Vibration Suppression Control of 3-mass Inertia System by using LMI Theory

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Abstract: The purpose of this paper is to propose an approach to suppress the vibration of three-mass inertia system based on the LMI theory, and confirm its validity through simulations under the condition of parameter variation. First, the existing H_∞ servo problem is modified to a structure to which the LMI theory can be applied by virtue of the internal model principle. By adopting this structure, we can divide given specifications for the vibration suppression problem into H_2 and H_∞ performance criteria. The results of simulation for the three-mass inertia system show that the proposed design approach is quite effective.

Keywords: 3-mass inertia system, robust servo, LMI theory

국문 요약: 본 논문은 3 관성시스템의 진동억제를 위한 강인한(robust) 제어기의 설계법을 제안하고 이를 simulation 을 통하여 확인하는 데 있다. 일반적으로 관성 시스템의 제어 문제는 결국, 시스템 자체에서 발생하는 진동을 최대한 억제하면서 빠른 시간에 원하는 위치에 출력을 가져가는 데 있다. 이 경우 문제로 되는 것은 시스템의 모델링 과정에서 발생하는 플랜트의 불확실성과 parameter 변동이다. 여기서는 일반적인 강인한 제어기 설계 이론의 하나인 H_∞ 이론이 가지는 단점인 제어기의 보수성을 극복하면서, 동시에 출력의 과도응답특성을 개선하기 위한 방법으로 H_2 이론을 병용하고 이를 LMI 이론으로 해석하였다. 이 과정에서 3 관성시스템에 LMI 이론을 적용하기 위한 일반화플랜트의 모형을 제시하고 이것의 유효성을, 모델의 불확실성과 parameter 변동을 동시에 고려한 simulation 을 통하여 확인하였다

1.Introduction

For a servo system design, the following three specifications are of practical interests: (1) internal stability of the closed-loop system which must be guaranteed; (2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation; (3) desired transient and steady-state properties such as robust tracking to reference inputs.

The H_∞ control is a suitable technique to achieve the first two specifications, because they can be naturally expressed as H_∞ norm constraints. However, since the H_∞ control is based on the maximum singular value of the transfer function matrix from disturbance to evaluation signals, it is inevitable that the response should be rather conservative. Therefore, it is required to alleviate this phenomenon in order to meet the third specification. Recently, it has been proved that, by introducing H_2 specification into the H_∞ design, we could simultaneously benefit from the H_2 and H_∞ control design [1]. This approach can be achieved by using the so- called LMI (Linear Matrix Inequalities) theory, and is generally called a mixed H_2/H_∞ control. In consequence, a designer can arbitrary determine the trade-off between H_2 (e.g. noise rejection) and H_∞ (e.g. robust stability) performance of the closed loop system.

2. Mixed H_2/H_∞ Optimal Design Problem by LMI

The basic block diagram used in this paper is given in Fig.1, in which the generalized plant $\widetilde{P}$ is given by the state-space equations

$$\widetilde{P}:\begin{cases}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1 w + \mathbf{B}_2 u \\ z_\infty = \mathbf{C}_\infty \mathbf{x} + d_{\infty 1} w + d_{\infty 2} u \\ z_2 = \mathbf{C}_2 \mathbf{x} + d_{21} w + d_{22} u \\ y = \mathbf{C}_y \mathbf{x} + d_{y1} w\end{cases} \qquad (1)$$

where $\mathbf{x} \in R^n$ is state vector, u is the control input, w is an exogenous input (such as a disturbance input, sensor noise etc.), y is the measured output and $\mathbf{z} = [z_\infty \; z_2]^T$ is a vector of output signal related to the performance of the control system (z_∞ is related to the H_∞ performance and z_2 is related to the H_2 performance).

