

Motion and Wave Transmission Effect on Floating OWC(Oscillating Water Column) Wave Energy Conversion System

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진동수주형 파력발전 시스템에서 운동과 파랑회절의 영향

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Abstract

부유식 파랑에너지 변환시스템(Oscillating Water Column)에서 에너지 변환은 입력파와 챔버, 챔버내 공기의 상호작용으로 이루어진다. 이 논문은 파랑에너지를 기계적 운동으로 변환하는 기계적 특성을 해석한다.

단일 진동수 규칙파가 입력되었을 때에 파에 의하여 챔버의 상하운동이 선형적으로 발생하며, 이 상하운동은 챔버내의 압력 변화에 영향을 받는다. 상하운동과 챔버내로 투과한 파, 그리고 챔버내 압력에 의해 발생하는 파에 의해 챔버내의 상대운동을 정하고, 그 상대운동에 의한 공기의 압축 팽창과 온도상승을 근사적 열역학적 방정식으로 해석하여 오리피스를 통한 유량과 압력을 기준으로 에너지 변환효율을 결정하였다. 얻어진 식은 간단하면서도 관련요소의 영향을 전반적으로 표현한다. 개구율 변화에 따른 운동응답을 비교하였다.

1. Introduction

Floating-type Wave Power Devices have a few advantages: They can be installed at any place in the ocean as long as enough wave power persists; They can be utilized as wave dissipation devices connected multitudinously; and they are relatively free from the tidal effect contrast to the onshore devices. Since the offshore water depth is comparatively higher than onshore, wave breaking is less and more energy can be transformed into electricity.

Analysis and modelling of OWC devices can be divided into two stages. The first is the system modelling of energy transformation from the wave to the air flow through the orifice of the

wells-turbine; and the second is the energy transformation from the air flow of the wells-turbine to electrical power. The present paper extends author's previous work²⁾ focused on the former. Introducing the transmission of the wave across the chamber, an attempt to develop the more realistic model has been suggested in this research.

2. Theoretical Derivation of the Governing Equation

The analysis procedure is described below. The governing equation for the gas in the chamber is derived from mass continuity and the thermal equations. These equations are approximated for small variation of chamber volume. The chamber motion is modelled as a linear dynamic system, excited by the incident wave and the internal pressure change. The equations are expressed in complex form and solved to give conversion efficiency.

2.1 Thermal Equation of the Air Chamber

The continuity equation for the air mass in the chamber is

$$\frac{d(\rho_a V)}{dt} + \dot{m} = 0 \quad (1)$$

where ρ_a is the density of the air in the chamber, V is the volume, and \dot{m} is the rate of the air mass flux through the nozzle. The air mass flux is linearly proportional to the air velocity w_p at the nozzle as

$$\dot{m} = \rho_a c_d \varepsilon A_w w_p \quad (2)$$

where c_d is the contraction coefficient, ε the orifice contraction ratio (A_0/A_w), A_w the free

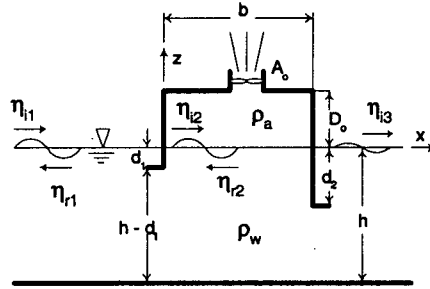


Fig. 1 Definition Sketch

surface area of the water column, and A_0 the area of the orifice. The air velocity, w_p , is determined in terms of the temperature difference as

$$w_p = \beta \phi \sqrt{2c_p |T_a - T_o|} \quad (3)$$

where $\beta = 1$ for outflow and -1 for inflow, T_o is the absolute temperature of the open air, c_p is specific heat at constant pressure ($=1005$ J/kgK), and ϕ is the velocity coefficient of air nozzle. Substitution of the gas equation $\rho_a = p_a / RT_a$ to Eq. 1 yields the flow rate equation as

