CONCURRENT SIMULATION TECHNIQUE USING THE PROPORTIONAL RELATION FOR THRESHOLD-TYPE ADMISSION CONTROL

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ABSTRACT

In various applications of queueing systems, admission control is often employed. It is known that the threshold-type of admission control is optimal in many practical applications despite its simplicity. However, determining the optimal threshold value is hard in general, because analytical expressions for the stationary queue length distributions are not easily available in most queueing systems. In this paper, to quickly determine the optimal threshold value under threshold-type admission control, we develop a concurrent simulation technique, which can save large amount of CPU time required in simulation, compared to the standard simulation procedure.

1 INTRODUCTION

In various applications of queueing systems, admission control which decides whether arriving customers should be accepted in the queue or be rejected is often employed. One of the reasons necessitating admission control is that we usually have to ensure that certain acceptable performance levels are achieved [1]. Typically we wish to keep the average system time of customers as low as possible, while maintaining the acceptance rate of customers as much as possible. Controlling the incoming customer flow is an obvious way to accomplish this goal.

The most widely used admission policy is a simple threshold-type policy which admits all customers as long as the queue length is less than or equal to a threshold K^* and rejects them otherwise. It is known that despite its simplicity, this type of admission control is optimal in many practical applications including communication networks, computer systems and manufacturing systems [12]. The implementation of the threshold-type policy is simple and all it requires is a simple counter of customers. However, determining the optimal threshold K^* in a (finite) set $\mathcal{K} = \{K_0, K_1, \ldots, K_M\}$ is hard in general, since analytical expressions for the stationary queue length distributions are not easily available in most queueing systems. Thus, in many cases, we resort to simulation to determine the optimal threshold K^*. We then need to perform simulations for each queue under a threshold value K_i for $i = 0, \ldots, M$ to determine the optimal threshold K^*, and this requires considerably large amount of CPU time in general.

In this paper, to quickly find the optimal threshold value under the threshold-type admission control, we develop a concurrent simulation technique which can estimate the optimal threshold K^* from simulation only for the queue under the threshold value $K = \max\{K_0, K_1, \ldots, K_M\}$. It is thus expected that the utilization of the concurrent simulation technique enables us to save large amount of CPU time compared to the standard brute-force simulation procedure.

The most widely known concurrent estimation technique is a technique based on the sample path construction such as Augmented System Analysis (ASA) [3, 4, 5] and Time Warping Algorithm (TWA) [2] (also see [1] for reference). The technique constructs the corresponding sample paths which are associated with systems under threshold values $K_0, K_1, \ldots, K_M$ from an observation of a particular system under a threshold value K. As a result, from simulation run only for the system under a threshold value K, the technique can concurrently estimate performance measures in the systems under threshold values $K_0, K_1, \ldots, K_M$. The technique based on the sample path construction is, indeed, a powerful tool for concurrent simulation, but there exist some important classes of queueing systems where the technique is not easily applicable. For example, although queueing systems with feedback controlled and correlated input streams frequently arise in various applications such as communication systems, the technique is not applicable to the concurrent performance estimation of such queueing systems [9, 8]. Recently, to overcome the difficulty, Ishizaki et al. [9, 8] have proposed a technique which uses a *proportional relation* [7, 8, 9, 10, 11] holding between the stationary distributions of queueing systems under different buffer capacity (or threshold value). The basic idea that a proportional relation is used for concurrent estimation may date back to [6]. In [6], Gong and Gong have exhibited the idea and they have applied a technique using a proportional relation to the concurrent performance estimation of M/GI/1/K queues with respect to buffer capacity (or threshold value).

In this study, we consider a discrete-time single-server queueing system under the threshold-type admission control and we introduce a cost structure where rejecting a customer incurs a cost r and maintaining the system for all admitted customers incurs a cost b per unit time for each admitted customer in queue or in service. We then develop a concurrent simulation technique to find the optimal threshold K^* such that the expected cost per unit time minimizes. In particular, we develop a concurrent simulation technique using the proportional relation. To investigate the usefulness of the concurrent simulation technique, we also provide a simulation result. In the simulation result, we compare CPU time required by the concurrent simulation procedure developed in this study with that required by the standard simulation procedure.

