

# The submerged flexible membrane breakwaters in oblique seas.

S.T. Kee<sup>1</sup>

## 1. Abstract

The focus of this paper is on the numerical investigation of obliquely incident wave interactions with a system composed of fully submerged and floating dual buoy/vertical-flexible-membrane breakwaters placed in parallel with spacing. The fully submerged systems allow surface and bottom clearances to enable wave transmission over and under the system. The problem is formulated based on the two-dimensional multi-domain hydro-elastic linear wave-body interaction theory. The hydrodynamic interaction of oblique incident waves with the combination of the rigid and flexible bodies was solved by the distribution of the simple sources (modified Bessel function of the second kind) that satisfy the Helmholtz governing equation. Using this computer program, the performance of various dual systems varying buoy radiuses and drafts, membrane lengths, clearances, spacing, mooring-lines stiffness, mooring types, water depth, and wave characteristics is thoroughly examined. It is found that the fully submerged and floating dual buoy/membrane breakwaters can, if it is properly tuned to the coming waves, have good performances in reflecting the obliquely incident waves over a wide range of wave frequency and headings.

## 2. Introduction

The advantages of floating flexible membrane wave barriers over conventional fixed breakwaters include their reduced environmental impacts, ability of relocation, simple sacrificial design, free from bottom foundation consideration, and comparably low costs in deep water constructions. The vertical floating flexible membrane breakwater was investigated by Thomson et al.(1992), Aoki et al.(1994), Kim and Kee(1996, 1997), Williams(1996). Kim and Kee(1996, 1997) showed that the a good performance can be obtained in spite of appreciable sinusoidal motions of membrane because the vertical sinusoidal motions tends to generate only exponentially decaying local(evanescent) wave in the lee side.

The vertical flexible membrane system was composed of surface pierced buoys and vertical flexible membranes hinged at seafloor. So these breakwaters expect large wave loadings and possible blockage of aesthetic view, water circulation, sediment transport, fish passage, and surface vessel. In view of this, ideal porous horizontal membrane wave barriers have been investigated by Cho and Kim(2000). In practice, a fairly good performance as breakwaters in wide frequency region including long waves, a

---

<sup>1</sup> Full Time Instructor, Dept. of Civil Engng., Seoul National University of Technology, #172, Kongneung-dong, Nowon-Gu, Seoul, Korea (139-743) Tel: 82-2-970-6509, Fax: 82-2-948-0043

major fraction of the water column needs to be occupied by the system.

In the present paper, we investigated the performance of the fully submerged vertical flexible membrane breakwaters with clearances between bottom of system and seafloor in oblique seas. It is assumed that buoy and membrane motions are uniform in the longitudinal direction and small to allow linear theory. It is also assumed, for simplicity, which the buoy is rigid and the heave motion of the buoy is negligible due to large initial tension. The coupling of buoy and membrane motions was taken into consideration of buoy and membrane motions through an appropriate boundary condition at the joint. The velocity potentials of wave motion are fully coupled with membrane deformation. The membrane motion and the rigid body motion of a buoy become dynamically coupled with each other, thus the membrane motion and velocity potentials need to be solved simultaneously. Numerical results are presented to check the accuracy and validity of the present multi-domains boundary element program by the energy-conservation formula and conversance test.

### 3. Numerical Method

The geometry of the submerged flexible membrane breakwaters is depicted in Fig. 1. The system is idealized as two-dimensional allowing that wave and system motions are uniform in  $z$  direction, and Cartesian coordinates are employed. The system is subjected to an incident train of regular, monochromatic, small amplitude  $A$ , harmonic motion of frequency  $\omega$ , obliquely propagating with an angle  $\theta$  ( $0 < \theta < \pi/2$ ) to  $x$ -axis in water of arbitrary depth  $h$  as shown in Fig. 1. The ideal flow field can be described in terms of the total velocity potential for an oblique incident wave:

$$\Phi(x, y, z, t) = \text{Re}[\phi_o(x, y)e^{ik_z z - i\omega t}], \quad \phi_o = \frac{-igA \cosh k_o(y+h)}{\omega \cosh k_o h} e^{ik_o \cos\theta x} \quad (1)$$