Let T_{zw} be the closed transfer function from w to z for the system

$\tilde{P}$ closed by the output-feedback control law $u = K_r y$. Our goal is to compute a dynamical output feedback controller K_r

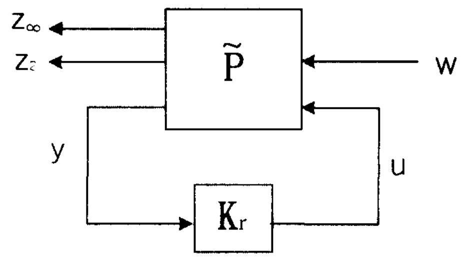

Fig.1 Block diagram of Mixed H_2/H_∞ control

$$K_r : \begin{cases} \dot{\mathbf{x}}_K = \mathbf{A}_K \mathbf{x}_K + \mathbf{B}_K y \\ u = \mathbf{C}_K \mathbf{x}_K + d_K y \end{cases} \quad (2)$$

that simultaneously meets H_2 and H_∞ performance on the closed-loop behavior.

The closed-loop system T_{zw} has the following description

$$T_{zw} : \begin{cases} \dot{\mathbf{x}}_{cl} = \mathbf{A}_{cl}\mathbf{x}_{cl} + \mathbf{B}_{cl} w \\ z_\infty = \mathbf{C}_{cl1}\mathbf{x}_{cl} + d_{cl1} w \\ z_2 = \mathbf{C}_{cl2}\mathbf{x}_{cl} + d_{cl2} w \end{cases} \quad (3)$$

The problem we concerned with can be summarized as minimizing the H_2 norm of the channel $w \rightarrow z_2(:T_2)$, while keeping the bound γ on the H_∞ norm of the channel $w \rightarrow z_\infty(:T_\infty)$. i.e.

$$\min \|T_2\|_2 \quad \text{subject to :} \quad \|T_\infty\|_\infty < \gamma$$

3. The Structure for LMI Design

In order to apply the mixed H_2/H_∞ control system to our system, we, first, divide the control objectives into each H_∞ and H_2 performance criterion, and then describe the two criteria as one formation. In other words, a new structure for the mixed H_2/H_∞ control is required to deal with two criteria simultaneously. Here, we introduce the following interconnection for robust control system, on which a controller satisfying two criteria is designed.

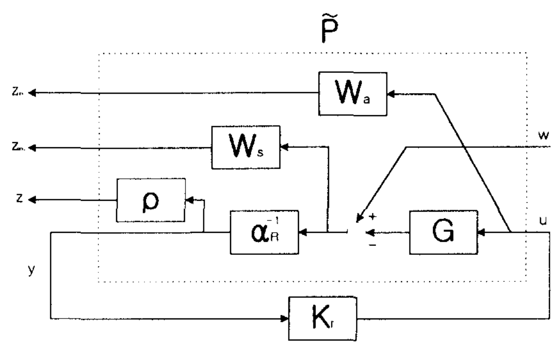

Fig. 2 The proposed generalized plant for Mixed H_2/H_∞ control

A. H_∞ *control Problem*: In Fig.2, $W_a(s)$ denotes a weighting function related to the plant uncertainty (this case, additive uncertainty) and $W_s(s)$ is a sensitivity weighting function. We can summarize the robust tracking H_∞ control problem as follows:

(S1) $K_r(s)$ stabilizes $\tilde{P}(s)$.

(S2) $\|T_\infty(s)\|_\infty = \left\| \begin{matrix} T_{z_{\infty1}w}(s) \\ T_{z_{\infty2}w}(s) \end{matrix} \right\|_\infty < \gamma$

where $T_{z_{\infty1}w}(s)$ denotes the transfer function from w to $z_{\infty1}$ and is related to robust stability requirement (for additive uncertainty)

$$\|T_{z_{\infty1}w}\|_\infty = \|(1+GK)^{-1}KW_a\|_\infty < \gamma \quad (4)$$

$$K = \alpha_R^{-1} K_r$$

By virtue of the Bound Real Lemma the H_∞ norm of $T_\infty(s)$ is smaller than γ if and only if there exists a symmetric positive definite matrix $\mathbf{X}_\infty$ with

