PROGRESSIVE ALGORITHM FOR RECONSTRUCTING A 3D STRUCTURE FROM A 2D SKETCH DRAWING

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ABSTRACT

This paper presents a progressive algorithm for reconstructing a 3D structure from a given 2D sketch drawing (edge-vertex graph without hidden line removal) according to the user's sketch order. While previous methods reconstruct a 3D structure at once, the proposed method progressively calculate a 3D structure by optimizing the coordinates of vertices of an object according to the sketch order. The progressive method reconstructs the most plausible 3D object quickly by applying 3D constraints that are derived from the relationship between the object and the sketch drawing in the optimization process. The progressive reconstruction algorithm is discussed, and examples from a working implementation are given.

1 INTRODUCTION

During the conceptual design stage of products, designers tend to draw their basic ideas of the mechanical parts mainly on papers with pencil. This tendency is natural because the interface with computer is not appropriate for designers to convey their basic ideas of products.

The method of representing three-dimensional information by using a line drawing is easy to input geometrical information. Once the 3D model is obtained, it can be manipulated/modified, and further detail can be sketched in to obtain more detailed and accurate object. The approach provides designers with the means to convey their ideas to a CAD system.

To reconstruct 3D geometrical information from a 2D sketch drawing, first, it is required to generate edge-vertex graph by analyzing strokes in the sketch drawing (Shpitalni and Lipson 1997). In addition, we should identify 2D actual faces (edge circuits) of a 3D object depicted as 2D edge-vertex graph (Oh and Kim 2001). Second, it is required to restore a 3D object, that is, the depth values of individual vertices of 3D object, by using the geometrical/topological relationship among 2D identified faces, edges and vertices (Lipson and Shpitalni 1996).

This paper discusses mainly about the restoration of a 3D object. Previous reconstruction methods reconstruct a 3D structure at once from the off-line sketch drawing by optimizing the objective function derived from image regularities of the sketch drawing. In addition, they require much time to process because they use the global relationship among all 2D entities.

In this paper, we describe an algorithm for reconstructing the most plausible object by optimizing the coordinates of vertices of an object face by face according to the sketch order. Figure 1 shows an overview of the progressive 3D reconstruction. First, the algorithm identifies 2D edge circuits that correspond to the actual faces of an object. And, it analyzes the optimal sketch order that serves as input for the system. Then, the proposed method refines the geometrical information of the rough partial object into that of the detailed complete object by adding faces to the partial object. In the optimization process, we restore a 3D structure quickly by applying 3D constraints that are derived from the relationship between the partial object and the sketch drawing. The algorithm reconstructs manifold or non-manifold object more quickly than previous ones from various sketch drawings.

2 RELATED WORKS

A sketch drawing lacks an entire dimension, therefore, additional methods must be found to extract the missing dimension. There are many methods on reconstruction from a single view, such as line labeling approach(Huffman 1971), gradient space approach(Mackworth 1973), linear system approach(Grimstead and Martin 1995), interactive construction approach(Fukui 1988), primitive identification approach(Peterson 1986), and minimum standard deviation approach(Braid and Wang 1991). Wang *et al.*(Wang and Grinstein 1993) and Lipson(Lipson 1988)

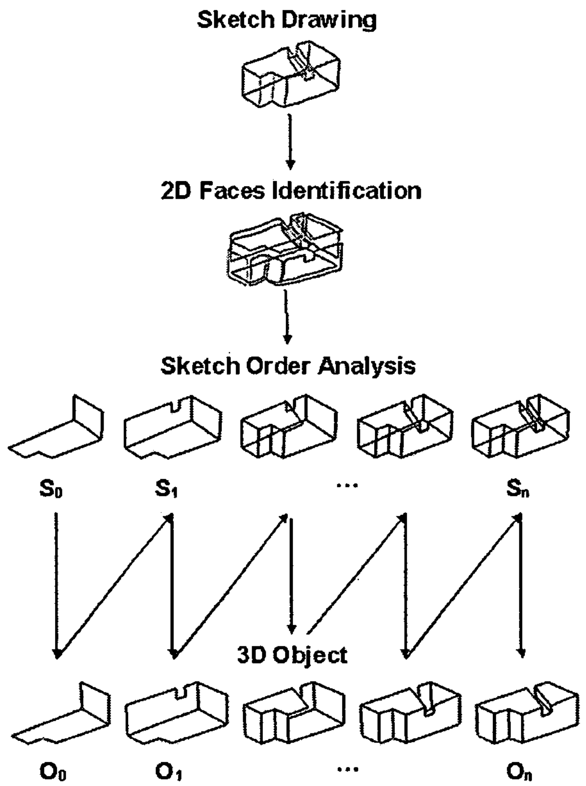


Figure 1: Overview of the progressive 3D reconstruction

surveyed detailed works on 3D object reconstruction from a single projection.

