Combined Discrete-Continuous Modeling for Supply Chain Simulation

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ABSTRACT

Many simulation models have been built to facilitate simulation technique in designing, evaluating, and optimizing supply chains. Simulation is preferred to deal with stochastic natures existing in the supply chain. Moreover simulation has a capability to find local optimum value within each component through entire supply chain. Most of supply chain simulation models have been developed on the basis of discrete-event simulation. Since supply chain systems are neither completely discrete nor continuous, the need of constructing a model with aspects of both discrete-event and continuous simulation is provoked, resulting in a combined discrete-continuous simulation. In this paper, an architecture of combined modeling for supply chain simulation is proposed, which includes the equation of continuous portion in supply chain and how these equations are used in the supply chain simulation models. A simple example of supply chain model dealing with the strategic level of supply chain presented in this paper shows the possibility and the prospect of this approach.

1. INTRODUCTION

Those who research and teach SCM (Supply Chain Management) agree with that the concept refers to a set of networked organizations working together to source, produce, and distribute products and services to the customer [6]. SCM is a new paradigm that maximizes enterprise profit by efficiently integrating and managing the flow of material and information through multiple stages of manufacturing, transportation, and distribution until it reaches the customer. That is to say, SCM can be called strategic planning to increase efficiency and earnings of organization by optimized speed and certainty and maximized value that is added up by each and every related process (refer to Figure 1).

< Figure 1 >

Enterprises have researched on how to do under SCM environment for satisfying customer's various needs. But there have not been good research result, on account of many uncertainty variables with stochastic properties in the supply chain and enormous supply chain model scale [10]. Existing analytical methods could not treat all variables with stochastic properties in the supply chain, therefore these methods are able to only present optimal values for partial supply chain. It is impossible to handle all the supply chain variables dynamically changed using analytic methods. Simulation is known as the best efficient method dealing with stochastic variables existing within the supply chain. And simulation is an effective analysis tool for dynamically changing internal supply chain variables. Moreover, simulation can work for the global optimization of planning entire supply chain with finding local optimum values within each component.

When a system is designed through simulation, simulation model is constructed based on the one of two methods. One is a discrete event simulation modeling method and the other is continuous simulation modeling method. Though product and information flow can have clearly continuous factors in supply chain, most supply chain problems are solved using discrete-event simulation modeling in the existing research. In this paper, the combined modeling method consisting of discrete and continuous events is proposed.

Ingalls discussed about why one would want to use simulation as the analysis methodology to evaluate supply chains, and its advantages and disadvantages against other analysis methodologies such as optimization, business scenarios, etc. [10]. Kritchanchai1, et al. reviewed the applicability of both approaches, specifically looking at discrete event simulation, an event-driven approach and System Dynamics, a time-driven approach [12]. Bernhard, et al. presented a taxonomy of research and development in System Dynamics modeling in supply chain management [2].

SCA(Supply Chain Analyzer) is a modeling and simulation software tool that can help companies make strategic business decisions for the design and operation of supply chain. It runs on a single platform and is built upon SIMPROCESS. This simulator runs discrete-event simulation using partial modules which are enriched operations of financial transaction and an optimization program developed at IBM Research called the Inventory Optimizer, and an optimization program, Supply Planning Optimizer that schedules replenishment orders or material shipping. But this simulator cannot be applied to distribution simulation and web-based simulation because of built on stand-alone platforms [1].

eSCA is the extended SCA as client/server structure based on internet environment. eSCA supported distribution simulation through synchronous working, and batch experiments, model catalog for reusable model and developed supply chain models. The architecture of eSCA consists of thin clients, web server and eServer. The salient features of eSCA are 1) client/server-based computing model for SCA, 2) a parallel and distributed simulation environment, 3) a knowledge-based model catalog [4].

CSCAT(Compaq Supply Chain Analysis Tool) is an ARENA discrete-event simulation that allows for the easy configuration of a supply chain and the analysis of dynamics of a supply chain. But these CSCAT is irrational general supply chain simulator because it was designed for Compaq's supply chain network and was used the module of Compaq's internal logic [9].

LOGSIM simulator is a simulation analysis tool for managing internal supply chain network of NOKIA. This simulator used simulation engine with PROMODEL and user interface as Visual Basic and report generator. However this simulator did not integrate process model and cost model into simulator [8].

2. SUPPLY CHAIN SIMULATION

The nature of supply chain integration is a systematic connection between the internal and external processes of company. Most strategic decisions for integrating supply chain are either to modify the supply chain's structure, or else to change the supply chain's policies. The following four steps are to help users to decide what techniques are appropriate for each type of decision (refer to Figure 2.) [7].

Step 1: Network Optimization

The objective of Step 1 is to arrive at an overall network structure that is efficient, meets all current demands, minimizes structurally cost based issues, and supports any other management constraints. To do this, a linear-mixed integer programming model is formulated and solved.

Step 2: Network Simulation

While Step 1 will produce an 'optimal' supply chain structure, it really ignores the issues of how the network will actually behave over time. As a result, it tells you what supply chain network design to select, but it cannot tell you exactly what will happen when that design is actually implemented. In order to predict exactly how a proposed supply chain design will operate, the design must be simulated in Step 2.

Step 3: Analysis & Decision Policy

Policy optimization is a cutting edge technology that can produce extremely valuable recommendations for policy improvement and better supply chain designs. However, along with this power, it has several weaknesses that must be watched with care. First, simulation-optimization requires a complete model with consistent and accurate input data and costs. Second, because each simulation run can take several minutes to complete, a full simulation optimization will take a lot of time. This combines with the third problem, that realistic policy optimizations can result in hundreds or even thousands of decision variables, which must be manipulated and optimized.

