EQUIVALENT CIRCUIT REPRENSENTATION OF MAGNETIC FIELDS BASED ON FINITE ELEMENT MODELLING

Y. Kagawa, A. Kimura† and N. Wakatsuki

Department of Electronics and Information Systems, Akita Prefectural University,
Honjo Akita 015–0055, Japan
† Department of System Information Engineering, Hachinohe Institute of Technology,
Hachinohe 031–8501, Japan

ABSTRACT

A lumped electrical circuit is an approximate representation of the field within a curtain frequency range. The finite element modelling is a synonym of the equivalent circuit. The electric conduction field and electric potential wave field have been modelled by an admittance network and an LC low-pass filter network. Here in the present paper, the equivalent magnetic circuit representation is created for a magnetostatic field by the finite element modelling in two dimension.

1 INTRODUCTION

Electrical circuit theory and electromagnetic field theory have historically been developed independently. The electrical circuit is essentially a lumped model to the electromagnetic field, which is however only valid at low frequency range or at the longer wavelength than the size of the objective field. The finite element approach provides a numerical analysis method on the one hand and a discrete modelling of the spatially distributed field with respect to the nodes of the meshes or elements into which the field is divided on the other hand. The lumped element models have been examined for electrostatic, electroconductive and potential wave or acoustic fields[1][2]. The triangular and pyramidal element field are equivalently expressed by the lumped parameter components between the nodes.

In the present paper, the modelling has been extended into the magnetostatic field in two dimension, made for the magnetic potential. Extension is also made to the equivalent circuit expression with respect to the magnetic flux density.

2 MAGNETOSTATIC FIELD

We confine ourselves to the two-dimensional field as shown in Fig. 1. The functional is formed from energy functions as

$$\mathcal{L} = \frac{1}{2}\int_S \nu \left\{ \left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2 \right\} dxdy - \int_S JA\,dxdy \tag{1}$$

where $\nu(= 1/\mu$, μ is magnetic permittivity) is the magnetic resistivity, $A(= A_z)$ is a component of the magnetic vector potential $\boldsymbol{A}(= (A_x, A_y, A_z))$. $\boldsymbol{A}$ is defined as $\boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A}$, in which $\boldsymbol{B}$ is magnetic flux density. For the two-dimensional field

$$B_x = \frac{\partial A}{\partial y},\ B_y = -\frac{\partial A}{\partial x} \tag{2}$$

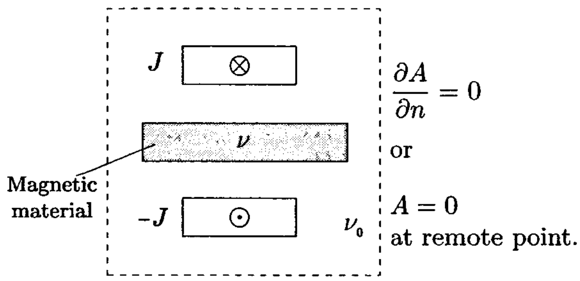

Fig. 1: Magnetostatic field in two-dimension

3 MAGNETOSTATIC FIELD IN TERMS OF MAGNETIC VECTOR POTENTIAL

Element

Fig. 2 shows an triangular element in which the potential at an arbitrary point is given as the interpolation of the nodal potentials $\{A\}_e(= \{A_1\,A_2\,A_3\})$. When an linear interpolation function is used, the potential is now expressed by

$$A(x,y) = \sum_{i=1}^{3} N_i A_i = \{N\}^T\{A\}_e \tag{3}$$

N_i is the interpolation function given as

$$N_i = \frac{1}{2\Delta_e}(b_i + c_i x + d_i y) \tag{4}$$

where

$$b_i = x_j y_k - x_k y_j,\ c_i = y_i - y_k,\ d_i = x_k - x_j \quad (5)$$

The subscripts permute in the order of i, j and k. Δ_e is the area of the triangular element.

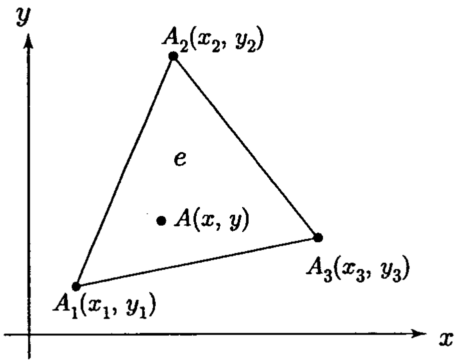

Fig. 2: Triangular element

In the similar manner, the current density is

$$J(x,y) = \sum_{i=1}^{3} N_i J_i = \{N\}^T\{I\}_e \quad (6)$$

The functional $\mathcal{L}_e$ for the element is now expressed discretized so that

$$\mathcal{L}_e = \frac{1}{2}\{A\}_e^T[S]_e\{A\}_e - \{A\}_e^T\{I\}_e \quad (1')$$