In this paper, our progressive reconstruction algorithm is based on the minimum standard deviation approach.

Marill(Marill 1991) suggested an optimization-based reconstruction for depth information of vertices using the minimum standard deviation of angles(MSDA). His method inflates the flat sketch into three-dimensional object by minimizing target objective function derived from MSDA at junctions. However, this technique yields implausible reconstruction for non-orthogonal models in experiments.

Braid and Wang(Braid and Wang 1991) refined Marill's approach by improving the optimization procedure with conjugate gradients.

Leclerc and Fiscler(Leclerc and Fiscler 1992) amended Marill's method by using face planarity, however, their method limited object types.

Lamb(Lamb and Bandopadhay 1990) implemented interactive reconstruction system based on line labeling. The algorithm allows user to avoid ambiguity, and it identifies principal axis and symmetry of face.

Lipson and Shpitalni (Lipson and Shpitalni 1996) reconstructed 3D object containing flat and cylindrical faces based on optimization method that formalizes various image regularities. Experiments we performed using their technique for various objects including manifold or non-manifold yielded plausible reconstruction.

3 PROBLEM DEFINITION

A sketch drawing is a single 2D projection of a 3D object that may be manifold or non-manifold. The goal of the sketch reconstruction is to restore the original 3D object with the information derived from the projection only.

3.1 Assumptions

1. **Sketch drawing** : The input projection represents a wireframe model of a general object, and it consists of a single 2D line drawing only which is given as a graph of connected entities.

2. **3D object** : It can be manifold or non-manifold depicted in the sketch drawing. However, no information is provided to the reconstruction system about the 3D object itself, its type or its position to the viewpoint.

3. **General viewpoint** : The projection is drawn from a general viewpoint that reveals all edges and vertices. That is, none of the edges or vertices of the object coincide accidentally, and none of them accidentally appear to be joined in the projection.

3.2 Requirements

1. **The most plausible object** : A 2D sketch might be a projection of an infinite number of possible 3D objects. The reconstruction must arrive at the most plausible 3D object described by a given projection, that is, the object that human observers are most likely to select.

2. **Sketch order analysis** : In the off-line sketch drawing, there are tremendous orders in drawing an object. The algorithm is required to analyze optimal sketch order to reconstruct the most plausible object.

4 RECONSTRUCTION PROCESS

4.1 Preprocessing

It is very important to identify edge circuits that correspond to the actual faces of a 3D object for reconstructing the most plausible 3D object. In the case of off-line sketch drawing that consists of edge-vertex graph, face identification problem is to select k faces among all plausible m faces by 2^m combinatorial searches. In this paper, we identify actual faces by applying maximum rank equation and face adjacent theorem (Shpitalni and Lipson) to reduce search space as shown in Figure 2.

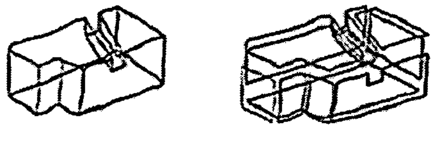
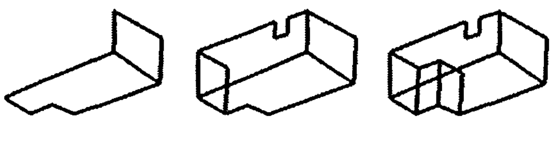
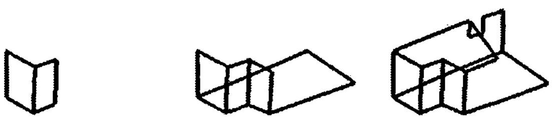


Figure 2: Off-line sketch drawing and face identification



(a) Normal sketch order



(b) Abnormal sketch order

Figure 3: Sketch order analysis

In this paper, we analyze the sketch order of faces to reconstruct a 3D object progressively from the off-line sketch drawing. There are tremendous sketch orders in drawing an object. If there are n actual faces of an object, there are about (actually smaller than) $n!$ sketch orders of sketching if we draw face by face. We analyze the initial sketch and the successive sketch order according to the following constraints.