Step 4: Design for Robustness

The objective of this step is to ensure that the final selection of the supply chain's network structure and policies will operate well under a wide variety of situations.

< Figure 2 >

Figure 3 illustrates the basis process flow that is useful for supply chain simulation. This procedure first reads all data required by the graphic user interface. This includes products, market, sales data, and detailed data on operation of each facility in the supply chain. And then, customer demand is calculated through forecasting method based on historical data. After that, optimization modules (supplier selection, location, inventory, transportation etc.) are running with the configuring and planning parameter in the database. Through the stages as mentioned above, a supply chain simulation process is basically defined.

< Figure 3 >

3. COMBINED SUPPLY CHAIN SIMULATION MODELING

Required activities of enterprises for supply chain management can be classified into three levels, operation level, tactical level, strategic level as follows [9]:

1) Operation level: This is typically a short time horizon with a limited scope, possibly one plant, at part number level. Resources and demands are fixed or known. Variance, though critical, can usually be dealt with an exception. Traditionally, mathematical optimization method is used in this level such as LP(Linear Programming), IP(Integer Programming), MIP(Mixed Integer Programming).

2) Tactical level: In this level, time horizons are longer, perhaps up to several month in length. Range of resource is expanded from a machine to a factory. Interested field is which products are manufactured, or which factory produces the products, or which supplier is selected, etc. In this level, the demand forecast can be simply a guess. If the demand forecast is a guess with stochastic characteristics, simulation is the best choice.

3) Strategic level: Time horizons are even longer, up to several years in length at this level. The strategic plan is developed at an aggregate level, perhaps at the level of product divisions or product families. Basically, there are no fixed resources. Manufacturing sites can be opened or closed, any capital can be procured, product deployments are completely opened, etc. Making decision is difficult because customer demands are very uncertain. Therefore, stochastic modeling and simulation are needed to build strategy plan.

Discrete-event simulation concerns the modeling of a system as it evolves over time by a representation, n which the state variables change instantaneously at separate points in time. However, discrete-event simulation modeling in supply chain model methodology gives supply chain analyst difficulties as follows:

① Do not reflect the continuous nature of the process being modeled.
② Do not represent the interaction that may occur among those components.
③ Having complication for more detailed model.
④ Having too simplicity for small scaled model.

Combined model is defined as an integrated model to incorporate both discrete and continuous variables within the same model. To model such systems, the system must represent both the discrete and continuous components, as well as the interactions that may occur among those components. Combined modeling gives supply chain analyst conveniences as follows [3]:

① Explaining the dynamic behavior of an existing system.
② Confirming validity of proposed system modification.
③ Predicting new types of system behavior.
④ Benchmarking competitive improvement strategies.
⑤ Checking out of novel adaptive control systems.
⑥ Convenient to approximate a discretely changing variable by using continuously changing variables.

Continuous models are based on a defined relationship for the state of the system over time. If the state of the system are denoted by variable x, then objective is to find a function f such that

$x = f(t, \lambda, x_0)$
where
t : denote time
λ : denote parameters of the model

This relationship is known as a state equation. In most case, the complex system's components aren't enough insight to directly develop state equations for system. However a relationship can be developed for the rate of change of x with respect to time, called the derivative of x.

In general, a continuous or combined model contains one or more state equations to describe the system state. If the model contains all state equations, then system state is known for all time in terms of the model's initial conditions and parameter [15]. Continuous model is mainly applied to chemical industry. The state of each process may be presented as the state function over time, and the change rate of state is calculated as differentiating the state function.

Modeling methodology is applied to designing supply chain, varied with levels for supply chain management. In operation level, optimal value is given by the analytical optimization method. It can be said that using discrete event simulation is appropriate choice for the tactical level [10]. Large size simulation model, based on the entire supply chain, is needed for enterprise plan in strategic level. If this simulation model is designed to use the discrete event modeling, large input data is required and the continuous nature of supply chain cannot be reflected. These facts have negative influence on supply chain modeling. However, if combined modeling approach is used, continuous nature of supply chain can be properly considered in designing supply chain.

The actual model of supply chain is large scale and requires large quantities of input data. But, if combined modeling approach is used, it can represent continuous features, interactions, flow rate of product and information in the model simply or more precisely.

< Table 1 >

Classifying elements into two groups (discrete or continuous) depends on the attributes of element. This proposition is applied to classifying supply chain component (Supplier, Factory, Distributor, Retailer, Customer, etc.) and connection between components. Information about customer order, information flow in each supply chain component, inventory level in distributor, each factory for product etc. are considered as continuous elements. Transportation is considered as discrete element in this paper (Refer to Figure 4).

< Figure 4 >

Two interactions are occurred between discrete and continuous elements in combined model. The first is the changes made to the continuous component from the discrete component. This type interactions superimposes a discrete jump on a continuous-change-response variable. The second type of interaction consists of changes made to the discrete component from the continuous component. To model interactions involving changes to continuous variable from the discrete component, changing value of the continuous variable from the discrete component in the discrete model is conducted by assigning a value to the state or derivative variable [15].

4. STRUCTURE OF SUPPLY CHAIN SIMULATION MODEL

We will concentrate on material flow from supplier to consumer, and on order information flow from consumer toward suppliers. The considered structure of supply chain is shown in Figure 5. The purpose of Figure 5 is not to exhibit a complete representation of all functions, but rather to describe fundamental activities such as shipping delay, information delay between supply chain components. These activities are controlled and constrained by the management policies and structure of supply chain.