$$\frac{d}{dt} \left(\frac{p_a V}{RT_a} \right) + \dot{m} = 0 \quad (4)$$

where R is the gas constant ($=287.2$ J/kgK for air). The conservation of energy in the air chamber is described as

$$p_a \frac{dV}{dt} + c_v \frac{d}{dt} \left(\frac{p_a V}{R} \right) + \dot{m} c_p T_a = 0 \quad (5)$$

where c_v is the specific heat at constant volume (=717.1 J/kgK).

For simplicity we assume that the incident harmonic wave has a single frequency, and that the motion response of the ocean water column is also linear to the wave. Further simplification is applied to the volume change, which is a small variation with respect to the total volume so that we can approximate the thermal gas equation.

2.2 Approximation of the Equations

Since Eqs. 4 and 5 are non-linear, it is not easy to find a full solution. To get an approximate solution we assume that the incident wave has a single frequency, the variations of V , T_a and p_a with respect to time t are quite small, and they can be expressed by the following sinusoidal equations as

$$V^* = V - V_0 = -a_2 A_w \sin(\sigma t + \phi_2) \quad (6)$$

where V^* the oscillatory fraction of V , σ the angular velocity of the incident wave, and a_2 and ϕ_2 the relative motion amplitude and the phase of the chamber to the internal water surface elevation. It is assumed that a_2 is much less than the depth of the air chamber D_0 . The consequent variations of temperature and pressure in the chamber are presented with phase shift ϕ_p as

$$T^* = T_a - T_0 = \alpha T_0 \sin(\sigma t + \phi_2 + \phi_p) \quad (7)$$

$$p^* = p_a - p_0 = \nu p_0 \sin(\sigma t + \phi_2 + \phi_p) \quad (8)$$

where α and ν are small compared to 1.0. To get an approximation to α and ν , Eqs. 6, 7, and 8 are substituted into Eqs. 4 and 5. The values of α , ν , and ϕ_p are determined by putting $\sigma t + \phi_2 = 0$ or $\pi/2$ in terms of the relative amplitude a_2 and the frequency σ as

$$\alpha = (\gamma - 1) \frac{a_2}{D_0} \cos \phi_p \quad (9)$$

$$\nu = \gamma \frac{a_2}{D_0} \cos \phi_p \quad (10)$$

$$\cos \phi_p = \pm \sqrt{1 + K^2} - K \quad (11)$$

where

$$K = (c_a \phi \epsilon)^2 (\gamma - 1) c_p \frac{T_a}{(\sigma D_0)^2} \frac{D_0}{a_2} \quad (12)$$

$$\gamma = c_p / c_v \quad (13)$$

It is noted that the above equations are nonlinear with respect to the relative amplitude.

2.3 Incident Wave and Air Response in the Chamber

The waves are described by the wave potentials that are defined according to the region. The regions are divided into the region I, II, and III. The region I is the seaward side of the chamber where the incident wave η_{i1} are incoming to the chamber, and the reflected wave η_{r1} is outgoing from the chamber. The region II is inside the chamber where the ongoing wave η_{i2} and reflected wave η_{r2} are present. The region III is the lee side of the chamber where outgoing wave η_{i3} only presents. Assuming the incident wave height is small compared to the wave length so that we can use linear wave potentials to describe the wave phenomenon as

$$\Phi_{i1} = \text{Re } a_{i1} \frac{g}{\sigma} \frac{\cosh k(z+h)}{\cosh kh} \exp i(\sigma t - kx) \quad (14)$$

$$\Phi_{r1} = \text{Re } a_{r1} \frac{g}{\sigma} \frac{\cosh k(z+h)}{\cosh kh} \exp i(\sigma t + kx) \quad (15)$$