where  $k_z = k_o \sin\theta$  is the wave number  $k_o$  component in the  $z$  direction, and is related to the angular frequency through the dispersion relation  $\omega^2 = k_o g \tanh k_o h$  with  $g$  being the gravitational acceleration. The complex velocity potentials,  $\phi_1$ ,  $\phi_2$  and  $\phi_3$ , in three fluid domains 1, 2 and 3 (see FIG. 1), then satisfy the Helmholtz equation  $\nabla^2 \phi_l - k_z^2 \phi_l = 0$ , ( $l=1,2,3$ ) as governing equation and the following linearized free-surface ( $\Gamma_F$ ), bottom ( $\Gamma_b$ ), and radiation conditions  $\Gamma_c$ :

$$-\omega^2 \phi_l + g \frac{\partial \phi_l}{\partial y} = 0 \quad (\text{on } \Gamma_F), \quad \frac{\partial \phi_l}{\partial n} = 0 \quad (\text{on } \Gamma_b), \quad \lim_{|x| \rightarrow \infty} \left( \frac{\partial}{\partial x} \pm ik_x \right) \begin{pmatrix} \phi_1 \\ \phi_3 \end{pmatrix} = 0 \quad (\text{on } \Gamma_c) \quad (2)$$

where  $n = (n_x, n_y)$  is the unit outward normal vector. Under large initial tension, we assume, for simplicity that the heave motion of the buoy is negligible. Then the boundary condition on the floating buoy is

$$\frac{\partial \phi_l}{\partial n} + i\omega\{\eta_1 n_x + \eta_3 n_\theta\} + \frac{\partial \phi_o}{\partial n} = 0 \quad (3)$$

where  $n_\theta = xn_y - yn_x$ , and the symbols  $\eta_1$ ,  $\eta_3$  represent complex sway and roll responses of a buoy respectively.

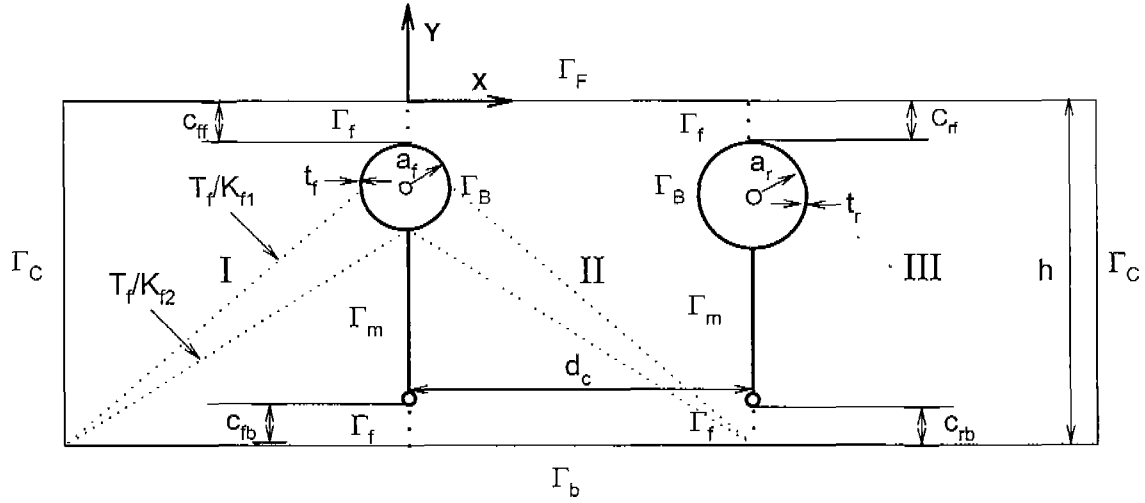


Fig 1. Definition Sketch for Fully Submerged Flexible Membrane Breakwaters

In addition, the disturbance potentials must satisfy the following linearized kinematic/dynamic boundary conditions on the membrane surface  $\Gamma_m$ , and the continuity of hydrodynamic pressure and normal fluid velocity on the vertical fictitious boundary  $\Gamma_f$ :

$$\frac{\partial \phi_l}{\partial x} = \frac{\partial \phi_{l+1}}{\partial x} = -i\omega \xi, \quad \frac{d^2 \xi}{dy^2} + \lambda^2 \xi = \frac{\rho i \omega}{T} (\phi_{l+1} - \phi_l) \quad \text{at } \Gamma_m, \quad \phi_l = \phi_{l+1}, \quad \frac{\partial \phi_l}{\partial x} = \frac{\partial \phi_{l+1}}{\partial x} \quad \text{at } \Gamma_f \quad (4)$$

