선형저수지 요소를 가진 침투모형에 의한 수막시간 및 유효강우량의 산정

Ponding Time and Effective Rainfall Estimation Using Infiltration Model with Linear Reservoir Element

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1. Introduction

Infiltration is the process that describes the water entry into soil. The process is very important to determine the distribution of water-normally in the form of rainfall. Despite the wealth of information included in the published material, there are a few topics that are not adequately treated. These include the changes in the infiltration capacity during the periods of low-intensity rainfall, the time of ponding under steady rainfall intensity and the process of production of rainfall excess over relatively large parts of watersheds.

The purpose of this study is to derive ponding time under steady rainfall intensity and estimate effective rainfall using a simple infiltration model with two linear reservoir elements. Some part of the model or of computations based on its structure have appeared in the literature; Diskin et al. (1995, 1996). The model considers that the moisture content in the upper soil layer is not totally depleted at the beginning of actual rainfall events. In the derivation of ponding time assumed the rainfall intensity to be steady and within the range between final infiltration capacity and initial infiltration capacity. The ponding time obtained with this model is compared with the literature values for some soil types.

2. Infiltration model

The infiltration model in this study is mainly based on the model by Diskin et al. (1995) and comprises two elements. One is a storage element receiving one input and producing one output. The other is an inlet-regulating element also receiving one input, but two outputs. Each element has one state variable that determines the magnitude of its outputs. The two

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elements are linked by a feedback path transmitting the information about the state of the storage element to the regulating element. The two elements are also related by the fact that one of the outputs of the inlet-regulating element is the input to the storage element.

From the relationships between the elements, Diskin et al. (1995) suggested expressions for moisture storage at the end of each computation time interval. The expressions are derived with the assumption that the value of the initial moisture storage in the upper soil layer is zero. However, at the beginning of actual rainfall events, the initial moisture storage cannot be completely depleted. Thus, modified expressions for the infiltration process with non-zero initial moisture condition are presented in this study.

Assuming the storage element to be a linear reservoir, the output of this element can be express like following:

$$g(t) = f_c S(t)/S_m \tag{1}$$

where $g(t)$ is the rate of percolation from upper soil layer, f_c minimum infiltration capacity rate, $S(t)$ moisture storage in upper soil layer, and S_m is maximum value of moisture storage in upper soil layer.

The value of the state variable of the regulating element is denoted by $f(t)$ and is assumed to be determined by the value of $S(t)$ transmitted to the regulating element via the feedback loop. The relationship between the two state variables is assumed to be decreasing linear:

$$f(t) = f_o + \frac{f_o - f_c}{S_m - S_o}(S_o - S(t)) \tag{2}$$

where f_o is maximum infiltration capacity rate and S_o is initial moisture storage in upper soil layer.

The value of the state variable of the storage element, $S(t)$, is changing at a rate which depends on the magnitude of the input and the output from the storage element. The relationship within the element is an expression of the principle of conservation of mass:

$$q(t) - g(t) = dS/dt \tag{3}$$

where $q(t)$ is actual rate of infiltration. Despite the use of linear relationships in the descriptions of the elements, the model in the complete unit is not linear.

In natural state, the moisture content in upper soil layer cannot be completely depleted. Thus, the moisture content at its maximum value is denoted by S_o as shown above, which Diskin et al. (1995) assumed to be zero. The maximum value of infiltration capacity is obtained at this condition. When the moisture is at its maximum value, the infiltration capacity is at its smallest value.

Considering a typical time interval Δt ($= t_{i+1} - t_i$), Eq. (3) can be written in the following form:

$$S_e - S_b = (q_b + q_e)\Delta t/2 - (g_b + g_e)\Delta t/2 \tag{4}$$

The notation used in that values of variables at the beginning of the time interval are denoted by the subscript b and the corresponding values at the end of interval by the subscript e.

Three cases are available considering the relationship between rainfall intensity and infiltration capacity at the beginning and the end of each time interval

case (a): $R > f_b$ and $R > f_e$ $\rightarrow$ $q_b = f_b$ and $q_e = f_e$ (R: rainfall intensity)

$$S_e - S_b = (f_b + f_e)\Delta t/2 - (g_b + g_e)\Delta t/2 \tag{5}$$

Using the definitions in Eqs. (1) and (2),

$$S_e = \frac{\left\{1 - \frac{1}{2}\Delta t\left(\frac{f_c}{S_m} + \frac{f_o - f_c}{S_m - S_o}\right)\right\}S_b + \left(f_o + \frac{f_o - f_c}{S_m - S_o}\right)\Delta t}{1 + \frac{1}{2}\Delta t\left(\frac{f_c}{S_m} + \frac{f_o - f_c}{S_m - S_o}\right)} \tag{6}$$

case (b): $R < f_b$ and $R < f_e$ $\rightarrow$ $(q_b + q_e)/2 = R$

$$S_e - S_b = R\Delta t - (g_b + g_e)\Delta t/2 \tag{7}$$

Using the definitions in Eq. (1),

$$S_e = \frac{S_b\left(1 - \frac{f_c\Delta t}{2S_m}\right) + R\Delta t}{1 + \frac{f_c\Delta t}{2S_m}} \tag{8}$$

case (c): $R < f_b$ and $R > f_e$

The infiltration capacity will become equal to the rate of rainfall and at some time inside the time interval. The moisture storage at this instant is denoted by S_r and can be obtained by substituting $f(t) = R$ in Eq. (2).

