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E_N^2 상의 비선형 퍼지 제어 시스템에 대한 완전 제어 가능성

The exact controllability for the nonlinear fuzzy control system in E_N^2.

권영철, 강점란, *박종서

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Abstract

This paper we study the exact controllability for the nonlinear fuzzy control system in E_N^2 by using the concept of fuzzy number of dimension 2 whose values are normal, convex, upper semicontinuous and compactly supported surface in R^2.

keyword : fuzzy control system, fuzzy number of dimension 2, exact controllability,

I. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva ([3]) studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala ([6]) proved the existence and uniqueness of fuzzy solution for the following equation:

$$\begin{cases} \dot{x}(t) = f(t, x(t)), \\ x(0) = x_0, \end{cases}$$

where f is a continuous mapping from $R^+ \times R \rightarrow R$ and x_0 is a fuzzy number in E^1. Diamond and Kloeden ([1]) proved the fuzzy optimal control for the following system:

$$\dot{x}(t) = a(t)x(t) + u(t), \quad x(0) = x_0$$

where $x(\cdot), u(\cdot)$ are nonempty compact interval-valued functions on E^1.

Recently, Young-Chel Kwun, Jum-Ran Kang and Seon-Yu Kim ([5]) proved the existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition in E_N^1 using by Kuhn-Tucker theorems.

The purpose of this paper is to investigate the exact controllability of the nonlinear fuzzy control system in E_N^2. Let E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X and Y ([4]).

We consider the exact controllability for the following nonlinear fuzzy control system:

$$\text{(F.C.S.)} \begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)) + u(t), \\ x(0) = x_0, \end{cases}$$

where $a: [0, T] \to E_N$ is fuzzy coefficient, initial value $x_0 \in E_N^2$ and $f: [0, T] \times E_N^2 \to E_N^2$ is nonlinear function and $u(t) \in E_N^2$ is control function.

II. Properties of fuzzy numbers of dimension 2

We consider a fuzzy graph $G \subset R \times R$ that is a functional fuzzy relation in R^2 such that its membership function $\mu_G(x, y)$, $(x, y) \in R^2$, $\mu_G(x, y) \in [0, 1]$, has the following properties:

1. $\forall x_0 \in R$, $\mu_G(x_0, y) \in [0, 1]$ is a convex membership function.
2. $\forall y_0 \in R$, $\mu_G(x, y_0) \in [0, 1]$, is a convex membership function.
3. $\forall \alpha \in [0, 1]$, $\{(x, y) \in R^2 : \mu_G(x, y) = \alpha\}$ is a convex surface.
4. $\exists$ $(x_1, y_1) \in R^2$, $\mu_G(x_1, y_1) = 1$.

If conditions are satisfied, the fuzzy subset $G \subset R^2$ is called a fuzzy number of dimension 2.

Let E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X and Y.

We denote by fuzzy number of dimension 2 in E_N^2

$$A = (a_1, a_2)$$

where a_1, a_2 is projection of A to axis X and Y respectively. And a_1 and a_2 are fuzzy number in R.

The α-level set of fuzzy number of dimension 2 in E_N^2 defined by

$$[A]^\alpha = \{(x_1, x_2) \in R^2 : (x_1, x_2) \in [a_1]^\alpha \times [a_2]^\alpha\}$$

where operation $\times$ is Cartesian product of the sets.

Let $A, B \in E_N^2$, two fuzzy numbers of dimension 2 A and B are called equal $A = B$, if

$$A = B \Leftrightarrow [A]^\alpha = [B]^\alpha \text{ for all } \alpha \in (0, 1].$$

If $A, B \in E_N^2$ then for $\alpha \in (0, 1]$,

$$[A *_2 B]^\alpha = [a_1 *_1 b_1]^\alpha \times [a_2 *_1 b_2]^\alpha,$$

where $*_2 = +_2, -_2, \cdot_2 \in E_N^2$

and $*_1 = +_1, -_1, \cdot_1 \in E_N^1$.

Let $[a_1]^\alpha \times [a_2]^\alpha$, $0 < \alpha \le 1$, be a given family of nonempty rectangle areas.