The remainder of the paper is organized as follows. Section 2 describes a queueing model and cost function considered in this paper. Also, in Section 2, a problem to find the optimal threshold value is formulated for the queueing model and cost function. In Section 3, we exhibit a proportional relation holding between the stationary queue length distributions in the queueing systems with different threshold values. Using the proportional relation, we can concurrently estimate the expected costs in the queueing systems with different threshold values through simulation only for a particular queueing system. We consider estimators for the expected cost and provide the estimators for the concurrent and standard simulations in Section 4. To investigate the usefulness of the concurrent simulation technique, we provide a simulation result in Section 5. Conclusion is drawn in Section 6.

2 QUEUEING MODEL

In this section, we describe a queueing model and cost function considered in the paper. A problem to find the optimal threshold value under the threshold-type admission control is then formulated for the queueing model and cost function.

In this paper, we consider a queueing model studied in [11]. By setting $\{\delta_n\}$ in (2.1) (and (2.2)) appropriately, the queueing model can represent various discrete-time single-server queueing system such as queues with geometrically and interrupted service, queues with preemptive priority, queues with periodic service assignment and so on.

The queueing model is a discrete-time queueing system where time is divided into unit-time intervals called *slots*. The queueing system consists of a single server and a buffer, and threshold-type admission control is employed in the queueing system. The threshold value is denoted by K $(0 < K \leq \infty)$. The system is fed by two arrival streams: A controlled stream and uncontrolled one. The uncontrolled stream is described as a sequence $\{A_n\}_{n\in\mathbb{Z}}$ on $\mathbb{Z}_+$, where A_n denotes the batch size (the number of customers) arriving from the uncontrolled stream in the nth slot. On the other hand, the controlled stream is subject to a feedback control based on the queue length information, i.e., the batch size B_n $(\in \mathbb{Z}_+)$ arriving from the controlled stream in the nth slot is probabilistically determined by the queue length in the $(n-1)$st slot. The potential services of customers are governed by a 0-1 sequence $\{\delta_n\}_{n\in\mathbb{Z}}$, and the server is available in the nth slot if $\delta_n = 1$ and not available otherwise. Let X_n be the random variable on $\{0, \ldots, K\}$ representing the queue length (including one in service if any) in the nth slot. The system admits all customers as long as the queue length (including one in service if any) is less than or equal to the threshold K and rejects them otherwise. The dynamics of the stochastic sequence $\{X_n\}_{n\in\mathbb{Z}}$ is then represented by the following recursion:

$$X_{n+1} = \min\left[(X_n - \delta_n)^+ + A_{n+1} + B_{n+1}, K\right], \quad (2.1)$$

where $x^+ = \max(x, 0)$. From (2.1), the number of rejected customers in the $(n+1)$st slot is given by

$$Z_{n+1} = \left[(X_n - \delta_n)^+ + A_{n+1} + B_{n+1} - K\right]^+. \qquad (2.2)$$

Throughout the paper, we impose the following assumption:

Assumption 1

(i) $\{A_n\}_{n\in\mathbb{Z}}$ and $\{\delta_n\}_{n\in\mathbb{Z}}$ are jointly stationary.

(ii) Given the value of X_n, B_{n+1} is conditionally independent of all other random variables, and $\mathrm{P}(B_{n+1} = k \mid X_n = j)$ is invariant in $n \in \mathbb{Z}$ for all $j = 0, \ldots, K$ and $k \in \mathbb{Z}_+$. $\sum_{k\in\mathbb{Z}_+} \mathrm{P}(B_1 = k \mid X_0 = j) = 1$ for each $j = 0, \ldots, K$.

(iii) $\{X_n\}_{n\in\mathbb{Z}}$ is stationary and ergodic.

Next, we consider queueing systems which are identical to one considered so far in this section but their threshold values are $K_0, K_1, \ldots, K_M$, where without loss of generality, we assume that $0 < K_M < K_{M-1} < \cdots < K_1 < K_0 \le +\infty$. In what follows, we refer to the system with threshold value K_i as K_i-system for $i = 0, 1, \ldots, M$ and, to emphasize the threshold value, we put the superscript (K_i) on the quantities associated with the K_i-system.

We now make the following assumption, which is a key assumption for a proportional relation, which will be established in the next section.