$$S_r = S_o + \frac{f_o - R}{\left(\frac{f_o - f_c}{S_m - S_o}\right)} \tag{9}$$

The time at which this equality will occur, relative to the beginning of the computation time interval, is obtained by substituting in Eq. (8) $S_e = S_r$ and $\Delta t = \Delta t_1$.

$$\Delta t_1 = \frac{S_r - S_b}{R - \frac{(S_b + S_r)f_c}{2S_m}} \tag{10}$$

Thus, the contents of the storage element at the end of time interval are obtained by using Eq. (6) with $\Delta t_2 = \Delta t - \Delta t_1$ replacing Δt and S_r replacing S_b.

$$S_e = \frac{\left\{1 - \frac{1}{2}\Delta t_2\left(\frac{f_c}{S_m} + \frac{f_o - f_c}{S_m - S_o}\right)\right\}S_r + \left(f_o + \frac{f_o - f_c}{S_m - S_o}S_o\right)\Delta t_2}{1 + \frac{1}{2}\Delta t_2\left(\frac{f_c}{S_m} + \frac{f_o - f_c}{S_m - S_o}\right)} \tag{11}$$

3. Computation of ponding time

Ponding can occur only if the rainfall intensity, R_c, which is assumed to be constant, is in the range:

$$f_c < R_c < f_o \tag{12}$$

For the range of the constant rainfall intensity specified in Eq. (12), the ponding storage, S_p, is a function of the rainfall intensity. This relationship can be derived directly from $f_p = R_c$ at the instant of ponding. Also, at the same time, the value of f_p and S_p are related by Eq. (2). Solving Eq. (2) for $S(t)$ and substituting S_p for $S(t)$ leads to the following:

$$S_p = S_o - \frac{(R_c - f_o)(S_m - S_o)}{f_o - f_c} \tag{13}$$

It is clear that the expression for the ponding storage is derived without reference to the ponding time. The ponding time can be evaluated by deriving an equation for the upper soil layer moisture storage as a function of time. The time corresponding to the value of the ponding storage derived in Eq. (13) is the ponding time.

During the ponding time, $q(t) = R_c$. Thus, solving Eq. (3) for $S(t)$ leads to the following expression:

$$S(t) = \frac{R_c S_m}{f_c} + Const. \times \exp\left(-\frac{f_c}{S_m}t\right) \tag{14}$$

Separate the variables and integrate (I.C. $S(t) = S_o$ at $t = 0$):

$$t = \left(-\frac{S_m}{f_c}\right)\ln\left(\frac{f_c S - R_c S_m}{f_c S_o - R_c S_m}\right) \tag{15}$$

At the instant of ponding time is the ponding time, t_p, and the reservoir state variable is at the ponding storage, S_p. The relationship between these two variables is given by:

$$t_p = -\frac{S_m}{f_c}\ln\left\{1 - \frac{f_c(R_c - f_o)(S_m - S_o)}{(f_o - f_c)(S_o f_c - R_c S_m)}\right\} \tag{16}$$

4. Sample applications

The following numerical example is presented as in illustration of the possible application of the model. The case considered is the production of rainfall excess in a small watershed for a specified rainfall event. The hyetograph of the rainfall is known and the parameters of the model are also assumed to be known. Values of the rainfall intensity during the assumed event are listed in Table 1. The following set of model parameters were adopted for the calculations: $f_o = 20.5$ mm/h, $f_c = 4.6$ mm/h, and $S_m = 25.6$ mm. For the first set of calculations the initial moisture content of the upper soil layer was taken to be $S_o = 0.0$ mm. From Table 1, the infiltration capacity values, as produced by the model, show the expected variations. The values decrease when the rainfall intensity is higher that the smallest infiltration capacity (f_c) and recover when the rainfall intensity is lower than the capacity rate of when the rain stops. The results of the second set of computations are given in the second part of Table 1. Computations start with the assumed initial value of $S_o = 17.0$ mm for the storage state variable.