in which  $\lambda = \omega \sqrt{m/T}$ , with  $T$  and  $m$  being the membrane tension and mass per unit length for front and rear membrane system, respectively. To solve the present boundary value problem, a three-domain boundary integral equation method using simple sources along the entire boundary is developed. The fundamental solution (Green function) of the Helmholtz equation and its the normal derivative of  $G$  are given by

$$G = -\frac{1}{2\pi} K_0(k_z r), \quad \frac{\partial G}{\partial n} = \frac{1}{2\pi} k_z K_1(k_z r) \frac{\partial r}{\partial n}, \quad K_0(k_z r) = -\gamma - \ln\left(\frac{k_z r}{2}\right) \quad \text{As } r \rightarrow 0 \quad (5)$$

where  $K_0$  is the modified zeroth-order Bessel function of the second kind and  $r$  is the distance from the source point  $(x', y')$  to the field point  $(x, y)$ ,  $\gamma = 0.5772$  is known as Euler's constant using recurrence formula of the Bessel function.

By applying Green's second identity in each of the fluid regions to the unknown potentials  $\phi_l$ ,  $l = 1, 2, 3$  and imposing the relevant boundary conditions, the integral equations in each fluid domain can be written as

$$\begin{aligned}
& C\phi_l + \int_f [k_z K_1(k_z r) \frac{\partial r}{\partial n} + v K_o(k_z r)] \phi_l d\Gamma + \int_c [k_z K_1(k_z r) \frac{\partial r}{\partial n} + i k_x K_o(k_z r)] \phi_l d\Gamma \\
& + \int_b \phi_l k_z K_1(k_z r) \frac{\partial r}{\partial n} d\Gamma + \int_m [\phi_l k_z K_1(k_z r) \frac{\partial r}{\partial n} - s_l (i\omega\xi) K_o(k_z r) - K_o(k_z r) \frac{\partial \phi_o}{\partial n}] d\Gamma \\
& + \int_a [\phi_l k_z K_1(k_z r) \frac{\partial r}{\partial n} - i\omega K_o(k_z r) \{\eta_1 n_x + \eta_3 n_\theta\} - K_o(k_z r) \frac{\partial \phi_o}{\partial n}] d\Gamma \\
& + \int_f (\phi_l k_z K_1(k_z r) \frac{\partial r}{\partial n} + K_o(k_z r) \frac{\partial \phi_l}{\partial n}) d\Gamma = 0 \quad (6)
\end{aligned}$$

where the infinite-depth wavenumber  $v = \omega^2/g$ ,  $s_l = 1$ ,  $s_{l+1} = -1$ , and  $C =$  solid angle constant. In (39), the potentials  $\phi_l$  do not include incident waves. To solve (6), the entire boundary is discretized into a large finite number of segments. On each segment the potential is assumed to be constant, and the singularities  $G$  and  $\partial G/\partial n$  are integrated analytically (Au & Brebbia, 1982). The integral equations (6) can then be transformed to the corresponding algebraic matrix equation. For instance, if each of half fluid domain is discretized by  $N = N_f + N_c + N_b + N_m + N_B + N_f$  segments, there are  $3N$  unknowns for  $\phi_1$ ,  $\phi_2$  and  $\phi_3$ ,  $2N_m$  unknowns for displacements  $\xi$  of dual membranes, and four more unknowns two  $\eta_1$  and  $\eta_3$  for two buoys. Therefore,  $3N + 2N_m + 4$  linear simultaneous equation has to be solved. The discrete form of the membrane equation of motion for  $j$ -th element is given by

$$\rho i \omega (\phi_{1j} - \phi_{2j}) l_j - T_j (\partial \xi / \partial \zeta)_j + T_{j+1} (\partial \xi / \partial \zeta)_{j+1} = -m l_j \omega^2 \xi_j, \quad (\partial \xi / \partial \zeta)_j = (\xi_j - \xi_{j-1}) / \Delta \zeta_j \quad (7)$$

where the symbol  $l_j$  is the length of the  $j$ -th segment, and  $\Delta \zeta_j = (l_j + l_{j+1})/2$ . As mentioned before, the heave response is neglected assuming large initial tension. The coupled equation of motion for sway and roll of a buoy is then given by

$$M(-\omega^2)X = F_p - (K_{HS} + K_m)X - F_T, \quad X = [\eta_1 \eta_3]^T, \quad M = \begin{bmatrix} m_o & -m_o y_c \\ -m_o y_c & I \end{bmatrix} \quad (8)$$

where  $m_o$  is the mass of the buoy,  $y_c$  the vertical coordinate of the center of mass,  $I$  the roll moment of inertia,  $F_p$  the potential forces and moments on the buoy,  $K_{HS}$  the restoring forces and moments due to the hydrostatic pressure,  $K_m$  sway and roll mooring stiffness, and  $F_T$  the force and moment on the buoy by tension at the connection point between membrane and buoy. Since  $\xi$  and  $\eta$  are unknown and coupled, (6) cannot be solved for  $\phi_l$ ,  $l=1,2,3$  independently. Thus, the total disturbance (diffraction+radiation) potential (6) and buoy (8) and membrane motions (7) have to be solved simultaneously.