$$\begin{pmatrix} \mathbf{A}_{cl}^T\mathbf{X}_\infty + \mathbf{X}_\infty\mathbf{A}_{cl} & \mathbf{X}_\infty\mathbf{B}_{cl} & \mathbf{C}_{cl\infty}^T \\ \mathbf{B}_{cl}^T\mathbf{X}_\infty & -\gamma & d_{cl\infty} \\ \mathbf{C}_{cl\infty} & d_{cl\infty} & -\gamma \end{pmatrix} < 0 \quad (5)$$

$$\mathbf{X}_\infty > 0 \quad (6)$$

where all the matrices $\mathbf{A}_{cl}, \mathbf{B}_{cl}, \mathbf{C}_{cl\infty}$ and $d_{cl\infty}$ are defined in (3).

B. *H_2 control Problem*: The traditional H_2 optimization attempts to minimize the energy of the system output when the system is faced with white Gaussian noise input. So, in order to design a controller adept at handling noises, H_2 optimization should be considered. That is, the H_2 norm minimization of the transfer function $T_{z_2w}(s)$ from w to z_2 in Fig.2 is to be taken as a controller design problem, where ρ is a varying parameter.

C. *Mixed H_2/H_∞ Control*: The mixed H_2/H_∞ controller K_r must satisfy both of the following criteria simultaneously [2]

$$\|T_{z_\infty w}\|_\infty < \gamma \quad (7)$$

$$\|T_{z_2w}\|_2 < \nu \quad (8)$$

4. Controller Design for three-mass Inertia System

We consider the problem given in [3]. The model treated in the problem is a coupled three-inertia system that reflects the dynamics of mechanical vibrations. A controller, by which robust performance (both in time and frequency-domain) condition must be satisfied, is required in order to solve the problem.

By using these parameters, the equations of motion can be described as

$$
\begin{aligned}
j_1\ddot{\theta}_1 &= -d_1\dot{\theta}_1 - k_a(\theta_1-\theta_2) - d_a(\dot{\theta}_1-\dot{\theta}_2) + \tau + \tau_{d_1} \\
j_2\ddot{\theta}_2 &= k_a(\theta_1-\theta_2) + d_a(\dot{\theta}_1-\dot{\theta}_2) - d_2\dot{\theta}_2 - f_b(\theta_2,\theta_3) - d_b(\dot{\theta}_2-\dot{\theta}_3) + \tau_{d_2} \\
j_3\ddot{\theta}_3 &= f_b(\theta_2,\theta_3) + d_b(\dot{\theta}_2-\dot{\theta}_3) - d_3\dot{\theta}_3 + \tau_{d_3} \\
f_b(\theta_2,\theta_3) &= k_b(\theta_2-\theta_3)
\end{aligned}
\tag{9}
$$

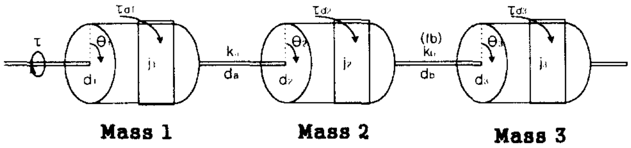

Fig.3 Coupled three-inertia system

We want to design a controller by which several design specifications are to be satisfied on condition that all of the 11 parameters are subject to change within the range of variation and there must exist hardware constraints.

4.1 Feedback Controller Design by Mixed H_2/H_∞ Control

In this problem, there are 11 physical parameters that are assumed to be changing within the given range of variation. If all the variations are reflected in the controller design, the obtained controller may be considerably conservative as well as complex. Therefore, we first find out principal parameters (j_3 and k_a) that strongly affect the resonant frequency of the plant by plotting the frequency response curve, and then a robust controller design dependent on these parameters is carried out by making use of the proposed structure for mixed H_2/H_∞ control.