< Figure 5 >

The four components - retailer, distributor, factory, supplier - are treated in the similar way and their detailed flows are shown in Figure 6 - 9. Equations are developed to represent the relationships between supply chain components. Levels are represented as variables, which measure associated quantities in equations. Theses levels are counted by running simulation model. For example, in the retailer section, levels are central to the distribution and sales task, and exist in the orders, information, and material flow channels. The levels appear to be

- Backlog of orders received from the consumer but not yet filled

· Inventory of goods in stock.

The major rates of flow that appear to be pertinent to the objectives will be as follows:

· Outgoing shipping rate to customer · Outgoing order rate from retailer to distributor · Outgoing order rate from distributor to factory · Outgoing order rate from factory to supplier · Incoming shipping rate of goods from supplier to factory · Incoming shipping rate of goods from factory to distributor · Incoming shipping rate of goods from distributor to retailer	· Inventory level at retailer section · Inventory level at distribution section · Inventory level at factory section · Inventory level at supplier section

- **Retailer section**

< Figure 6 >

Definition
index
j : former time
k: present time, k = j+1
Given value
I D R : desired inventory level at retailer (volume)
Parameter
UOR : unfilled order at retailer (rate)
I A R : actual inventory level at retailer
ORD : order quantity, from Retailer to Distributor

There are unfilled order rate at a retailer at a point of time k such as

$$UOR_k = 1 - \left(\frac{\text{shipped quantity up to time k}}{\text{order quantity up to time k}} \right) = 1 - \left(\frac{SQR_j + SQR_{j+1}}{OQR_j + OQR_{j+1}} \right)$$

and inventory level at retailer at 0time k as

$$IAR_k = \frac{\text{shipped quantity up to time k}}{\text{received quantity up to time k}} = \frac{SQR_j + SQR_{j+1}}{RQR_j + ROQ_{j+1}}$$

Order quantity from Retailer to Distributor is reflected by inventory optimization module. A numerical formula is as follows:

$ORD_k = IDR_k - \{IAR_k - (1 - UOR_k)\}$, where IAR_k, UOR_k are in volume.

that is $ORD_k = IDR_k - RQR_j - RQR_{j+1} + OQR_j + OQR_{j+1}$

- **Distribution section**

< Figure 7 >

Given value
I D D : desired inventory level at distributor (volume)
Decision variables
UOD: unfilled order at retailer (rate)
I A D : actual inventory level at retailer
ODF: order quantity from distributor to factory

UOD_k, IAD_k, ODF_k can be obtained by the same methods in retailer section.

- **Factory section**

< Figure 8 >

Given value
I D F : desired inventory level at factory (volume)
Constant
LT : manufacturing lead time
MC: manufacturing capacity
Parameter
UOF: unfilled order at factory (rate)
I A F : actual inventory level at factory
OFS: order quantity from factory to supplier
MQF: manufacturing quantity at factory
MDT: manufacturing delay time

UOF_k, IAF_k can be obtained by the same methods in retailer section. It is decided as follows:

$$MQF_k = \begin{bmatrix} MQF_k & \text{if } MQF_k \leq MC \\ MC & \text{if } MQF_k > MC \end{bmatrix} \quad \textit{MC is manufacturing capacity}$$

Manufacturing quantity is reflected by inventory optimization module. A numerical formula is as follows:

$$MQF_k = IDF_k - \{IAF_k - (1 - UOF_k)\} \quad \text{where } IAF_k, UOF_k \text{ is in volume}$$
$$\text{and } MQF_k = OFS_k.$$

It is replenished at present quantity with the difference in quantity, if capacity at the previous time is less than production quantity. Therefore MQF_k is decided as follows:

$$MQF_k = \begin{bmatrix} IDF_k - RQM_j - RQM_{j+1} + OQF_j + OQF_{j+1} & \text{if } MQF_j \le MC \text{ at the former time} \\ MC & \text{if } MQF_j > MC \text{ at the former time} \end{bmatrix}$$

A formula for MDT_k is as follows:

$$MDT_k = LT(\frac{ROM_j + ROM_{j+1}}{MQF_j + MQF_{j+1}})$$

- **Supplier section**

< Figure 9 >

Given value
I D F : desired inventory level at factory (volume)
Constant
LT : manufacturing lead time
MC: manufacturing capacity
Parameter
UOS: unfilled order at supplier (rate)
I A S : actual inventory level at supplier
MQS: manufacturing quantity at supplier
MDT: Manufacturing delay time

UOS_k, IAS_k, MQS_k can be obtained by the same methods in retailer section. MQS_k is decided as follows:

$$MQS_k = \begin{bmatrix} IDS_k - RQS_j - RQS_{j+1} + OQS_j + OQS_{j+1} & \text{if } MQS_j \le MC \text{ at the former time} \\ MC & \text{if } MQS_j > MC \text{ at the former time} \end{bmatrix}$$

A formula for MDT_k is as follows:

$$MDT_k = LT(\frac{ROS_j + ROS_{j+1}}{MQS_j + MQS_{j+1}})$$

5.EXPERIMENT

The purpose of experiment is to show the benefits of combined modeling, based on above formulas, respect to discrete-event modeling in supply chain simulation.