in the region I,

$$\Phi_{i2} = \text{Re } a_{i2} \frac{g}{\sigma} \frac{\cosh k(z+h)}{\cosh kh} \exp i(\sigma t - kx) \quad (16)$$

$$\Phi_{r2} = \text{Re } a_{r2} \frac{g}{\sigma} \frac{\cosh k(z+h)}{\cosh kh} \exp i(\sigma t + kx) \quad (17)$$

in the region II, and

$$\Phi_{i3} = \text{Re } a_{i3} \frac{g}{\sigma} \frac{\cosh k(z+h)}{\cosh kh} \exp i(\sigma t - kx) \quad (18)$$

in the region III, where the wave oscillation frequency σ and the wave number k are used. The exponentially decaying terms in the potentials are neglected. The velocity and pressure at the regions are written as

$$u_I = -\frac{\partial \Phi_{i1}}{\partial x} - \frac{\partial \Phi_{r1}}{\partial x}, \quad u_{II} = -\frac{\partial \Phi_{i2}}{\partial x} - \frac{\partial \Phi_{r2}}{\partial x}, \quad u_{III} = -\frac{\partial \Phi_{i3}}{\partial x} \quad (19)$$

$$p_I = \rho \frac{\partial (\Phi_{i1} + \Phi_{r1})}{\partial t}, \quad p_{II} = \rho \frac{\partial (\Phi_{i2} + \Phi_{r2})}{\partial t} + p_a^*, \quad p_{III} = \rho \frac{\partial \Phi_{i3}}{\partial t} \quad (20)$$

These wave potentials have to satisfy the mass continuity between I and II, and between II and III as

$$\int_{-h}^{-d_1} u_I dz \Big|_{x=0} = \int_{-h}^{-d_1} u_{II} dz \Big|_{x=0} \quad (21)$$

$$\int_{-h}^{-d_2} u_{II} dz \Big|_{x=B} = \int_{-h}^{-d_2} u_{III} dz \Big|_{x=B} \quad (22)$$

respectively, and the pressure continuity as

$$p_I \Big|_{x=0} = p_{II} \Big|_{x=0} \quad (23)$$

$$p_{II} \Big|_{x=B} = p_{III} \Big|_{x=B} \quad (24)$$

From the velocity and pressure continuity, we get the relations

$$-a_{i1} + a_{r1} = -a_{i2} + a_{r2} \quad (25)$$

$$-a_{i2} \exp(-ikB) + a_{r2} \exp(ikB) = -a_{i3} \exp(-ikB) \quad (26)$$

and

$$a_{i1} + a_{r1} = a_{i2} + a_{r2} + \frac{1}{i\rho_w g} f_1(kh, kd_1) p_c^* \quad (27)$$

$$a_{i2} \exp(-ikB) + a_{r2} \exp(ikB) + \frac{1}{i\rho_w g} f_2(kh, kd_2) p_c^* = a_{i3} \exp(-ikB) \quad (28)$$

where

$$f_1(kh, kd_1) = \frac{2 \cosh kh \sinh k(h-d_1)}{\cosh k(h-d_1) \sinh k(h-d_1) + k(h-d_1)} \quad (29)$$

$$f_2(kh, kd_2) = \frac{2 \cosh kh \sinh k(h-d_2)}{\cosh k(h-d_2) \sinh k(h-d_2) + k(h-d_2)} \quad (30)$$

and p_c^* is the amplitude of the harmonically varying pressure with $p^* = p_c^* e^{i\sigma t}$.