The parameter variations of j_3 and k_a can be described by additive uncertainty such as

$$
\begin{aligned}
j_3 &= j_{3_o} + W_{j_3}\delta_{j_3} \\
k_a &= k_{a_o} + W_{k_a}\delta_{k_a}
\end{aligned}
\tag{10}
$$

where j_{3_o}, k_{a_o} are nominal values, W_{j_3}, W_{k_a} are constant values representing the range of variation, and

$$|\delta_{j_3}| \le 1,\ |\delta_{ka}| \le 1 \tag{11}$$

If we define the input and output of variations as

$$z_{\infty_1} = [z_{j_3} \quad z_{k_a}]^T,\ w_S = [w_{j_3} \quad w_{k_a}]^T \tag{12}$$

and then, parameter variation from w_S to z_{∞_1} can be described as a structured perturbation using

$$\Delta = \mathrm{diag}[\delta_{j_3} \quad \delta_{ka}] \tag{13}$$

The purpose of a controller is, if possible, to make $\mathbf{z}_{\infty_2} = [\theta_1-\theta_2 \quad \theta_2-\theta_3 \quad u]^T$ small in the presence of parameter variations and disturbance. This can be achieved by letting the closed-loop transfer function T_{∞_2} from $w_T = [\tau_{d_1} \quad \tau_{d_3}]^T$ to $\mathbf{z}_{\infty_2}$ have robust performance. Therefore, if it is possible to design a controller by which we keep the H_∞-norm of the closed-loop transfer function T_∞ from $\mathbf{w} = [w_{j_3} \quad w_{k_a} \quad \tau_{d_1} \quad \tau_{d_2}]^T$ to $\mathbf{z} = [z_{j_3} \quad z_{k_a} \quad \theta_1-\theta_2 \quad \theta_2-\theta_3 \quad u]^T$ low, the design specifications can be satisfied.

As a next step, by introducing H_2 specifications into the H_∞ design, we can allow to take noise transmission aspects into account. That is, a controller, by which H_2-norm of the transfer function T_2 from **w** (refer to Fig.4) to z_2 is to be minimized under the condition of H_∞ norm, can be designed by applying the mixed H_2/ H_∞ control to the structure shown in Fig.4.

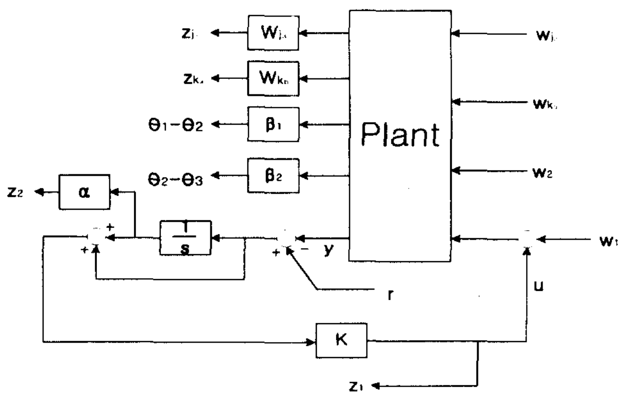

Fig. 4 Structure for Mixed H_2/H_∞ controller design

4.2 Simulation Results

The parameter values used in the simulation are classified as follows: (1) <u>nominal case</u>: an ideal case without parameter variations, (2) <u>Case1</u>: the moments of inertia have their minimum values and the torsional and viscous-friction coefficients are varied maximally within the range of variation, (3) <u>Case2</u>: the moments of inertia have their maximum values and the torsional and viscous-friction coefficients are varied minimally, (4) <u>Case3</u>: all parameters have their minimal value.

The values of W_{j_3}, W_{k_a} which express the magnitude of variations on j_3, k_a are given as 0.004 (20% variation) and 8 (10% variation), respectively. And constants α, β_1, β_2 are determined as 10, 0.08, 0.05, respectively through several trial and errors.

(1) Tracking ability (vibration suppression)

The reference tracking ability is shown in Fig. 5, where θ_3 and τ represent plant output and control input, respectively. From the results of simulation, we know that the design specification were sufficiently satisfied in the presence of the parameter variations and disturbance.