1. **Initial sketch (S_0) analysis** : It is general for human beings to draw orthogonal adjacent faces of an object first. Also, human beings start to sketch with large faces. Here, we assume that the initial sketch drawing (S_0) contains at least two adjacent faces and 6 points as shown in leftmost of Figure 3.

2. **Successive sketch order ($S_i, i > 0$)analysis** : In general, human beings tend to draw adjacent faces of the previous sketch drawing. To get i^{th} sketch order (S_i), we select a face that is adjacent to the faces of $(i-1)^{th}$ sketch order (S_{i-1}) and that has high ratio of previously sketched edges as shown in Figure 3(a).

Our sketch order analysis is natural and coincides with the general order of drawing. In this respect, sketch order of Figure 3 (b) is too odd to produce the most plausible object depicted in the sketch drawing.

4.2 Initialization

First, we identify the prevailing axis system by analyzing the distribution of strokes by means of an angular distribution graph (ADG). ADG is constructed by sampling the angle of every entity in the sketch drawing. Then, we generate the objective function by weighting coefficients of various image regularities(Lipson and Shpitalni 1996). We optimize the objective function with respect to the Z coordinates of points $P(X, Y)$ in the initial sketch drawing (S_0) to reconstructed the initial object (O_0). We use Brent minimization algorithm(Brent 1973) to solve full n-dimensional nonlinear optimization problem. Where n is the number of points in the initial sketch drawing. ADG provides a good initial guess for most typical engineering parts exhibiting some degree of orthogonality.

Once the initial object has been reconstructed, we can define the projection matrix T that transforms 3D vertex v expressed in homogeneous coordinates into normalized homogeneous 2D point p on the sketch plane. In experiment, we find that the matrix T is good enough to establish relationship between the sketch drawing and an object although the sketch drawing is inaccurate. The matrix T is defined as Eq. 1(Rogers and Adams 1976).

$$\begin{pmatrix} p_u \\ p_v \\ 1 \end{pmatrix} = \begin{pmatrix} t_0 & t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 & t_7 \\ t_8 & t_9 & t_1 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \\ 1 \end{pmatrix} \quad (1)$$

We calculate the projection matrix as follows. Assume that there is a normalized 2D line $l(a, b, c)$ in sketch drawing. Then, we can define a 3D plane A that projects into the 2D line l by Eq. 2.

$$A = \begin{pmatrix} a & b & c \end{pmatrix} T = \begin{pmatrix} at_0 + bt_4 + ct_8 \\ at_1 + bt_5 + ct_9 \\ at_2 + bt_6 + ct_{10} \\ at_3 + bt_7 + c \end{pmatrix} \quad (2)$$

Let 2D lines $l_h(0, 1, -p_v)$ and $l_v(1, 0, -p_u)$ be orthogonal in 2D point $p(p_u, p_v)$. Then, we can define 3D plane A_h and A_v that project into 2D line respectively with Eq. 2. We can derive Eq. 3 from the plane A_h and vertex v because 3D vertex v should lie on the plane A_h. Similarly, we can derive Eq. 4 from the plane A_v and the vertex v.

$$A_h v = \begin{pmatrix} 0 & 1 & -p_v \end{pmatrix} T v = 0 \qquad (3)$$

$$A_v v = \begin{pmatrix} 0 & 1 & -p_u \end{pmatrix} T v = 0 \qquad (4)$$

Each 2D point with a known 3D position thus provides two linear equations. To solve 11 coefficients of i^{th} projection matrix T_i, the sketch drawing should have at least 6 points. We can solve the linear equations Eq. 3 and Eq. 4 by using singular value decomposition(W. H. Press and Flannery 1992).