If

(2.1) $[a_1]^\beta \times [a_2]^\beta \subset [a_1]^\alpha \times [a_2]^\alpha$ for $0 < \alpha \le \beta$

and

(2.2) $\lim_{k \to \infty} [a_1]^{\alpha_k} \times \lim_{k \to \infty} [a_2]^{\alpha_k} = [a_1]^\alpha \times [a_2]^\alpha$

whenever (α_k) is nondecreasing sequence converging to $\alpha \in (0, 1]$, then the family $[a_1]^\alpha \times [a_2]^\alpha$, $0 < \alpha \le 1$, represents the α-level sets of a fuzzy number of dimension 2 $A \in E_N^2$.

Conversely, if $[a_1]^\alpha \times [a_2]^\alpha$, $0 < \alpha \le 1$, are the α-level sets of a fuzzy number of dimension 2, then the conditions (2.1) and (2.2) holds true.

We define the Hausdorff distance between subsets A and B of R^2 by

$$d_H(A, B) = \max\{d_H^*(A, B), d_H^*(B, A)\}.$$

The metric d_∞ on E_N^2 is defined by

$$d_\infty(A, B) = \sup\{d_H([A]^\alpha, [B]^\alpha) : \alpha \in (0, 1]\}$$

for all $A, B \in E_N^2$.

III. The exact controllability

In this section, we show the exact controllability for the following nonlinear fuzzy control system:

(F.C.S.) $\begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)) + u(t) \ , \\ x(0) = x_0, \end{cases}$

with fuzzy coefficient $a: [0, T] \rightarrow E_N$, initial value $x_0 \in E_N^2$ and control $u: [0, T] \rightarrow E_N^2$ and inhomogeneous term $f: [0, T] \times E_N^2 \rightarrow E_N^2$ satisfies a global Lipschitz condition, there exists a finite constant $k > 0$ such that

$$d_H([f(s, \xi_1(s))]^\alpha, [f(s, \xi_2(s))]^\alpha) \le k d_H([\xi_1(s)]^\alpha, [\xi_2(s)]^\alpha)$$

for all $\xi_1(s), \xi_2(s) \in E_N^2$

The (F.C.S.) is related to the following fuzzy integral system:

(F.I.S.) $\begin{cases} x(t) = S(t)x_0 + \int_0^t S(t-s) f(s, x(s))\, ds \\ \qquad + \int_0^t S(t-s) u(s) ds \ , \\ x(0) = x_0 \ \in E_N^2, \end{cases}$

where $S(t)$ is fuzzy number of dimension 2 and

$$[S(t)]^\alpha = [S_1(t)]^\alpha \times [S_2(t)]^\alpha$$
$$= [S_{1l}^\alpha(t), \ S_{1r}^\alpha(t)] \times [S_{2l}^\alpha(t), \ S_{2r}^\alpha(t)]$$

where $S_{il}^\alpha(t)$ $(i=1,2)$ is $\exp\{\int_0^t a_l^\alpha(s) ds\}$

and $S_{ir}^\alpha(t)$ $(i=1,2)$ is $\exp\{\int_0^t a_r^\alpha(s) ds\}$.

And $S_{ij}^\alpha(t)$ $(i=1,2, \ j=l, r)$ is continuous.

That is, there exists a constant $c > 0$ such that $|S_{ij}^\alpha(t)| \le c$ for all $t \in [0, T]$.

Definition 3.1 The (F.I.S.) is exact controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (F.I.S.) satisfies $x(T) = {}_\alpha x^1$ (i.e., $[x(T)]^\alpha = [x_1(T)]^\alpha \times [x_2(T)]^\alpha$ $= [(x^1)_1]^\alpha \times [(x^1)_2]^\alpha = [x^1]^\alpha$) where x^1 is target set.

We assume that the following linear fuzzy control system with respect to nonlinear fuzzy control system (F.C.S.):

(F.C.S. 1) $\begin{cases} \dot{x}(t) = a(t)x(t) + u(t) \ , \\ x(0) = x_0 \ \in E_N^2 \end{cases}$

is exact controllable. Then

$$x(T) = S(T)x_0 + \int_0^T S(T-s) u(s) ds = {}_\alpha x^1$$

and

$$[x(T)]^\alpha = [S_{1l}^\alpha(T)(x_1)_{0l}^\alpha + \int_0^T S_{1l}^\alpha(T-s) u_{1l}^\alpha(s) ds,$$
$$S_{1r}^\alpha(T)(x_1)_{0r}^\alpha + \int_0^T S_{1r}^\alpha(T-s) u_{1r}^\alpha(s) ds]$$
$$\times [S_{2l}^\alpha(T)(x_2)_{0l}^\alpha + \int_0^T S_{2l}^\alpha(T-s) u_{2l}^\alpha(s) ds,$$
$$S_{2r}^\alpha(T)(x_2)_{0r}^\alpha + \int_0^T S_{2r}^\alpha(T-s) u_{2r}^\alpha(s) ds]$$
$$= [(x^1)_{1l}^\alpha, \ (x^1)_{1r}^\alpha] \times [(x^1)_{2l}^\alpha, \ (x^1)_{2r}^\alpha]$$
$$= [(x^1)_1]^\alpha \times [(x^1)_2]^\alpha = [x^1]^\alpha \ .$$