In experimental model, retailer section is similar to distribution section. Also, factory section is similar to supplier section. In processing experiment, order quantity is firstly generated from customer. Then fill rate and new order quantity to distributor are calculated in retailer section. For example, if inventory level is 500, order quantity from customer is 300, and desired inventory level is set as 600, then the fill rate for customer will be 100 %, and order quantity for distribution section will be 400(=600-(500-300)). And then, this quantity will be sending to distributor section. In distribution section, the similar process is repeated with 400 as order quantity from retailer section.

In factory section, two major processes about parameter are.

First, the process about manufacturing order quantity is. Manufacturing order quantity is decided considering the capacity of factory and the received quantity from distributor. If the manufacturing order quantity is larger than the capacity of factory, factory can produce products as much as its capacity. Otherwise, factory will produce products as much as manufacturing order quantity.

Second, the process about order quantity for supplier is. Order quantity for supplier is calculated considering the ratio about the final product and their parts. In general, the ratio or the relation between the product and their parts is shown in BOM (Bill Of Material). If the quantity of order from distributor is 300, the present inventory level of factory is 500, the desired inventory level of factory is 600, factory capacity is 700, and the products to their parts ratio is 1 to 3, then manufacturing order quantity will be 400(=600-(500-300)). This quantity is less than factory capacity. Therefore, the quantity of 400 should be produced. Also order quantity for supplier will be 1200(3*400 = 1200). The same process is applied to supplier section.

In experimental model, the mainly input parameters are set as following Table 2.

<Table 2>

Figure10 is the plot diagram about the inventory level using combined model approach. It reveals that all inventory level can be reflected continuous property in supply chain.

<Figure 10>

In this experiment, discrete-event model is developed to compare the inventory level given by combined modeling approach. Simulation results for each approach are as followed:

<Table 3>

Average inventory level for each approach is shown in Table4.

<Table 4>

Comparison of inventory level for each approach is shown in Figure 11.

<Figure 11>

In Figure 11, the inventory level, given by discrete-event modeling approach, is bigger than the inventory level, given by combined modeling approach. In other words, the simulation results of discrete-event modeling approach are overestimated than that of combined modeling approach. This is mainly due to the difference between the discrete and continuous characteristic. These overestimated results are interpreted as unnecessary inventory. Unnecessary inventory is a major barrier to disturb the supply chain. Therefore, combined modeling approach, can more accurately measure the inventory level, is effective in supply chain simulation.

6. CONCLUSION

Most of supply chain simulation models have been developed using discrete-event simulation. Since supply chain systems are neither completely discrete nor continuous, the need of constructing a model with aspects of both discrete-event and continuous simulation is provoked.

In this paper, an architecture of combined modeling for supply chain simulation is proposed, which includes the equation of continuous portion in supply chain and how these equations are used in the supply chain simulation models. And, the simple experiment is done to show the benefits of combined modeling, respect to discrete-event modeling, in supply chain simulation.

For further research, developing the process for generating graphical output data such that decision maker can see how the supply chain acts over time during simulation. Besides, it is necessary to develop modeling process in more detailed way for the specific industry. Combined simulation modeling helps manages to observe the supply chain more macroscopically and to save time in the execution of supply chain simulation model.

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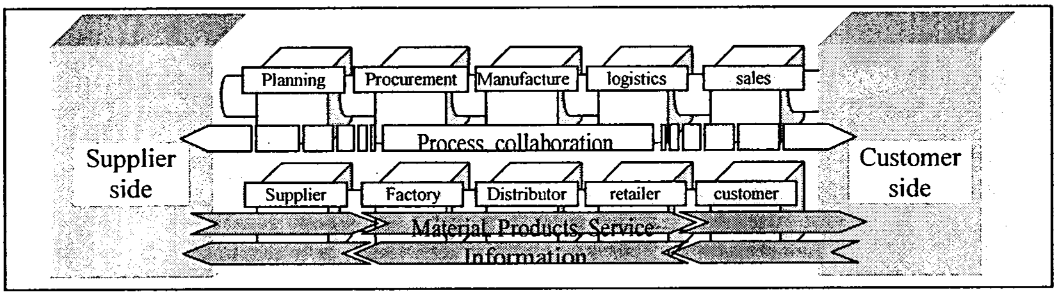
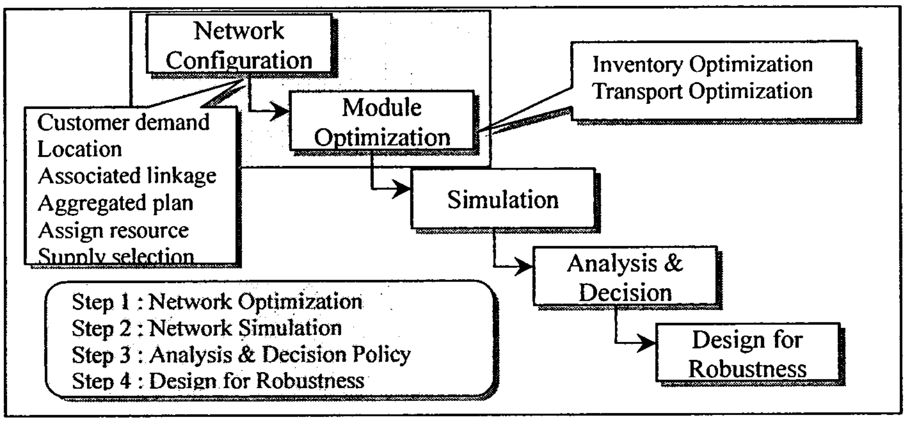
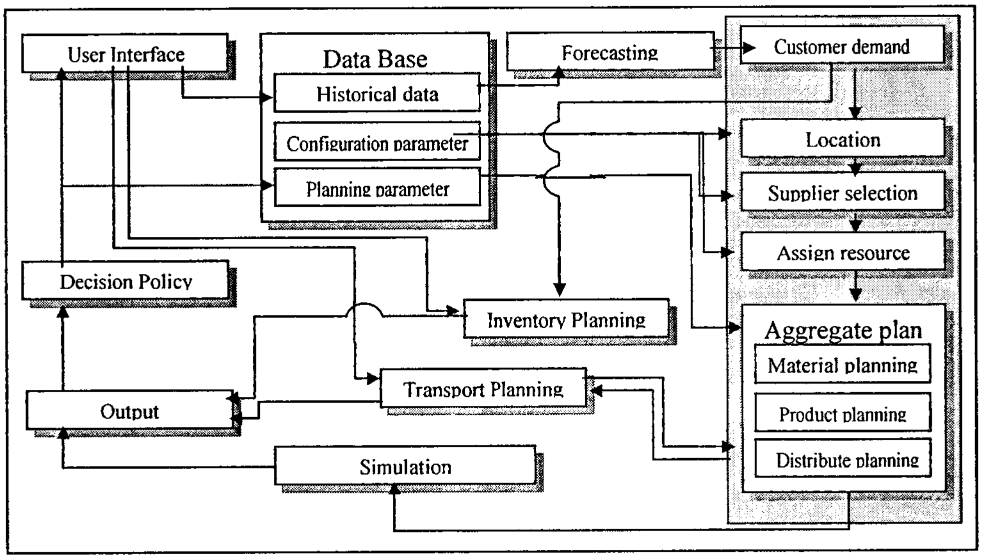