4.3 Reconstruction

To estimate the positions for the inserted vertices progressively, we derive three constraints from the relationship between an object and the sketch drawing. In the reconstruction process, each vertex should satisfy three constraints. Three constraints are as follow:

1. **Vertex constraint**: By Eq. 1, the vertex constraint means that each vertex v of a 3D object should be transformed into the corresponding 2D point p of the sketch drawing by the projection matrix T (Figure 4). That is, each vertex v satisfies $Tv - p = 0$. Let there are n vertices of a 3D object. Then, the vertex constraint can be defined as Eq. 5.

$$F_{vertex} = \sum_{i=0}^{n} \{(Tv_i - p_u^i)^2 + (Tv_i - p_v^i)^2\} \qquad (5)$$

where $(p_u^i, p_v^i)$is the coordinates of the corresponding point p_i of i^{th} vertex v_i; T denotes the projection matrix extracted in the previous reconstruction step.

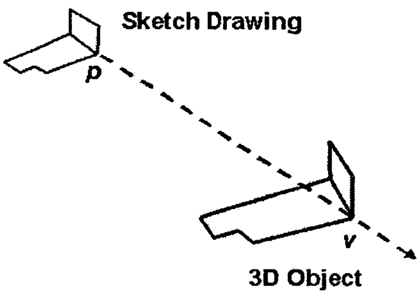


Figure 4: Vertex constraint

2. **Edge constraint** : Given a 2D line $l(\overline{p_1 p_2})$, we can generate a 3D plane A by Eq. 2. Then, the correspondence end-vertices of a 3D line $L(\overline{v_1 v_2})$ should lie on a plane A. We evaluate the edge constraint by summing the distance from each vertex to the plane. However, there are infinite plausible lines that satisfy the vertex constraint and the edge constraint as shown in Figure 5. Let there are n edges of a 3D object. Then, the edge constraint can be defined as Eq. 6.

$$F_{edge} = \frac{\sum_{i=0}^{n-1} \{(l_i T v_1)^2 + (l_i T v_2)^2\}}{n} \qquad (6)$$

where, v_1 and v_2 are two end-vertices of a 3D line L; l is the normalized corresponding 2D line; T denotes the projection matrix extracted in the previous reconstruction step.

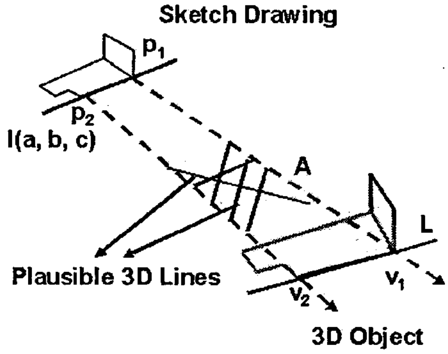


Figure 5: Edge constraint

3. **Face constraint** : The face constraint means that 3D vertex v should be lie on the 3D plane(face). With the constraints on vertex, edge and face, we can constraint the optimal 3D line (Figure 6) among the infinite plausible lines (Figure 5). Let there are n faces of a 3D object and each face have m vertices. The face constraint can be defined as Eq. 7.

$$F_{face} = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \{a_i x_i^j + b_i y_i^j + c_i z_i^j + d_i\}^2}{n} \qquad (7)$$

where, a_i, b_i, c_i and d_i are the coefficients of i^{th} plane equation and satisfy $\sqrt{a_i^2 + b_i^2 + c_i^2} = 1$; $(x_i^j, y_i^j, z_i^j)$are the coordinates of j^{th} vertex of i^{th} face.

To estimate the positions for the vertices progressively as shown in Figure 7(a), we generate $(i+1)^{th}$ partial object(O^{i+1}) by optimizing the objective function that is derived from the previously reconstructed partial object(O^i), projection matrix(T^i), and current sketch

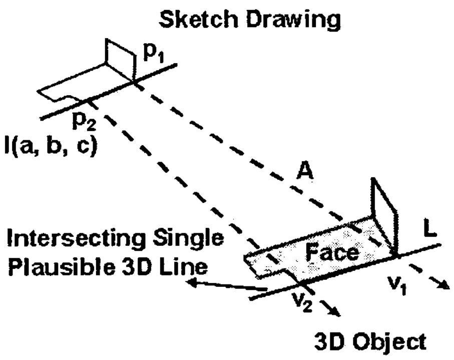
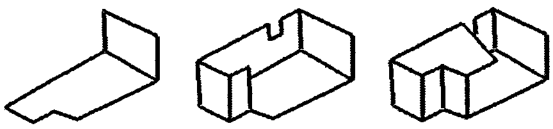
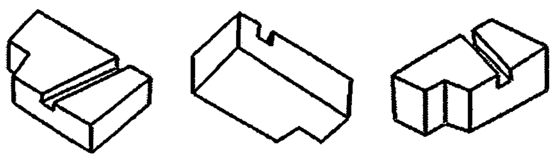


Figure 6: Face constraint

drawing(S^{i+1}). The objective function must be optimized with respect to the coordinates of vertices. In this paper, we define the objective function ($F_{progressive}$) as the sum of three 3D constraints as Eq. 8.