Assumption 2

(i) The stationary sequences $\{(A_n^{(K_i)}, \delta_n^{(K_i)})\}_{n\in\mathbb{Z}}$ are stochastically identical for $i = 0, 1, \ldots, M$, and $\{(A_{n+1}^{(K_i)}, \delta_n^{(K_i)})\}_{n\in\mathbb{Z}}$ $(i = 0, 1, \ldots, M)$ is regenerative in the sense that, if $\{A_{n+1}^{(K_i)} = 0, \delta_n^{(K_i)} = 1\}$ occurs, then $\{(A_{l+1}^{(K_i)}, \delta_l^{(K_i)})\}_{l>n}$ is independent of $\{(A_{l+1}^{(K_i)}, \delta_l^{(K_i)})\}_{l\le n}$.

(ii) $\mathrm{P}(X_0^{(K_i)} = 0) > 0$ for $i = 0, 1, \ldots, M$.

Assumption 2(i) says that the lengths of periods during which the event $\{A_{n+1}^{(K_i)} = 0, \delta_n^{(K_i)} = 1\}$ continues are independent and geometrically distributed. In case where $\{A_n^{(K_i)}\}_{n\in\mathbb{Z}}$ and $\{\delta_n^{(K_i)}\}_{n\in\mathbb{Z}}$ are mutually independent, the uncontrolled stream regenerates when $A_n^{(K_i)} = 0$, which means that the lengths of off-periods in the uncontrolled stream (i.e. the periods of $\{A_n^{(K_i)} = 0\}$) are independent and geometrically distributed but it allows enough generality of the on-periods (i.e. the periods of $\{A_n^{(K_i)} = 1\}$). In our model, we can consider the case where the state space $\{0, \ldots, K_i\}$ is divided into some disjoint subsets and the stationary and ergodic queue length process satisfying Assumption 2(ii) is a.s. in one of such subsets and never enters to any other subsets. Assumption 2(ii) ensures that the queue length processes in K_i-systems $(i = 0, 1, \ldots, M)$ belong to a common subset.

Finally, we formulate a problem to find the optimal threshold value K^*. To establish a cost structure for the problem, we assume that rejecting a customer incurs a cost r and maintaining the system for all admitted customers incurs a cost b per unit time for each admitted customer in queue or in service. The expected cost per unit time in K_i-system is then expressed as

$$V^{(K_i)} = b\mathrm{E}[X_0^{(K_i)}] + r\mathrm{E}[Z_1^{(K_i)}]. \qquad (2.3)$$

Under this formulation, the problem is to find i $(i = 0, \ldots, M)$ such that the expected cost $V^{(K_i)}$ per unit time minimizes.

3 PRELIMINARY RESULT

In this section, we establish a proportional relation holding between the stationary queue length distributions in the queueing systems with different threshold values. The utilization of the proportional relation will enable us to concurrently estimate the expected costs in the queueing systems with different threshold values through simulation only for a particular queueing system. An estimator using the proportional relation will be provided in the next section.

The following theorem shows that a proportional relation holds between the stationary queue length distributions in the queueing systems with different threshold values. More precisely, the stationary queue length distribution in K_i-system $(i = 0, \ldots, M)$ is expressed as that in K_0-system multiplied by a constant and the constant is also expressed in terms of the stationary queue length distribution of K_0-system.

Theorem 1 Under Assumptions 1 and 2, for $i = 0, \ldots, M$ we have

$$\mathrm{P}(X_0^{(K_i)} = j) = c^{(K_i)}\mathrm{P}(X_0^{(K_0)} = j), \qquad j = 0, \ldots, K_i - 1,$$

and

$$\mathrm{P}(X_0^{(K_i)} = K_i) = 1 - c^{(K_i)}\mathrm{P}(X_0^{(K_0)} \le K_i - 1),$$

where $c^{(K_i)}$ is expressed as

$$c^{(K_i)} = 1/\mathrm{P}(X_0^{(K_0)} \le K_1 \mid \delta_0^{(K_0)} = 1, A_1^{(K_0)} = 0)$$

if $\mathrm{P}(\delta_0^{(K_0)} = 1, A_1^{(K_0)} = 0) > 0$ and otherwise $c^{(K_i)} = 1$.
Proof: See [11]. ∎

The following proposition is a direct conclusion of Theorem 1, and it is immediately obtained from the result shown in Section 4 in [11].