<Figure 1: Concept of supply chain management >



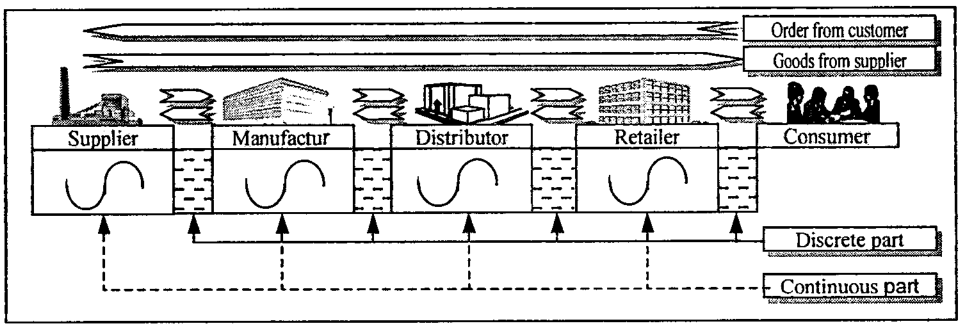
< Figure 2: Steps for decision on supply chain >



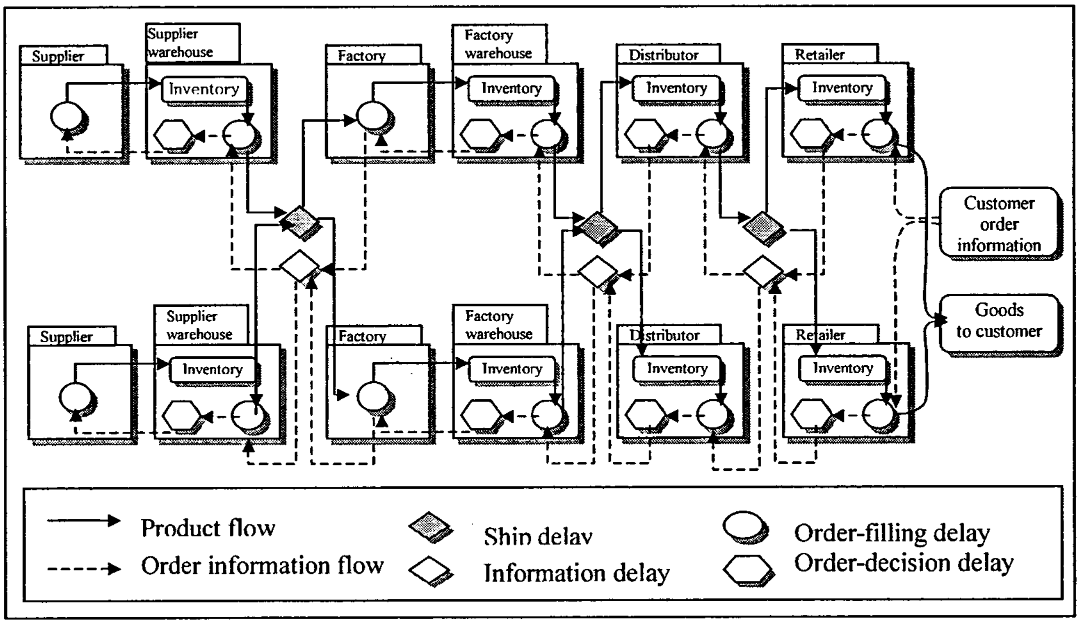
< Figure 3: Simulation process for supply chain>