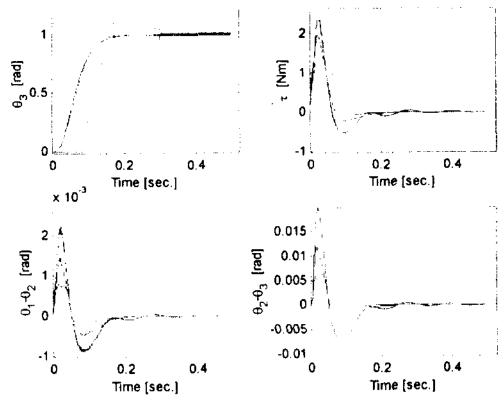

Fig. 5 Step Responses to the reference input

(2) Complementary Sensitivity Function

The gain plots of the complementary sensitivity function are shown in Fig.6. We know that, although the condition - the gain must be under 20[dB] over all the frequencies considered – is satisfied despite of variations, the other one – the gain must be under –20[dB] above 300[rad/sec] frequency – cannot be met in any cases. Actually, since it was already known that specification no. 4 and the others had a reciprocal relationship each other [3], it is impossible to meet all the specifications simultaneously.

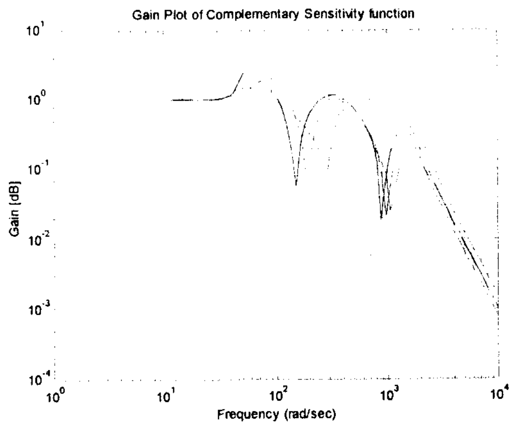

Fig.6 Gain plots of complementary sensitivity function

(3) Robust stability by μ-analysis

After closing the plant with the obtained controller to the plant, we take the additive uncertainty of the parameters j_3, k_a as input and output of the closed-loop system (see Fig.4). Then the system is robustly stable for all structured Δ(s) satisfying $\|\Delta\|_\infty < 1$ if and only if the interconnected system in Fig.7 is stable. This can be done by checking

$$\mu_m := \sup_{\omega} \mu_\Delta(\widetilde{P}(j\omega)) < 1 \tag{14}$$

That is, we can check robust stability for the perturbed system by evaluating (14). The μ-value is shown in Fig.7, when we let the class of model error as $\Delta \in \mathrm{diag}[C \quad C]$, where C denotes the set of complex numbers. Since the maximum μ-value is about 0.28 at ω=300[rad/sec], we can confirm that the robust stability is satisfied.

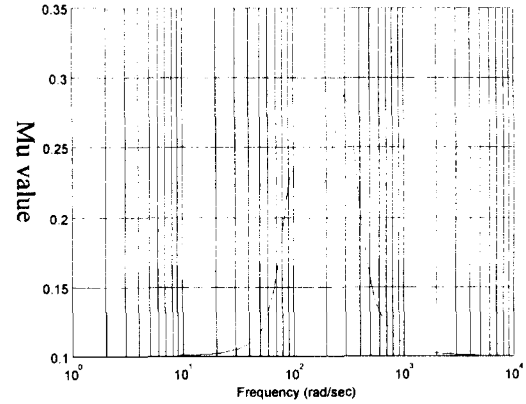

Fig. 7 Structured singular value

5. Conclusion

In this paper, we have proposed a generalized plant structure for applying LMI theory to control design to cope with some difficulties in the three-mass inertia system. And the effectiveness of the proposed structure was confirmed through simulations. For the purpose of designing a robust controller, the design objectives such as sufficient vibration suppression and robust tracking are first defined in terms of H_2 and H_∞ optimization theory, then the generalized plant for the mixed H_2/H_∞ control is determined and solved by using a LMI algorithm. Practical computation to get a controller is now quite easy thanks to some excellent software such as Matlab.

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