Proposition 1 Under Assumptions 1 and 2, for $i = 0, \dots, M$,

$$\begin{aligned} V^{(K_i)} &= b\Big[K_i - c^{(K_i)}\mathrm{E}[(K_i - X_0^{(K_0)})\mathbf{1}_{\{X_0^{(K_0)} \le K_i - 1\}}]\Big] \\ &\quad + r\Big[- \mathrm{E}[\delta_0^{(K_0)}] + c^{(K_i)}\mathrm{E}[\delta_0^{(K_0)}\mathbf{1}_{\{X_0^{(K_0)} = 0\}}] \\ &\quad + \mathrm{E}[A_1^{(K_0)}] \\ &\quad + [1 - c^{(K_i)}\mathrm{E}[\mathbf{1}_{\{X_0^{(K_0)} \le K_i - 1\}}]] \\ &\quad \cdot \mathrm{E}[B_1^{(K_0)} \mid X_0^{(K_0)} = K_1] \\ &\quad + c^{(K_i)}\mathrm{E}[B_1^{(K_0)}\mathbf{1}_{\{X_0^{(K_0)} \le K_i - 1\}}]\Big], \end{aligned} \tag{3.1}$$

where $\mathbf{1}$ denotes the index function and $c^{(K_i)}$ is given in Theorem 1.

Note that the expression (3.1) for the expected cost $V^{(K_i)}$ in K_i-system ($i = 1, \dots, M$) allows us to evaluate the expected cost in K_i-system from an observation of K_0-system. This makes it possible to concurrently estimate the expected cost in each K_i-system ($i = 0, 1, \dots, M$) through simulation for K_0-system (without simulation for each K_i-system). As a result, the concurrent simulation technique using (3.1) will save large amount of simulation time to find the optimal threshold value K^*, compared to the standard brute-force simulation procedure, which performs simulation for each K_i-system ($i = 0, 1, \dots, M$).

4 ESTIMATORS

In this section, we consider estimators for the expected cost $V^{(K_i)}$ per unit time in K_i-system. We hereafter assume that the quantities $\mathrm{E}[\delta_0^{(K_0)}]$, $\mathrm{E}[A_1^{(K_0)}]$ and $\mathrm{E}[B_1^{(K_0)} \mid X_0^{(K_0)} = K_1]$ are given. Also, we assume that $\delta_0^{(K_0)}$ and $\mathbf{1}_{\{X_0^{(K_0)} = 0\}}$ are independent.

First, for the concurrent simulation, we consider an estimator which can concurrently estimate $V^{(K_i)}$ for $i = 0, \dots, M$ from simulation only for K_0-system. Let $\hat{V}^{(K_i)}(N)$ ($i = 0, \dots, M$) denote the estimator for $V^{(K_i)}$ where N denotes the simulation time. The estimator $\hat{V}^{(K_i)}(N)$ for the concurrent simulation is easily obtained from (3.1) and it is given by

$$\begin{aligned} \hat{V}^{(K_i)}(N) &= b[K_i - \hat{c}^{(K_i)}(N)\hat{f}_0^{(K_i)}(N)] \\ &\quad + r\Big[\mathrm{E}[\delta_0^{(K_0)}](- 1 + \hat{c}^{(K_i)}(N)\hat{f}_1^{(K_i)}(N)]\Big) \\ &\quad + \mathrm{E}[A_1^{(K_0)}] \\ &\quad + [1 - \hat{c}^{(K_i)}(N)\hat{f}_2^{(K_i)}(N)] \\ &\quad \cdot \mathrm{E}[B_1^{(K_0)} \mid X_0^{(K_0)} = K_1] \\ &\quad + \hat{c}^{(K_i)}(N)\hat{f}_3^{(K_i)}(N)\Big], \end{aligned} \tag{4.1}$$

where $\hat{c}^{(K_i)}(N)$ is given by

$$\begin{aligned} \hat{c}^{(K_i)}(N) &= \left[\sum_{n=0}^{N-1} \mathbf{1}_{\{A_{n+1}^{(K_0)} = 0, \delta_n^{(K_0)} = 1\}}\right] \\ &\quad \cdot \left[\sum_{n=0}^{N-1} \mathbf{1}_{\{X_n^{(K_0)} \le K_i, A_{n+1}^{(K_0)} = 0, \delta_n^{(K_0)} = 1\}}\right]^{-1} \end{aligned}$$