(a) Progressive reconstructed partial objects



(b) The final reconstructed complete object

Figure 7: Reconstruction results

$$F_{progressive} = F_{vertex} + F_{edge} + F_{face} \quad (8)$$

A vector V containing the x, y, z coordinates of the vertices can represent a 3D configuration of an object. An objective function $F_{progressive}(V)$ can be computed for any 3D configuration by evaluating above three constraints. The process of manipulating the V while seeking the best reconstruction is a full $3 \times n$-dimensional nonlinear optimization problem, where n is the number of vertices of current partial object. We use Brent minimization algorithm(Brent 1973). Because the problem requires much time in the minimization, in practice, we reconstruct a 3D structure of an object face by face each time.

Table 1: Evaluation of the progressive algorithm

Sketch Drawing	♯ of vert.	♯ of *edge*	♯ of *face*	Eval. $F_{progre.}$	Time (sec)
Fig. 8(a)	20	30	12	0.30161	7
Fig. 8(b)	28	39	11	1.41668	8
Fig. 8(c)	32	56	26	0.35640	21
Fig. 9(a)	48	72	26	0.72953	30
Fig. 9(b)	68	100	34	2.14005	58
Fig. 9(c)	124	186	64	0.51826	74

Although we divide the problem into small ones, the initial guess of solution is main issue of the nonlinear optimization problem. We use ADG as an initial guess for reconstructing an initial object. Consequently, we use the $(i-1)^{th}$ partial solution as an initial guess of the i^{th} solution. This initial guess is good enough to restore a 3D structure of an object efficiently as shown in Figure 7.

5 EXPERIMENTAL RESULTS

To estimate the efficiency of the proposed algorithm, we applied the method to various sketch drawings. The experiment is done on a PC with Pentium III processor (600 MHz).

We first apply the algorithm to the synthetic sketch drawing that is acquired by projecting the synthetic objects parallelly as shown in Figure 8(a). Our algorithm can be applied to an inaccurate sketch drawing (parallel freehand sketch drawing) as shown in Figure 8(b) as well as an accurate sketch drawing. In the case of perspective projection (Figure 8(c)), the algorithm generate somewhat distorted object because the initial object reconstruction used image regularities (assuming parallel projection). In experiments, we assume that the initial object is given.

We illustrate more complicated examples that are too complex to sketch in as shown in Figure 9. We use the projections of synthetic 3D object as the sketch drawings. The results are acceptable enough to convey the concept of products.

To determine the accuracy of reconstruction results, we show the reconstructed 3D objects with arbitrarily rotations with hidden-line removal or rendering. Normally, we can reconstruct 3D object as accurate as the sketch drawing itself. Table 1 shows the evaluation of the progressive algorithm for various projections and objects. The evaluation shows that the proposed algorithm reconstructs the most plausible objects from the sketch drawings quickly.

6 CONCLUSION

We have shown that the proposed algorithm effectively tackles the three-dimensional sketch reconstruction problem in a case where depth information is difficult to establish by restoring 3D structure progressively. The proposed algorithm reconstructs 3D object face by face according to the sketch order to improve the efficiency.

As a result, we have been able to develop an integrated and automated approach to reconstructing a complete mechanical part model from the sketch drawing. By using the progressive approach, we can generate a manifold/non-manifold object more quickly than other ones from various sketch drawings with orthographic, parallel or perspective projections.

Future works are as follows. First, we are interested in refining an inaccurate sketch drawing. Second, we will use quadric models to extend our approach to general mechanical part modeling. Finally, we will adapt the progressive approach to the on-line sketch drawing.