Defined the fuzzy mapping $\tilde{g}: \mathcal{P}(R^2) \rightarrow E_N^2$ by

$$\tilde{g}^\alpha(v) = \begin{cases} \int_0^T S^\alpha(T-s) v(s) ds \ , & v \subset \overline{\Gamma_u} \ , \\ 0 \ , & \text{otherwise}. \end{cases}$$

Then there exists $\tilde{g}_i: \mathcal{P}(R) \rightarrow E_N$ $(i=1,2)$ such that

$$\tilde{g}_i^\alpha(v_i) = \begin{cases} \int_0^T S_i^\alpha(T-s) v_i(s) ds, & v_i(s) \subset \overline{\Gamma_{u_i}}, \\ 0 \ , & \text{otherwise} \end{cases}$$

where u_i is projection of u to axis X and Y respectively and there exists $\tilde{g}$ $(j=l, r)$

$$\tilde{g}_{il}^\alpha(v_{il}) = \int_0^T S_{il}^\alpha(T-s) v_{il}(s) ds,$$
$$v_{il}(s) \in [u_{il}^\alpha(s), \ u_i^1(s)],$$
$$\tilde{g}_{ir}^\alpha(v_{ir}) = \int_0^T S_{ir}^\alpha(T-s) v_{ir}(s) ds,$$
$$v_{ir}(s) \in [u_i^1(s), u_{ir}^\alpha(s)].$$

We assume that $\tilde{g}_{il}^\alpha$, $\tilde{g}_{ir}^\alpha$ are bijective mappings.

Hence α-level of $u(s)$ are

$$[u(s)]^\alpha = [u_{1l}^\alpha(s), u_{1r}^\alpha(s)] \times [u_{2l}^\alpha(s), u_{2r}^\alpha(s)]$$
$$= [(\widetilde{g_{1l}^\alpha})^{-1}((x^1)_{1l}^\alpha - S_{1l}^\alpha(T)(x_1)_{0l}^\alpha),$$
$$(\tilde{g}_{1r}^\alpha)^{-1}((x^1)_{1r}^\alpha - S_{1r}^\alpha(T)(x_1)_{0r}^\alpha)]$$
$$\times [(\tilde{g}_{2l}^\alpha)^{-1}((x^1)_{2l}^\alpha - S_{2l}^\alpha(T)(x_2)_{0l}^\alpha),$$
$$(\tilde{g}_{2r}^\alpha)^{-1}((x^1)_{2r}^\alpha - S_{2r}^\alpha(T)(x_2)_{0r}^\alpha)].$$

Thus we can be introduced $u(s)$ of nonlinear system

$$[u(s)]^\alpha = [u^\alpha_{1l}(s), u^\alpha_{1r}(s)] \times [u^\alpha_{2l}(s), u^\alpha_{2r}(s)]$$
$$= [(\tilde{g}_{1l}{}^\alpha)^{-1}((x^1)^\alpha_{1l} - S^\alpha_{1l}(T)(x_1)^\alpha_{0l} - \int_0^T S^\alpha_{1l}(T-s) f^\alpha_{1l}(s, x(s))\,ds),$$
$$(\tilde{g}_{1r}{}^\alpha)^{-1}((x^1)^\alpha_{1r} - S^\alpha_{1r}(T)(x_1)^\alpha_{0r} - \int_0^T S^\alpha_{1r}(T-s) f^\alpha_{1r}(s, x(s))\,ds)\ ]$$
$$\times [(\tilde{g}_{2l}{}^\alpha)^{-1}((x^1)^\alpha_{2l} - S^\alpha_{2l}(T)(x_2)^\alpha_{0l} - \int_0^T S^\alpha_{2l}(T-s) f^\alpha_{2l}(s, x(s))\,ds),$$
$$(\tilde{g}_{2r}{}^\alpha)^{-1}((x^1)^\alpha_{2r} - S^\alpha_{2r}(T)(x_2)^\alpha_{0r} - \int_0^T S^\alpha_{2r}(T-s) f^\alpha_{2r}(s, x(s))\,ds)\ ].$$

Then substituting this expression into the (F.I.S.) yields α-level of $x(T)$.

