_{in} and passed distance h_{out} were measured as shown in Fig. 2 and Fig. 3. Fig. 1 showed a frame captured before impaction. Position of a solid particle could be easily found using grid lines in the image processing program.

Using the measured values and time interval between frames, the contact time can be measured as Eq. (2).

$$\frac{h_{in}}{v_{in}} + impact\ time + \frac{h_{out}}{v_{out}} = 0.001\,\text{sec} \qquad \text{----------(2)}$$

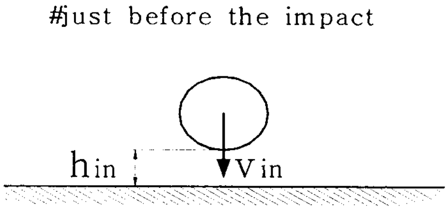

Fig. 2 Schematic features of the frame (just before the impact)

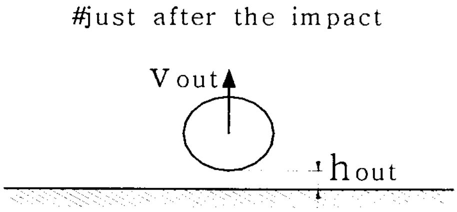

Fig. 3 Schematic features of the frame (just after the impact)

Spring coefficient and damping coefficient

Impaction of a fertilizer particle with could be modeled as one dimensional free vibration system with spring, damper and mass while it was in contact with a surface. Before and after impaction, the particle could be modeled a solid object moving under influence of gravity and drag force. Fig. 4 shows the impaction processes with time. In this study, deformation of a fertilizer particle was assumed to be elastic.

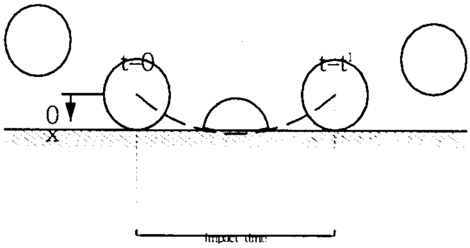

Fig. 4 Movement of a fertilizer in the Impact time

The position(x) of a mass in case of a free vibration system was described by Eq. (3). Here position x represented the location of center of mass. Eq. (3) and Eq. (4) was valid only while x was smaller than the radius of the particle.

$$x = e^{-\zeta\omega_n t}\left(x_0\cos\omega_d t+\frac{\dot{x}_0+\zeta\omega_n x_0}{\omega_d}\sin\omega_d t\right) \qquad \text{--------(3)}$$

$$\begin{aligned}\dot{x} = &-\zeta\omega_n e^{-\zeta\omega_n t}\left(x_0\cos\omega_d t+\frac{\dot{x}_0+\zeta\omega_n x_0}{\omega_d}\sin\omega_d t\right) \\ &+e^{-\zeta\omega_n t}\left(-x_0\,\omega_d\sin\omega_d t+(\dot{x}_0+\zeta\omega_n x_0)\cos\omega_d t\right)\end{aligned} \qquad \text{-------(4)}$$

Because the restitution coefficient of solid particle was measured, the initial condition x_0, $\dot{x}_o$, x_{t1}, $\dot{x}_{t1}$ could be calculated as Eq.(5) using the initial height and initial velocity.

$$\begin{aligned}&x_0=0,\ \dot{x}_0=\sqrt{2gh} \\ &x_{t_1}=0,\ \dot{x}_{t_1}=-\dot{x}_0\varepsilon=-\sqrt{2gh}\,\varepsilon, \quad \varepsilon\text{: restitution coefficient}\end{aligned} \qquad \text{-------(5)}$$

When the initial conditions were applied, position and velocity of a particle could be represented by Eq.(6) and Eq.(7)

$$x = \frac{\sqrt{2gh}}{\omega_d} \sin \omega_d t \qquad \text{--------(6)}$$

$$\dot{x} = e^{-\zeta \omega_n t} \sqrt{2gh} \left(- \frac{\zeta' \omega_n}{\omega_d} \sin \omega_d t + \cos \omega_d t \right) \qquad \text{--------(7)}$$

The contact time or the time when the particle bounced from a surface, t_1 should be π / ω_d or Eq. (8)

$$t_1 = \frac{\pi}{\omega_d} = \frac{\pi \sqrt{m}}{\sqrt{1-\zeta^2} \sqrt{k}} \quad \text{or} \quad k = \frac{\pi^2 m}{(1-\zeta^2) t_1^2} \qquad \text{--------(8)}$$