< Table 1: Modeling method for each level >

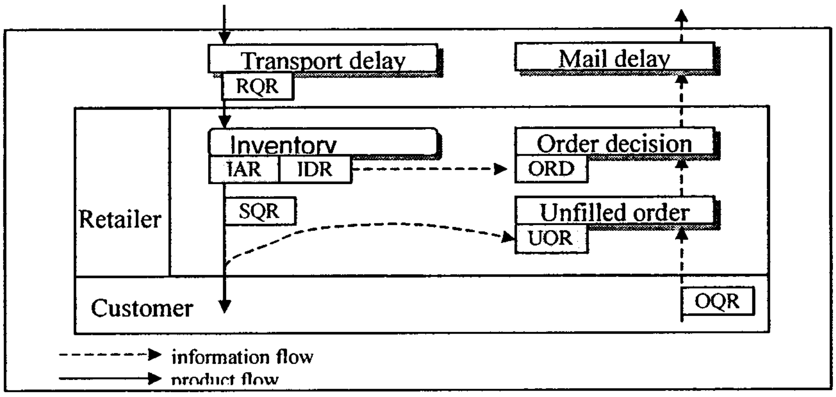
Level	Modeling method	Model Detail	Model scale
Operation level	Mathematical model	Very detailed	Small
Tactical level	Optimization/discrete event simulation	Normal	Normal
Strategic level	Combined modeling simulation	Simple	Very large



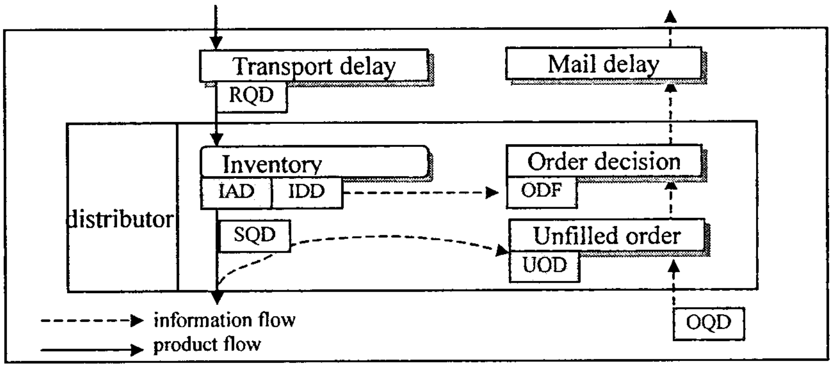
< Figure 4: Discrete and continuous portion in supply chain>



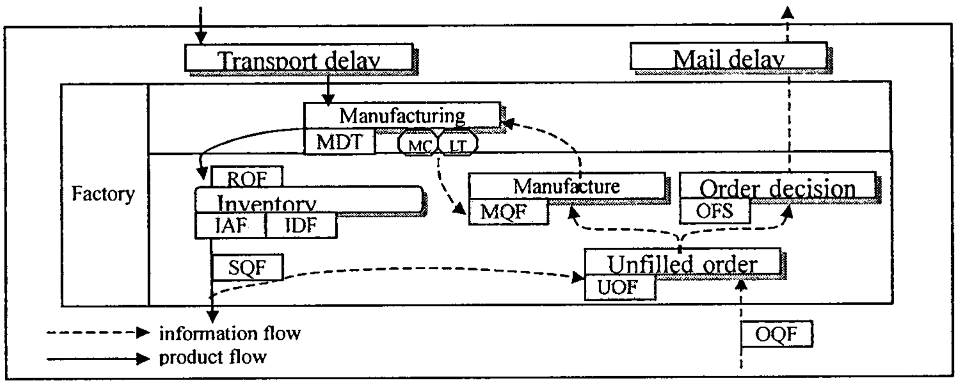
< Figure 5: Structure of supply chain system >



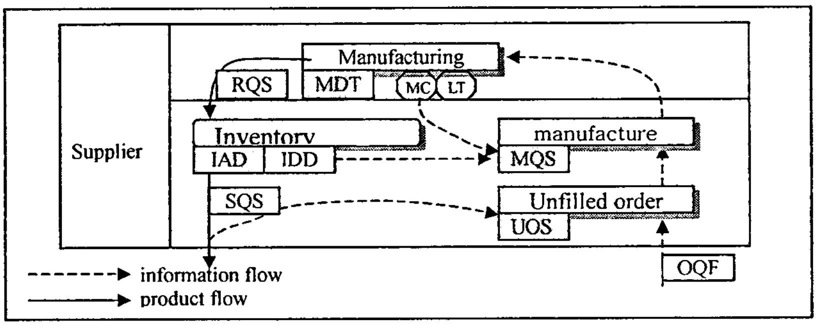
< Figure 6: Retailer section >



<Figure 7: Distributor section>



<Figure 8: Factory section>

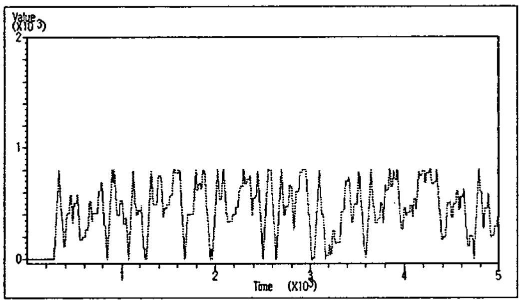


<Figure 9: Supplier section>

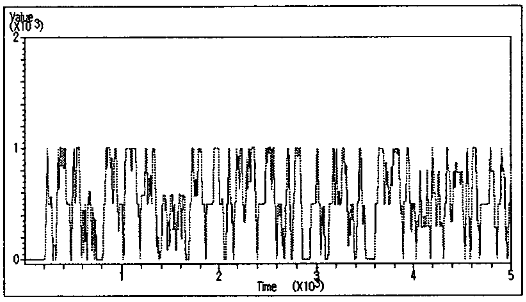
<table>
<tr><td rowspan="4">Supplier section</td><td>Manufacturing process time</td><td>TRIA(14,16,18)</td><td rowspan="12">Customer order generate;
EXPO(20)