We now set

$$\Phi x(t) =_\alpha S(t)x_0 + \int_0^t S(t-s) f(s, x(s))\,ds + \int_0^t S(t-s)\ \tilde{g}^{-1}(x^1 - S(T)x_0 - \int_0^T S(T-s) f(s, x(s))\,ds\ )ds$$

where the fuzzy mappings $\tilde{g}^{-1}$ satisfied above statements. Notice that $\Phi x(T) =_\alpha x^1$, which means that the control $u(t)$ steers the (F.C.S.) from the origine to x^1 in time T provided we can obtain a fixed point of the nonlinear operator Φ.

Assume that the following hypotheses:

(H1) (F.C.S. 1) is exact controllable.

(H2) Inhomogeneous term $f: [0, T] \times E^2_N \to E^2_N$ satisfies a global Lipschitz condition, there exists a finite constant $k > 0$ such that

$$d_H([f_i(s, x_i(s))]^\alpha, [f_i(s, y_i(s))]^\alpha) \le k\ d_H([x_i(s)]^\alpha, [y_i(s)]^\alpha)$$

for all $x_i(s), y_i(s) \in E_N$ and $f_i: [0, T] \times E_N \to E_N$ $(i = 1, 2)$ is projection of f.

Theorem 3.1 Suppose that hypotheses (H1), (H2) are satisfied. Then the state of the (F.I.S.) can be steered from the initial value x_0 to any final state x^1 in time T.

Proof. Omitted.

IV. Examples

Example 4.1

Consider the following fuzzy control system:

$$\begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)) + u(t), \\ x(0) = x_0 \end{cases}$$

where fuzzy coeffient $a(t) = \tilde{2}t$ and nonlinear term $f(t, x(t))$ is $(\tilde{2}tx(t)^2, \tilde{2}tx(t)^2)$. And initial value x_0 is $(\tilde{0}, \tilde{0})$. Target set is $x^1 = (\tilde{2}, \tilde{3})$. The α-level set of fuzzy numbers are following it. $[\tilde{0}]^\alpha = [\alpha - 1, 1 - \alpha]$, $[\tilde{2}]^\alpha = [\alpha + 1, 3 - \alpha]$, $[\tilde{3}]^\alpha = [\alpha + 2, 4 - \alpha]$.

The α-level of $u(s)$ of nonlinear system are

$$u^\alpha_{1l}(s) = \tilde{g}_{1l}{}^{-1}((\alpha+1) - S^\alpha_{1l}(T)(\alpha - 1) - \int_0^T S_{1l}(T-s)s\ x_{1l}(s)^2 ds\ ),$$
$$u^\alpha_{1r}(s) = \tilde{g}_{1r}{}^{-1}((3-\alpha) - S^\alpha_{1r}(T)(1 - \alpha) - \int_0^T S_{1r}(T-s)s\ x_{1r}(s)^2 ds\ ),$$

Then α-level of $x(T)$ is

$$[x_1(T)]^\alpha = [\alpha + 1, 3 - \alpha] = [\tilde{2}]^\alpha$$

Hence $[x(T)]^\alpha = [\tilde{2}]^\alpha \times [\tilde{3}]^\alpha = [x^1]^\alpha$.

V. References

[1] D. Dubois and H. Prade, Towards Fuzzy Differential Calculus Part 1 : Integration of fuzzy mappings, Fuzzy Sets and Systems, 8, 1-17, (1982).

[2] P. E. Kloeden, Fuzzy dynamical systems, Fuzzy Sets and Systems, 7, 275-296, (1982).

[3] O. Kaleva, Fuzzy differential equations, Fuzzy Sets and Systems, 24, 301-317, (1987).

[4] A. Kaufmann and M. M. Gupta, Introduction to fuzzy arithmetic, Van Nostrand Reinhold, (1991).

[5] Y. C. Kwun, J. R. Kang, S. Y. Kim, The existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition, Journal of Fuzzy Logic and Int. Systems, 10, No. 1, 6-11, (2000).

[6] S. Seikkala, On the fuzzy initial value problem, Fuzzy Sets and Systems, 24, 319-330, (1987).