Restitution coefficient(ε) and damping factor(ζ) could be defined as follows as the initial condition($\dot{x}_{t_1} = -\sqrt{2gh}\,\varepsilon$) and Eq. (9) were put into Eq. (8).

$$\varepsilon = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}} \pi} \quad \text{or} \quad \zeta = \pm \frac{\ln \varepsilon}{\sqrt{\pi^2 + (\ln \varepsilon)^2}} \quad (\zeta \geq 0) \qquad \text{------(9)}$$

Damping coefficient could be positive value of Eq. (10), because $\zeta = c / 2\sqrt{km}$ and the physical properties are always positive.

$$c = \pm 2\sqrt{km} \frac{\ln \varepsilon}{\sqrt{\pi^2 + (\ln \varepsilon)^2}} \quad (c \geq 0) \qquad \text{------(10)}$$

Using the above Eq.(8), Eq. (9) and Eq.(10), damping coefficient and spring constant could be determined based on the measured values, restitution coefficient and contact time.

RESULT AND DISCUSSION

Bulk density, true density(particle density), repose angle, friction coefficient, sphericity were measured and but not shown in this paper. True density of real chemical fertilizer showed a range of 1.2~1.7, but plastic pellets showed 0.877.

Table 1. Properties of fertilizers
(unit : g/㎤ or °)

Property	Mixture (17-21-17)	Compound (10-16-10)	Pellet
Particle density	1.595	1.734	0.877
Bulk density	0.859	0.933	0.551
Repose angle	36.9	36.0	31.7

*Condition : 25℃/29%RH(17-21-17), 25.9℃ /69%RH(10-16-10, pellet)

**MC(db) of fertilizer : 3%(17-21-17), 2.2%(10-16-10)

Table. 2 Friction coefficient of fertilizers

Material	Mixture (17-21-17)	Compound (10-16-10)	Pellet
PVC	0.47	0.45	0.43
Steel	0.51	0.50	0.48
Aluminum	0.59	0.46	0.47
Rubber	1.23	1.08	0.79
MC	0.63	0.52	0.50

*Condition : 25℃/29%RH(17-21-17), 25.9℃ /69%RH(10-16-10, pellet)

**MC(db) of fertilizer : 3%(17-21-17), 2.2%(10-16-10)

Friction coefficients of fertilizers with various materials PVC were close to those of plastic pellets. Coefficients of restitution of fertilizer on hard surface were smaller than those on soft surface such as PVC and MC. However coefficients of restitution of pellets did not show such tendency and were larger than those of fertilizers.

Contact time of fertilizer and plastic pellets were measured with 3 repetitions on MC, PVC and steel. Table 3 summarized the results. As shown contact time on plastics(MC and PVC) were nearly same, but showed difference between steel and plastics.

Table 3 Spring coefficient and damping coef. between fertilizer and spinner during the impact

	Items	Compound fertilizer (10-16-10)	Pellet
MC	impact time(sec)	0.0002323	0.0002474
	restitution coeff.	0.43	0.478
	spring coeff.(N/m)	4356.6048	13803.966
	damping coef.(Ns/m)	0.1509914	0.2021185
PVC	impact time(sec)	0.0002581	0.0002036
	restitution coeff.	0.45	0.534
	spring coeff.(N/m)	3858.5642	10257.137
	damping coef.(Ns/m)	0.1324056	0.1710161
Steel	impact time(sec)	0.0002917	0.0001673
	restitution coeff.	0.351	0.525
	spring coeff.(N/m)	2630.4851	242698.53
	damping coef.(Ns/m)	0.1449512	0.6147344

Spring constant and damping coefficient

Spring constants and damping coefficients of fertilizers and pellets were calculated and shown in Table 3. Spring constant and damping coefficient of pellets were larger than those of fertilizer,

Model development

A spinner was modeled as a disk with 4 vanes. Spinner was assumed to be made of MC and of which diameter was set to be 300 mm with 4 radial vanes with height of 30mm and width of 5mm. At center of a spinner a revolute joint was assumed. Fig. 5 showed the spinner modeled by RecurDyn.

A fertilizer particle was released from a fixed height with zero initial velocity over the spinner. At beginning two mass had independent motions. When the particle reached on any surface of the spinner, two masses were modeled to interact each other. Contact force between two mass was calculated. The surface friction between a particle and a rotating spinner caused the rotation of a particle. Fig. 5 showed a trajectory of a particle over the sinner surface. As shown in the figure, particle showed one bounce over the spinner and sliding along a vane.

Simulation

Accuracy of simulation depends on simulation parameters. In our application, error tolerance was very important because displacement change during impaction was infinitesimal. Time step and error tolerance were set to be 0.00005 and 0.0001. Measured properties of fertilizers and spinners were used.