if $\sum_{n=0}^{N-1} \mathbf{1}_{\{X_n^{(K_0)} \le K_1, A_{n+1}^{(K_0)} = 0, \delta_n^{(K_0)} = 1\}} > 0$ and otherwise $\hat{c}^{(K_i)}(N) = 1$, and $\hat{f}_0^{(K_i)}(N)$, $\hat{f}_1^{(K_i)}(N)$, $\hat{f}_2^{(K_i)}(N)$ and $\hat{f}_3^{(K_i)}(N)$ is given by

$$\hat{f}_0^{(K_i)}(N) = \frac{1}{N}\sum_{n=0}^{N-1}(K_i - X_n^{(K_0)})\mathbf{1}_{\{X_n^{(K_0)} \le K_i - 1\}},$$

$$\hat{f}_1^{(K_i)}(N) = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{1}_{\{X_n^{(K_0)} = 0\}},$$

$$\hat{f}_2^{(K_i)}(N) = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{1}_{\{X_n^{(K_0)} \le K_i - 1\}},$$

$$\hat{f}_3^{(K_i)}(N) = \frac{1}{N}\sum_{n=0}^{N-1}B_{n+1}^{(K_0)}\mathbf{1}_{\{X_n^{(K_0)} \le K_i - 1\}},$$

respectively.

Finally, we provide an estimator for the standard simulation procedure. Let $\tilde{V}^{(K_i)}(N)$ ($i = 0, \dots, M$) denote the estimator for $V^{(K_i)}$ where N denotes the simulation time. The estimator $\tilde{V}^{(K_i)}(N)$ is simply given by

$$\tilde{V}^{(K_i)}(N) = \frac{1}{N}\sum_{n=0}^{N-1} bX_n^{(K_i)} + rZ_n^{(K_i)}. \tag{4.2}$$

5 SIMULATION RESULT

In this section, we provide a simulation result to examine the usefulness of the concurrent simulation technique. In the simulation result, we compare the efficacy of the concurrent simulation technique with that of the standard simulation procedure in determining the optimal threshold value of the admission control.

The following queueing model is considered in the simulation result. The uncontrolled stream is generated by 20 uncontrolled sources and each source generates one customer with probability 0.01 in each slot. $A_n^{(K_0)}$ then has the binomial distribution:

$$\mathrm{P}(A_n^{(K_0)} = j) = \binom{20}{j}(0.01)^j(0.90)^{20-j}.$$

In the simulation result, for simplicity, we assume that there is no arrival from the controlled stream, i.e., $B_n^{(K_i)} = 0$ with probability one for $n = 1, 2, \ldots$. The service discipline is work-conserving, i.e., the server serves exactly one unit of work whenever there exists a customer in the system. The service time of a customer has the geometric distribution with mean 4. In this setting, $\delta_n^{(K_i)}$ is i.i.d. and $\mathrm{P}(\delta_n^{(K_i)} = 1)$ with probability 0.25 and $\mathrm{P}(\delta_n^{(K_i)} = 0)$ with probability 0.75. We set $b = 1.0$ and $r = 200$ in the cost function (2.3). Also, we set $M = 4$, $K_0 = 24$, $K_1 = 20$, $K_2 = 16$, $K_3 = 12$ and $K_4 = 8$. In the standard simulation, using the estimator given in (4.2), we perform simulation for the queueing systems with the threshold value of 24, 20, 16, 12 and 8. On the other hand, in the concurrent simulation, using the estimator given in (4.1), we perform simulation only for the queueing system with the threshold value of 24. In both simulations, we get one sample of $\hat{V}^{(K_i)}(N)$ and $\tilde{V}^{(K_i)}(N)$ during 4.0×10^4 busy cycles with initial condition $X_0^{(K_i)} = 0$. .

Table 1: Simulation results

K_i	standard	concurrent
8	$3.710 \pm 3.12 \times 10^{-3}$	$3.714 \pm 4.89 \times 10^{-3}$
12	$3.465 \pm 3.13 \times 10^{-3}$	$3.468 \pm 4.75 \times 10^{-3}$
16	$3.482 \pm 3.24 \times 10^{-3}$	$3.485 \pm 4.62 \times 10^{-3}$
20	$3.527 \pm 3.51 \times 10^{-3}$	$3.527 \pm 4.55 \times 10^{-3}$
24	$3.551 \pm 3.84 \times 10^{-3}$	$3.555 \pm 4.56 \times 10^{-3}$