## NUMERICAL RESULTS AND DISCUSSIONS

The computational domain is defined as in Figure 1. The vertical truncation boundary is located 3-4 waterdepths away from the membrane to ensure that local wave effects are negligible. First, the numerical results

were checked satisfactory against the energy-conservation formula i.e.  $R_f^2 + T_r^2 = 1$ . In Figures 2, the reflection coefficients of a fully submerged system with varying mooring type, stiffness, and clearances are plotted for the cases  $t_f/a_f = 0.02$ ,  $t_r/a_r = 0.02$ ,  $a_f/h = 0.2$ ,  $a_r/h = 0.2$ , and parameters as shown in Table 1. The efficiency drastically enhances as the mooring type and stiffness varies. In case 1, both buoy are restrained firmly by upper and lower moorings, and results in poor performance. It shows that incident wave is transmitted over the system except some narrow frequency band and high wave headings. As can intuitively be expected, the performance of the large-clearance case in short waves is poor. However, fairly good efficiency is shown in case 2 that both systems are restrained by only lower mooring, which allows motion of buoys. The response of the buoy restrained at its joint to the incident waves generate radiation wave, which interact with incident and scattering waves. It is interesting that the some low efficiency at  $kh \approx 2.5, 5$  due to system resonance disappears after slightly tuned by  $T_f/K_{f1} = 0.2$ , which performance shows in case 3. As shown in case 4, the front and rear system has smaller free surface and bottom clearances compared to that of case 2, respectively, overall efficiency of the fully submerged system with only lower mooring is improved for wider incident wave frequency and angles.