As an example, spinner material was selected as MC, PVC and Steel. Spinner diameter was 300mm, rational speed 500rpm, Release point was 43mm from the center of the spinner and its height was 50mm. Fig. 6 and 7.

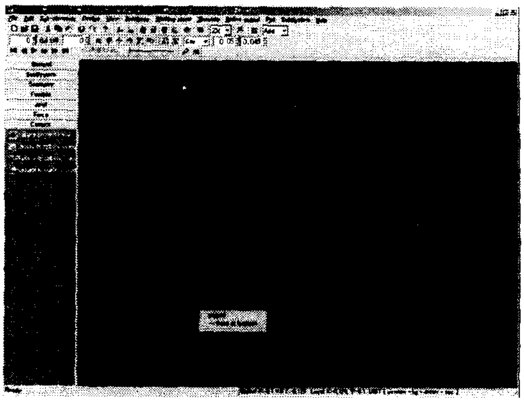

Fig. 5 Spinner model by Recurdyn

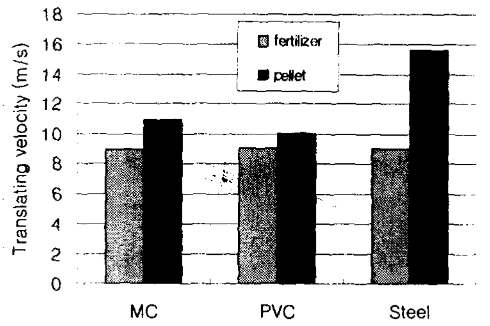

Fig. 6 Effect of material in trans. velocity

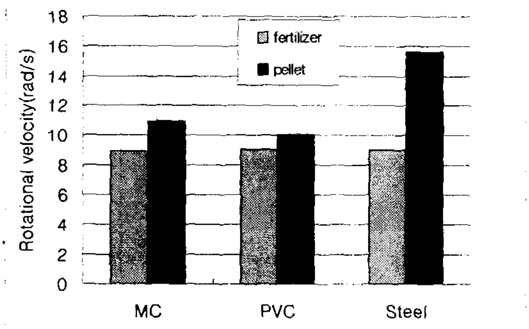

Fig. 7 Effect of material in Rotating velocity

Rotation speed effect was simulated at 200, 500, 800 and 1100rpm. Fig. 8 showed translational velocity of fertilizer at the departing moment from the spinner. Fig. 9 showed the effect of spinner size (200, 300 and 400mm) at 500 rpm, releasing height 43mm.

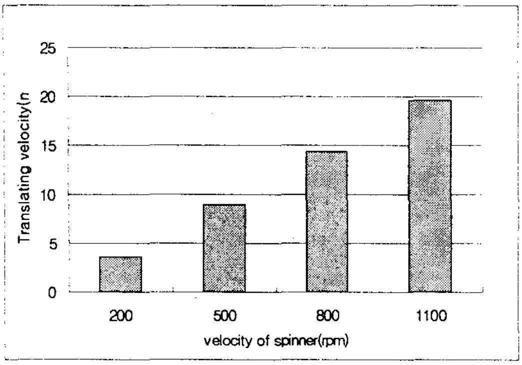

Fig. 8 Effect of spinner velocity in translation velocity

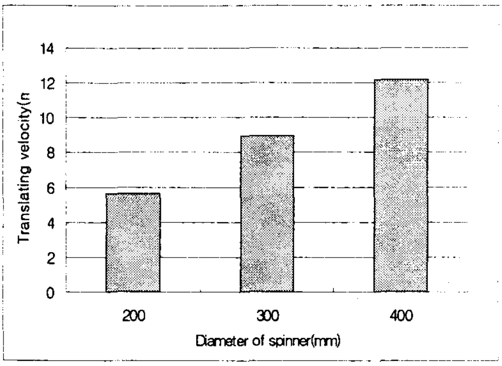

Fig. 9 Effect of spinner diameter in translation velocity

CONCLUSION

In order to model a broadcaster, RecurDyn® , a commercial software for dynamic analysis was adopted and some physical properties related to impaction such as restitution coefficient and contact time during the impact process was measured. Assumption of one dimensional vibration was taken.

Modeling procedure and simulation parameters were chosen and simulations on several cases were accomplished. This simulation techniques was assessed to be appropriate in predicting trajectories under various simulation condition. However, there was a possibility that fertilizer particles in groups might have different characteristics.

At this moment, the validity of this simulation was not evaluated, however possibility to model a particle trajectory with rotational and translational velocities would open a new horizon to model the performance of broadcaster with reflecting plates.

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