For the stationary condition $\delta\mathcal{L}_e = 0$, one obtains

$$[S]_e\{A\}_e = \{I\}_e \quad (7)$$

where

$$S_{ij} = S_{ji} = \frac{\nu}{4\Delta_e}(c_i c_j + d_i d_j) \quad (8)$$

It is easy to show that

$$S_{ii} = -\sum_{\substack{j=1 \\ (j\neq i)}}^{3} S_{ij} \quad (9)$$

and

$$I_i = \frac{1}{3}\Delta_e J \quad (10)$$

Equivalent Circuit

Fig. 3 shows in figure(a), the equivalent circuit expression of an element corresponding to the matrix equation(7), and in figure(b), the two connected elements. The matrix expression to figure(b) is

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 \\ S_{21} & S_{22}^{\mathrm{I}} + S_{22}^{\mathrm{II}} & S_{23}^{\mathrm{I}} + S_{23}^{\mathrm{II}} & S_{24} \\ S_{31} & S_{32}^{\mathrm{I}} + S_{32}^{\mathrm{II}} & S_{33}^{\mathrm{I}} + S_{33}^{\mathrm{II}} & S_{34} \\ 0 & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (11)$$

The circuit equation has the form

$$[S]\{A\} = [I] \quad (12)$$

in which the coefficient matrix is of admittance type, which makes thus the connection easy. The dimensions of the expression are $[\frac{\mathrm{Am}}{\mathrm{W_b}}]$, $[\frac{\mathrm{W_b}}{\mathrm{m}}]$ and [A] respectively.

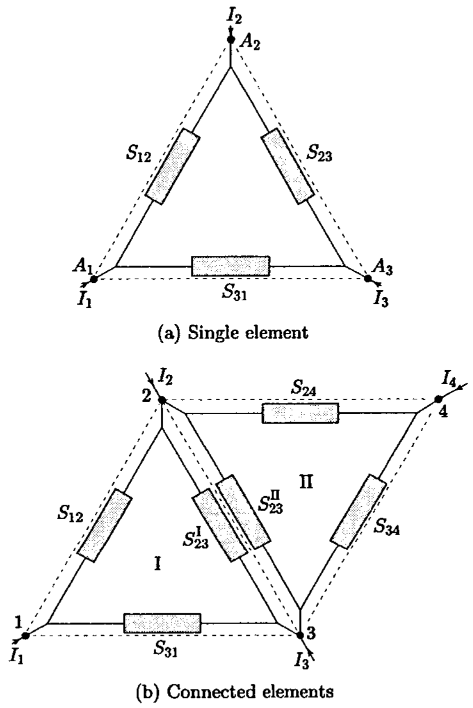

Fig. 3: Equivalent circuit

4 MAGNETOSTATIC FIELD IN TERMS OF MAGNETIC FLUX DENSITY

In usual finite element formulation of the magnetostatic field analysis, magnetic vector potential is taken as the dependent (unknown) variable, for which the field is solved. The flux density $\boldsymbol{B}$ is then obtained by differentiating the potential A (equation(2)). Z.H.Shaiki *et al* [4] presented an alternative approach with the magnetic flux densities as the unknown variables. They developed this approach as the better solution could be expected for the magnetic flux density solution. This can be the starting point to the alternative equivalent circuit expression.

We again confine ourselves to the two dimensional field. From Maxwell's equations, one has the magnetostatic equations as follows

$$\nu\left(\frac{\partial^2 B_x}{\partial x^2}+\frac{\partial^2 B_x}{\partial y^2}\right)+\frac{\partial J}{\partial y}=0 \qquad (13\text{-a})$$

$$\nu\left(\frac{\partial^2 B_y}{\partial x^2}+\frac{\partial^2 B_y}{\partial y^2}\right)-\frac{\partial J}{\partial x}=0 \qquad (13\text{-b})$$

It is interesting that the two equations are independent. The equations are discretized by means of the Galerkin method in which the triangular linear element as used in the previous section is used to give the expression of the form

$$[C]_e\{B_x\}_e=\{J_y\}_e \qquad (14\text{-a})$$

$$[C]_e\{B_y\}_e=-\{J_x\}_e \qquad (14\text{-b})$$

where the component of $[C]$ is

$$C_{ij}=C_{ji}=\frac{\nu}{4\Delta_e}(c_ic_j+d_id_j) \qquad (15)$$

C_{ij} is exactly the same as S_{ij} in equation(8). For the conductor region, however, the exciting terms should be replaced by the following expressions.

$$J_{yi}=\frac{1}{3}\Delta_e\frac{\partial J}{\partial y} \qquad (16\text{-a})$$

$$J_{xi}=\frac{1}{3}\Delta_e\frac{\partial J}{\partial x} \qquad (16\text{-b})$$

If J is constant across the conductor, $\partial J/\partial y=0$ within the conductor. This is not zero along the line non-parallel to the y axis between the conductor and the non-conductive regions.