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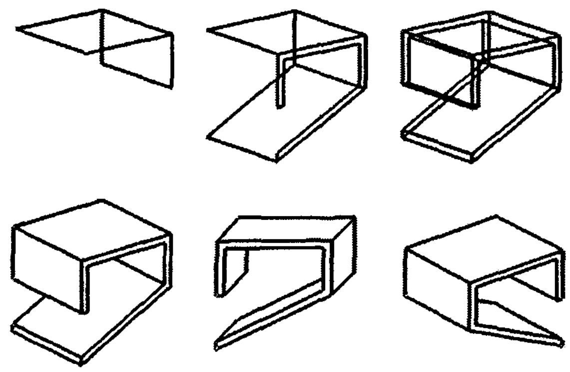
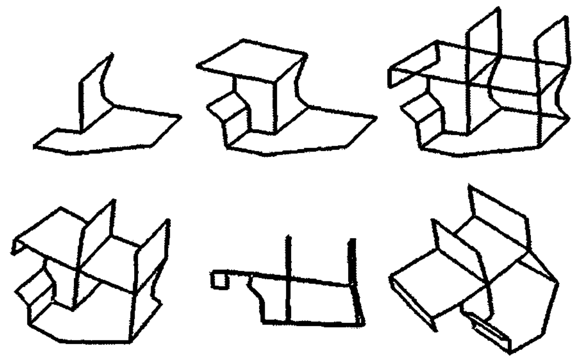
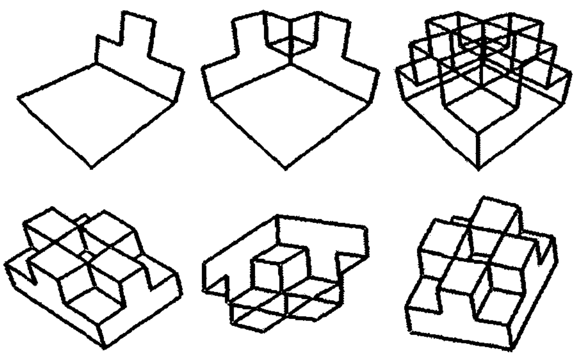


(a) Manifold object (orthogonal)

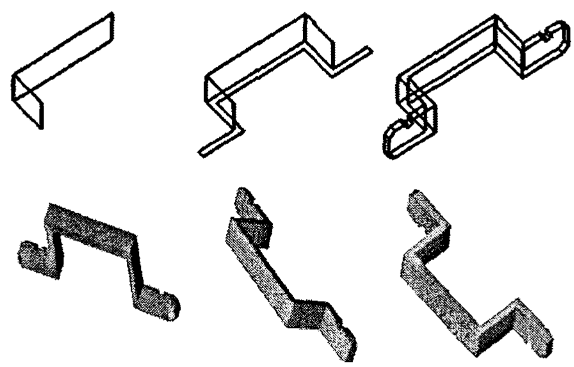


(b) Non-manifold object (parallel or freehand)

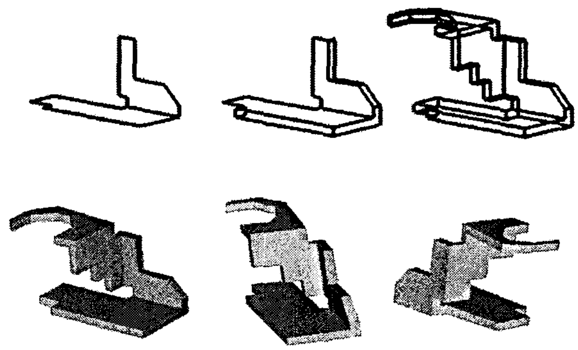


(c) Object with hole (perspective)

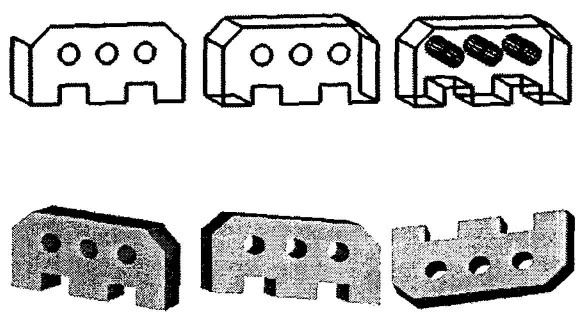
Figure 8: Optimal sketch orders and reconstructions from various projection types



(a) Sheet metal



(b) General mechanical part



(c) Product with hole

Figure 9: Optimal sketch orders and reconstructions of mechanical parts