Table 1. Computation of effective rainfall using sample data

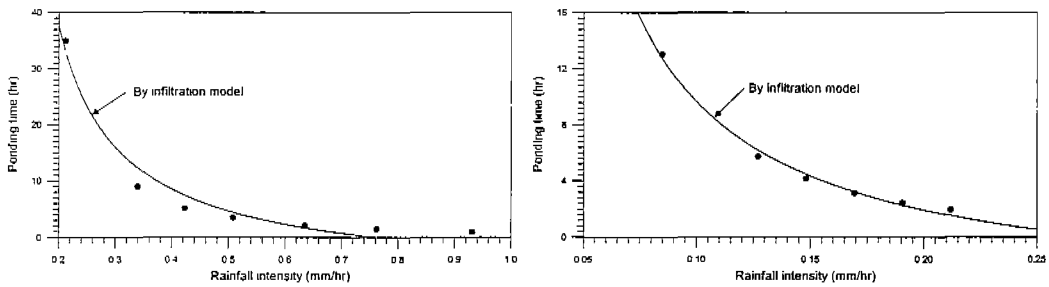
t	R mm	S_o = 0.0mm				S_o = 17.0mm			
		S mm/h	f mm/h	g mm/h	Re mm	S mm/h	f mm/h	g mm/h	Re mm
0.0	-	0.00	20.50	0.00	-	17.00	20.50	3.05	-
0.5	1.4	0.67	20.08	0.12	-	16.21	21.96	2.91	-
1.0	4.2	2.62	18.87	0.47	-	16.82	20.82	3.02	-
1.5	8.9	6.66	16.37	1.20	-	19.64	15.63	3.53	-
2.0	12.0	11.83	13.16	2.12	-	23.29	8.88	4.18	0.36
2.5	9.5	15.35	10.96	2.76	-	24.84	6.00	4.46	1.03
3.0	8.3	18.01	9.32	3.24	-	25.35	5.06	4.56	1.39
3.5	2.3	17.56	9.59	3.15	-	24.27	7.05	4.36	-
4.0	1.1	16.57	10.21	2.98	-	22.71	9.94	4.08	-
4.5	4.8	17.45	9.66	3.13	-	23.06	9.30	4.14	-
5.0	10.9	20.17	7.97	3.62	1.04	24.77	6.14	4.45	1.59
5.5	14.1	21.98	6.85	3.95	3.34	25.33	5.10	4.55	4.24
6.0	10.2	23.19	6.10	4.17	1.86	25.51	4.76	4.58	2.63
6.5	8.0	23.99	5.60	4.31	1.08	25.57	4.65	4.59	1.65
7.0	2.9	23.32	6.02	4.19	-	24.76	6.15	4.45	-
7.5	0.0	21.31	7.26	3.83	-	22.63	10.09	4.07	-
8.0	0.0	19.48	8.40	3.50	-	20.69	13.69	3.72	-
8.5	0.0	17.80	9.44	3.20	-	18.91	16.97	3.40	-
9.0	0.0	16.27	10.39	2.92	-	17.28	19.98	3.11	-

Synthetic data of ponding times for various rainfall intensities and a number of soil types were presented by Smith (1972). Values for minimum infiltration capacity for each soil type are also given in this literature. The synthetic data presented by Smith (1972) are for five different soils. Among these, Paudre sand and Nickel gravelly sandy loam were selected for the sample applications of ponding time equation of this study.

The agreement between the ponding time data (Smith, 1972) and Eq. (16) based on the infiltration model, was demonstrated by deriving the values of parameters of the equation that best fitted the synthetic data. Eq. (16) makes use of four parameters: f_o, f_c, S_o, S_m. In the search for optimal values of the parameters for each soil, the value of f_c was not changed using the value reported by Smith (1972) and S_o was assumed to be zero for the convenience of computation.

Table 2. Optimal parameters for soil types

Soil type	f_o (cm/min)	S_m (cm/min)	r.m.s deviation (min)
Paudre sand	0.7553	5.2949	4.25
Nickel gravelly sandy loam	0.2756	1.2340	0.10



(a) Paudre sand (b) Nickel gravelly sandy loam

Fig. 1. Fitting of an infiltration envelop curve to synthetic data

The values of the parameters that produced a good agreement with the synthetic data for two soils are listed in Table 2. Also given are values of the r.m.s deviation between the ponding times given by Smith (1972) and those calculated by the infiltration model of this study. The good results obtained are shown in Fig. 1.

5. Summary and conclusions

Using a relatively simple model for the infiltration process an equation for ponding time is derived considering intial moisture contents in upper soil layer. The ponding time is only for the range between maximum infiltration capacity and the minimum capacity. Values of ponding time computed by the equation derived herein were compared with synthetic data presented by Smith (1972). Good agreement can be obtained by optimizing two variables of the model. The good results obtained may be taken as an indication of the validity of the model used for the derivation of the equation for ponding time.

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