For the triangular element 123 illustrated in Fig. 4, $\partial J/\partial y$ is not zero for the line $\overline{23}$. Therefore the exciting vector is given so that

$$\{J_y\}_e=\{\ 0\quad \tfrac{1}{2}|d_1|J\quad \tfrac{1}{2}|d_1|J\ \} \qquad (17\text{-a})$$

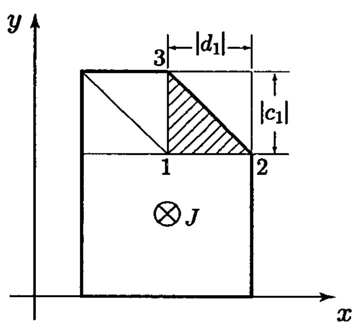

Fig. 4: Exciting current region

Similarly

$$\{J_x\}_e=\{\ 0\quad \tfrac{1}{2}|c_1|J\quad \tfrac{1}{2}|c_2|J\ \} \qquad (17\text{-b})$$

As the result, one has the two discretized expression corresponding to the field given by equations(13)

$$[S]\{B_y\}=\{J_y\} \qquad (18\text{-a})$$

$$[S]\{B_x\}=-\{J_x\} \qquad (18\text{-b})$$

The exciting terms on the right hand side have the non-zero terms only at the nodes corresponding to the nodes between the conductive and non-conductive regions. They could directly be obtained by differentiating the both sides of equation (7), with respect to dx or dy respectively, in which however the right hand side should carefully be treated. Equations(18) are two independent equivalent circuit similar to Fig. 3, in which A_i is replaced by B_{xi} or B_{yi} and I_i by J_{yi} or J_{xi}. They are again an admittance type expression. The dimensions of each term in equations(18) are respectively $\left[\frac{\mathrm{Am}}{\mathrm{W_b}}\right]$, $\left[\frac{\mathrm{W_b}}{\mathrm{m^2}}\right]$ and $\left[\frac{\mathrm{A}}{\mathrm{m}}\right]$.

5 MAGNETIC CIRCUIT

The equivalent magnetic circuit approach is frequently practiced by engineers for the device design, which has the form of

$$[R]\{\Phi\}=\{F\} \qquad (19)$$

$\Phi=BS$ is the magnetic flux, $F=NI$ is the magnetomotive force and $R=\frac{\nu L}{S}$ is the magnetic resistance, where S is the cross sectional area of the magnetic path, N is the number of turns of the coil wound around the magnetic path and I is the electric current of the coil wire, L is the length of the path. Equation(19) is an impedance type expression, and the dimensions are $\left[\frac{\mathrm{A}}{\mathrm{W_b}}\right]$, $[\mathrm{W_b}]$ and $[\mathrm{A}]$. The components of the resistance matrix in equation(19) is made of line segments. As the expression is of impedance type, it is not easy to evaluate the matrix components when two segments are connected in parallel, while the component evaluation is straightforward when two elements are connected in the finite element modelling, as the expressions(7) and (18) are of admittance type.

REFERENCES

[1] Y.Kagawa: Introduction to Finite Element Method for Electrical and Electronics Engineers, OHM-sha, Tokyo(1977)

[2] Y.Kagawa and T.Yamabuchi: Finite Element Equivalent Circuit for Acoustic Field, J.Acoust.Soc.Am **64**(4) pp1196–1200, Oct. 1978

[3] Y.Kagawa: Introductions to Magnetic Circuit Design, Applied Magnetism Technology Symposium Lecture Note **S–4** G4–1, pp1–8, April 15, 1993

[4] Z.H.Shaiki, H.Yamashita and E.Nakamae: A Novel Finite Element Analysis Method Using Magnetic Flex Density Directly as An Unknown Variable, RM–84–48, SA–84–19 Meeting on Electric Rotary and Stationary Machines, IEE, Japan, pp85–94, 1984

AUTHOR BIOGRAPHIES

Yukio Kagawa received the B.Eng., M.Eng., and D.Eng. degrees in 1958, 1960, and 1963, respectively, all from Tohoku University. Currently, he is Professor of Dept. of Electronics and Information Systems at Akita Prefectural Univ., Japan, and Professor Emeritus, Toyama and Okayama Universities. His interests include the application of numerical approach to the simulation of acoustic and electromagnetic fields and systems, in particular inverse problems. <Y.Kagawa@akita-pu.ac.jp>

Akio Kimura studied from 1993 to 1996 at the Graduate School of Natural Science and Technology, Okayama University. He is currently a Lecturer at the Hachinohe Institute of Technology. He has been working for the magnetic device optimization using Neural Network. <kimura@hi-tech.ac.jp>

Naoto Wakatsuki received the B.Eng. and M.Eng. degrees in 1993, 1995, respectively, all from Tsukuba University, Japan. Since 2001, he has been with the Dept. of Electronics and Information Systems at Akita Prefectural Univ., Japan. He is currently Research Asistant. His current interests includes the numerical simulation of piezoelectric devices. <wakatuki@akita-pu.ac.jp>