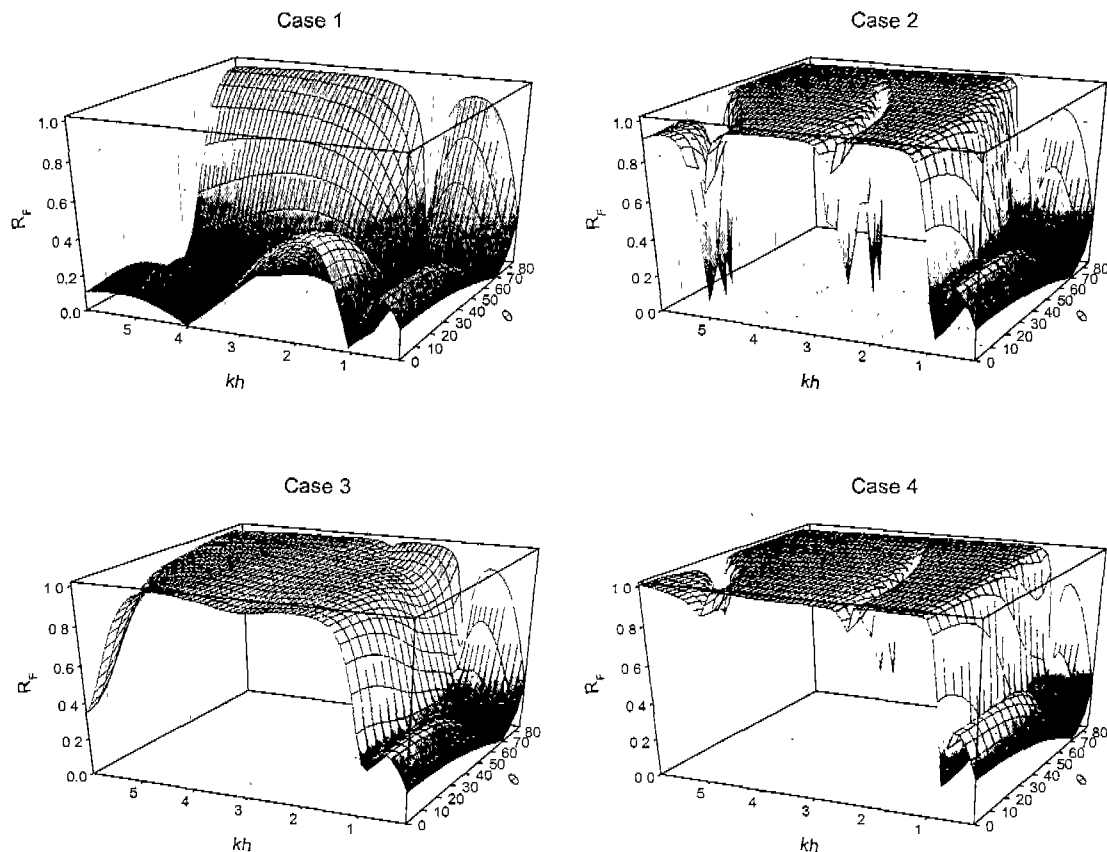


Fig 2. The reflection coefficients for  $t_f/a_f = 0.02$ ,  $t_r/a_r = 0.02$ ,  $a_f/h = 0.2$ ,  $a_r/h = 0.2$ , and the specified parameters in Table 1.

Case #	$T_f/K_{f1}$	$T_f/K_{f2}$	$T_r/K_{r1}$	$T_r/K_{r2}$	$C_{ff}/h$	$C_{fb}/h$	$C_{rf}/h$	$C_{rb}/h$
Case 1	0.1	0.1	0.1	0.1	0.125	0.125	0.125	0.125
Case 2	0	0.1	0	0.1	0.125	0.125	0.125	0.125
Case 3	0.2	0.1	0	0.1	0.125	0.125	0.125	0.125
Case 4	0	0.1	0	0.1	0.125	0.05	0.05	0.125

Table 1: Various values for mooring stiffness and clearances for the submerged flexible membrane breakwaters.

## SUMMARY AND CONCLUSIONS

The performance of fully submerged dual systems in oblique waves was tested with varying buoy radius and draft, water depth, membrane length, clearances, mooring-line characteristics, and wave conditions. The efficiency as breakwater depends critically on the three parameters: buoy radius to water-depth ratio, mooring type with different stiffness, free surface and bottom clearances. Therefore, a properly tuned system with sufficiently large membrane tension, restrained motion of buoys, and clearances allowing passage of flow needs to be provided to guarantee high performance. The performance in long oblique waves can be greatly improved controlling surface and bottom clearances. Using the developed computer program, an optimum design for a given sea condition can be determined through a comprehensive parametric study including various buoy shapes. To see the effects of large motions and high waves, a nonlinear time-domain program needs to be developed. The numerical results also need to be verified by large-scale experiments and/or field tests.

## REFERENCES

- Aoki, S., Liu, H., & Sawaragi, T. (1994) "Wave transformation and wave forces on submerged vertical membrane" *Proc. Intl. Symp. Waves - Physical and Numerical Modeling*, Vancouver, 1287-1296
- Au, M.C. & Brebbia, C.A. (1982) "Numerical prediction of wave forces using the boundary element method" *Appl. Math. Modelling*, Vol. 6, 218-228.
- I.H. Cho & M.H. Kim (2000) "Interactions of Horizontal Porous Flexible Membrane with Waves" *ASCE J. of Waterway, Port, Coastal & Ocean Engineering*, Vol.126, No.5, 245-253.
- Kee, S.T. & Kim, M.H. (1997) "Flexible membrane wave barrier. Part 2. Floating/submerged buoy-membrane system" *ASCE J. of Waterway, Port, Coastal & Ocean Engineering*, Vol. 123, No. 2, 82-90.
- Kim, M.H. & Kee, S.T. (1996) "Flexible membrane wave barrier. Part 1. Analytic and numerical solutions" *ASCE J. of Waterway, Port, Coastal & Ocean Engineering*, Vol.122, No.1, 46-53.
- Kim, M.H., Edge, B.L., Kee, S.T., and Zhang, L. (1996) "Performance evaluation of buoy-membrane wave barriers" *Proc. 25<sup>th</sup> Intl. Conf. on Coastal Engineering*, Orlando
- Thompson, G.O., Sollitt, C.K., McDougal, W.G. & Bender W.R. (1992) "Flexible membrane wave barrier" *ASCE Conf. Ocean V*, College Station, 129-148.
- Williams, A.N. (1996) "Floating membrane breakwater" *J. of Offshore Mechanics & Arctic Engr.*, Vol 118, 46-51.