Information delay;
10</td></tr>
<tr><td>Manufacturing capacity</td><td>75</td></tr>
<tr><td>Inventory capacity</td><td>1600</td></tr>
<tr><td>Desired Inventory level</td><td>1000</td></tr>
<tr><td rowspan="4">Factory section</td><td>Manufacturing process time</td><td>TRIA(12,14,16)</td></tr>
<tr><td>Manufacturing capacity</td><td>55</td></tr>
<tr><td>Inventory capacity</td><td>1600</td></tr>
<tr><td>Desired Inventory level</td><td>800</td></tr>
<tr><td rowspan="2">Distributor section</td><td>Inventory capacity</td><td>1000</td></tr>
<tr><td>Desired Inventory level</td><td>500</td></tr>
<tr><td rowspan="2">Retailer section</td><td>Inventory capacity</td><td>800</td></tr>
<tr><td>Desired Inventory level</td><td>400</td></tr>
</table>

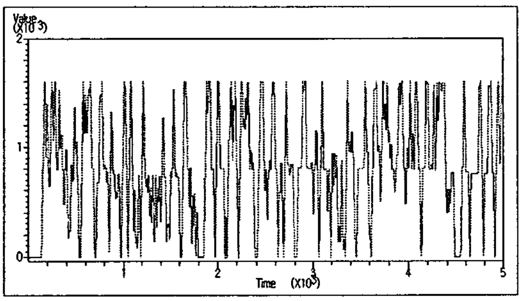
<Table2: Parameters in experimental model>



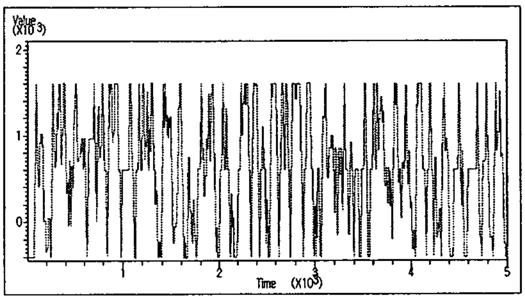
Retailer section



Distributor section



Factory section



Supplier section

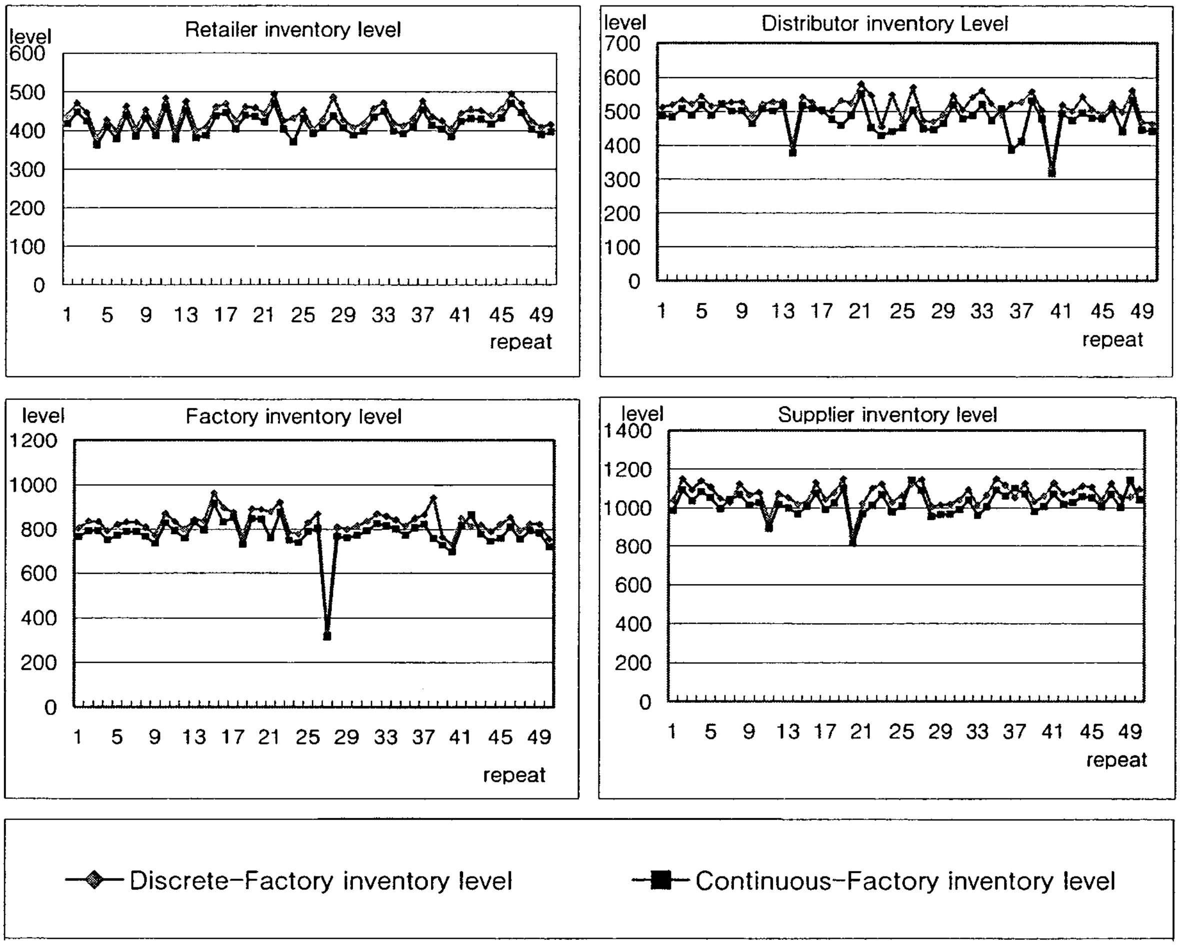
<Figure 10: Inventory level by combined model approach in each section>