Table 1 shows the estimations of $V^{(K_i)}$ $(i = 0, 1, 2, 3, 4)$ with 90% confidential interval, which are obtained through the standard and concurrent simulation procedures. 1000 samples are used in both estimations. From Table 1, we see that the optimal threshold value is 12 in this case. Also, we observe that under the condition that the number of the samples is fixed, the standard simulation procedure provides the estimates with lower variance than the concurrent simulation procedure. However, recall here that the standard simulation procedure will require longer *total* CPU time to get *one set* of the samples for queueing systems with the threshold value of 24, 20, 16, 12 and 8 than the concurrent simulation procedure, because the standard simulation procedure needs to perform simulation for the 5 queueing systems while the concurrent simulation procedure needs to perform simulation only for the queueing system with the threshold value of 24.

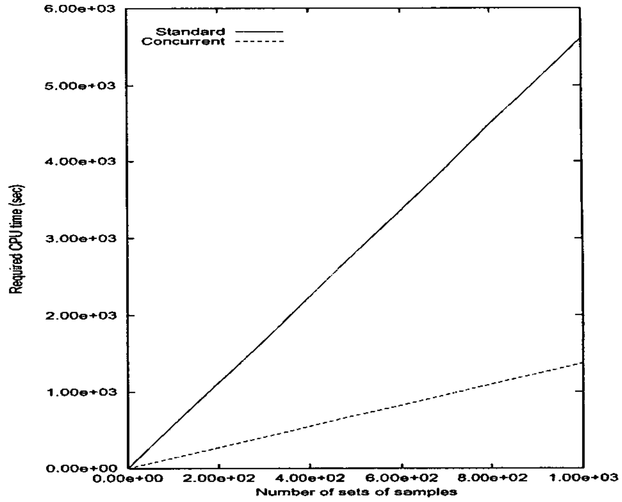

Figure 1: Comparison of total required CPU time

Next, we will compare the total required CPU times to get the set of the samples in both simulation simulation procedures. Fig. 1 depicts the total required CPU time in simulation as a function of the number of the sets of the samples. In Fig. 1, we observe that the CPU time which is required to get one set of the samples by the concurrent simulation is smaller than that by the standard simulation. In fact, the former is 1.378(sec) while the latter is 5.612(sec), and the latter is 4.07 times as large as the former (although 4.07 is slightly smaller than the value of $M + 1 = 5$).