The mean wave elevation in the chamber is in complex form as

$$a_0^c = \frac{1}{B} \int_0^B \frac{1}{g} \frac{\partial (\Phi_{i2} + \Phi_{r2})}{\partial t} \Big|_{z=0} dx = \frac{1}{kB} (-a_{i2} (e^{-ikB} - 1) + a_{r2} (e^{ikB} - 1)) \quad (31)$$

Then the real time elevation will be as

$$\eta(t) = \text{Re } a_0^c e^{i\sigma t} \quad (32)$$

2.4 Motion of the Air Chamber

The floating water chamber, positioned by mooring cables, is subject to the incident waves. The six degrees of device motion is affected by the internal air pressure as well as the incident wave. For simplification, only the heave motion is considered as the interaction and the motion response is linear as

$$m_H \ddot{z} + c_H \dot{z} + k_H z = a_1 k_H r_f \sin(\sigma t - \phi_1 + kB/2) + p^* A_w \quad (33)$$

where m_H is the mass of the chamber including the hydrodynamic mass effect, z is the heave motion of the chamber, c_H is hydrodynamic damping coefficient for heave, $k_H = \rho_w g A_H$ is the heave static restoring coefficient, g is gravitational acceleration, A_H is the cross sectional area of the chamber at the waterplane, and r_f is the factor of the wave shape to the exciting force. Using the relation Eq. 8 and Eq. 10, the solution to Eq. 33 is rewritten as

$$m_H \ddot{z} + c_H \dot{z} + k_H z = \text{Re } k_H r_f a_{i1} i e^{i(\sigma t - kB/2)} + \text{Re } A_w \frac{\gamma p_0}{D_0} \cos \phi_p a_2^c e^{i(\sigma t + \cos \phi_p)} \quad (34)$$

since

$$p^* = \text{Re } \frac{\gamma}{D_0} \cos \phi_p a_2^c e^{i(\sigma t + \cos \phi_p)} \quad (35)$$

The motion amplitude is obtained as

$$z^c = (-m_H \sigma^2 + ic_H \sigma + k_H)^{-1} \{ k_H r_f a_{i1} i e^{-ikB/2} + A_w \frac{\gamma p_0}{D_0} \cos \phi_p a_2^c e^{i \cos \phi_p} \} \quad (36)$$

where the motion amplitude z^c of the chamber be defined as $z(t) = \text{Re } z^c e^{i\sigma t}$. The relative motion of the air volume is $-a_2 A_w \sin(\sigma t + \phi_2) = -a_0 A_w \sin \sigma t + A_w z(t)$. This equation can be rewritten as

$$-\text{Re } a_2^c e^{i\sigma t} = -\text{Re } a_0^c e^{i\sigma t} + \text{Re } z^c e^{i\sigma t}$$

or

$$-a_2^c = -a_0^c + z^c \quad (37)$$

without the loss of generality. The equations 25, 26, 27, 28, 31, 36, and 37 comprise a simultaneous system equation for 8 unknowns. By fixing the incident waves amplitude a_{i1} , the system equation is solved iteratively.

length	(l_B)	0.28m	$\epsilon = 0.01$
breadth	(B)	0.08m	
height	(D_0)	0.2m	
front wall depth	(d_c)	0.02m	
wave amplitude =	(a_j)	0.01, 0.015m	
water depth	(h)	0.16	

Table 1. The Dimension of the Slopped Front Wall Caisson

3 Numerical Results

The numerical computation is carried out on a chamber of which dimensions(11) are shown in

Table 1.

The computational results of the motion responses are shown in Fig. 2. When the orifice contraction ratio ϵ is about 0.9, the motion of the chamber is not affected by the chamber air. When $\epsilon=0.01$, the motion response is highly dependent on chamber air interaction. This method, however, does not give reasonable results in the efficiency computations. Further research has to be done for applications

3 Conclusion

This paper proposes an analysis on the floating wave energy conversion system. The effect of the wave diffraction and chamber motion are focussed in the analysis formulation. Introducing the linear motion dynamics of the floating chamber to the approximated nonlinear thermal gas equation in the chamber, an explicit form of governing equation is derived and is feedback to the simplified wave dynamics. The wave profiles in and out of the chamber are formulated as the sum of ongoing and reflected waves. The result shows the difference in the chamber motion response due to the orifice ratio.

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