	Statistics of discrete-event model				Statistics of combined model			
Repeat	Retailer	Distributor	Factory	Supplier	Retailer	Distributor	Factory	Supplier
1	438.56	512.19	805.42	1038.3	416.64	486.61	765.23	986.39
2	470.59	519.71	835.57	1149.4	447.05	483.25	493.95	1092.03
3	445.65	533.68	834.96	1094.3	423.41	506.99	79326	1035.12
4	380.8	519.76	791.79	1139.8	361.81	487.99	752.34	1083
5	428.29	544.49	820.47	1108.2	407.52	517.14	791.28	1052.98
6	398.2	514.25	832.85	1049.8	378.32	488.44	789.52	997.41
7	461.43	519.65	831	1027.5	438.39	519373	767.89	1042.63
8	405.16	526.7	808.29	1123.1	384.95	500.34	734.67	1067.14
9	453.19	527.35	773.2	1066.6	430.58	501.09	827.72	1013.37
10	406.89	487.44	871.14	1079.3	386.58	463.11	793.73	1025.34
11	484.16	519.2	835.42	941.3	459.87	506.06	758.58	894.28
12	397.18	526.69	798.6	1069.2	377.36	500.36	832.11	1015.74
13	474.72	526.17	845.2	1051.7	450.98	51557	797.16	999.12
14	400.93	397.02	839.02	1019.3	380.87	377.16	914.31	968.34
15	407.58	512.97	962.27	1028.6	387.18	515.80	831.91	1005.76
16	460.36	524.8	895.3	1129.6	437.40	507.74	853.21	1073.03
17	468.43	497.8	875.7	1041.9	445.06	503.12	728.37	989.9
18	424.95	501.44	766.68	1078.2	403.72	476.39	847.84	1024.38
19	461.56	531.8	892.33	1148.9	438.52	458.47	844.03	1100.29
20	458.03	523.6	888.31	854.7	435.19	486.73	758.32	812.23
21	443.57	580.08	878.4	1020.6	421.43	551.01	875.62	969.76
22	493.94	548.1	921.6	1102.2	469.25	451.68	747.42	1012.23
23	424.61	455.3	786.7	1123.3	403.42	428.13	738.74	1067.23
24	431.8	548.2	777.7	1029.1	370.12	440.15	788.91	977.74
25	453.31	474.78	830.42	1062.89	430.68	450.99	803.09	1010.31
26	402.95	570.74	868.5	1120.1	391.68	502.04	314.44	1143.28
27	427.51	471.97	331.46	1145.9	406.13	448.31	767.78	1088.70
28	486.4	468.91	808.14	1005.5	436.34	445.45	760.29	955.23
29	426.88	488.58	800.41	1014.46	405.53	464.14	773.47	964.49
30	408.16	545.84	814.08	1017.7	387.73	518.55	793.67	966.91
31	418.36	503.68	835.33	1041.9	397.46	478.43	825.42	989.9
32	456.87	542.1	868.76	1094.9	434.09	488.01	815.06	1040.16
33	471.93	560.2	857.94	1010.9	448.35	519.36	800.71	960.45
34	418.11	522	842.61	1065.9	397.24	473.22	772.32	1005.44
35	411.5	489.3	812.94	1148.6	390.74	506.24	805.86	1091.17
36	429.99	521.1	848.33	1116.3	408.48	386.54	822.10	1061.58
37	475.99	526.9	865.27	1054.3	452.24	410.92	758.21	1100.29
38	434.69	558.03	942.5	1125.3	412.99	530.12	726.39	1069.13
39	424.29	501.95	764.6	1033.2	403.11	476.51	692.70	981.635
40	403.92	333	728.98	1058.9	383.76	316.26	816.16	1005.86
41	445.26	518.66	852.2	1126.8	423.02	492.73	864.74	1070.46
42	453.08	498.73	812.5	1071.4	430.46	473.68	778.76	1018.02
43	451.69	542.1	819.66	1081.2	429.17	495.29	745.14	1027.24
44	438.17	507.74	789.6	1113.3	416.32	479.59	758.84	1057.73
45	455.11	489.6	822.1	1104.32	432.36	477.83	809.97	1050.14
46	494.35	523.1	852.44	1036.9	469.89	506.24	754.29	1006.67
47	468.97	498.5	793.85	1125	445.56	439.33	794.63	1068.94
48	424.87	560.43	823.9	1053.6	403.62	532.37	772.89	1001.02
49	409.06	468.77	823.41	1055.9	388.61	445.27	782.35	1143.23
50	416.48	464.61	755.62	1095.7	395.68	441.39	717.81	1041.2

<Table 3: Simulation results for inventory level>

	Discrete-event model				Combined model			
	Retailer	Distributor	Factory	Supplier	Retailer	Distributor	Factory	Supplier
Average inventory level	438.57	511.53	822.67	1069.92	415.54	477.44	779.06	1024.47

<Table 4: Average inventory level by each approach>



<Figure 11.Comparison of discrete-event modeling with combined modeling>