Next, under the condition that the total required CPU

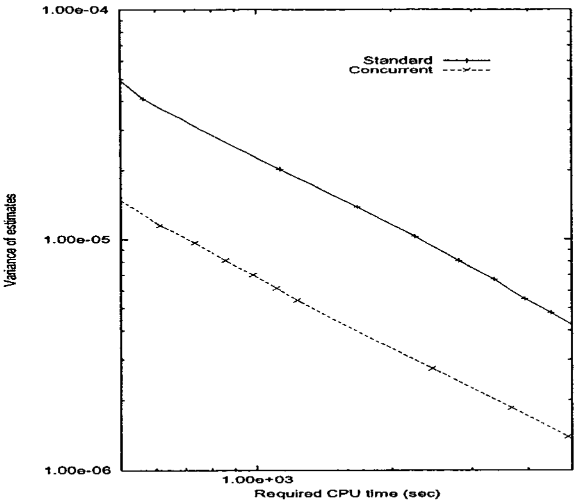

Figure 2: Variance of estimates for K_0-system

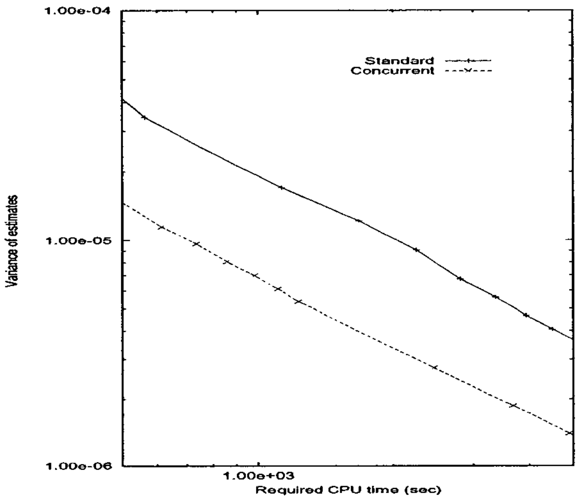

Figure 3: Variance of estimates for K_1-system

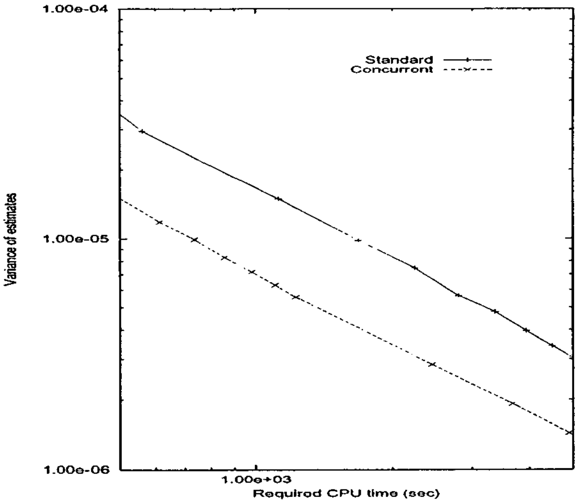

Figure 4: Variance of estimates for K_2-system

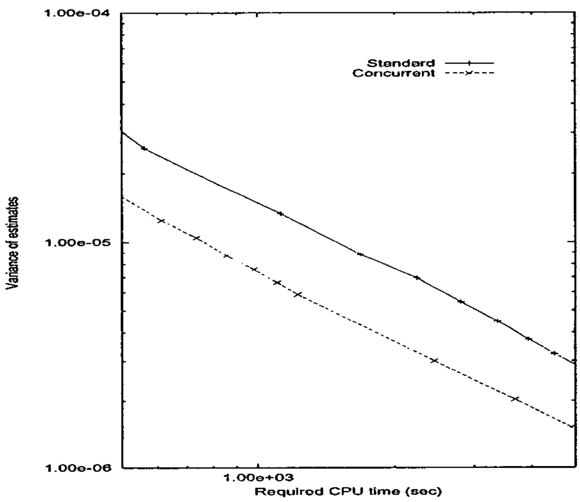

Figure 5: Variance of estimates for K_3-system

time is fixed, we will compare the variance of each estimate obtained through the concurrent simulation procedure with that obtained through the standard simulation procedure. Figs. 2, 3, 4, 5 and 6 display the variance of estimates for the queueing systems with the threshold value of 24, 20, 16, 12 and 8 as a function of the total required CPU time within a range from 500(sec) to 5000(sec). In these figures, we observe that under the condition that the total required CPU time is fixed, the concurrent simulation procedure provides estimates with lower variance than the standard simulation procedure. We then see that the utilization of the concurrent simulation technique enables us to save large amount of CPU time compared to the standard simulation procedure.

6 CONCLUSION

In this paper, to quickly determine the optimal threshold value under threshold-type admission control, we have developed a concurrent simulation technique which can find the optimal threshold K^* from simulation only for the queue under the threshold value $K = \max\{K_0, K_1, \ldots, K_M\}$. The concurrent simulation technique is based on the proportional relation holding between the stationary queue length distributions in queue-

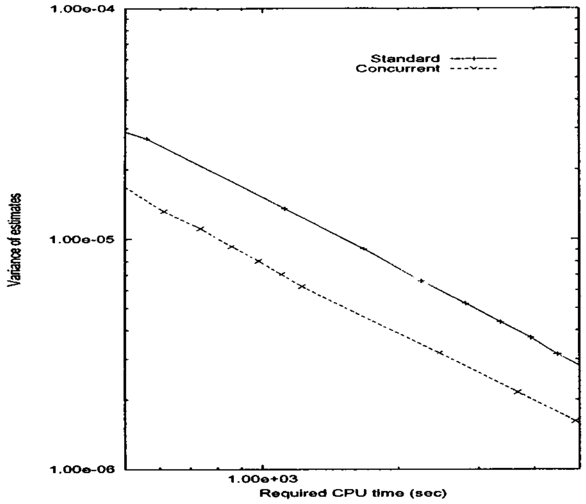

Figure 6: Variance of estimates for K_4-system

ing systems with different threshold values. The simulation result exhibits that in determining the optimal threshold value, the concurrent simulation technique can save large amount of CPU time compared to the standard simulation procedure. It is expected that the concurrent simulation technique is useful, especially, when the value of M is large.

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