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An Improved Pipe Hoop Stress Formula

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Abstract

The ASME B31.4 [1] and B31.8 [2] codes use the thin wall formula to predict hoop stress in a pipe. To account for the external pressure, the above codes simply subtract the external pressure from the internal pressure. The thin wall formula using this differential pressure does not give the same hoop stress as the thick wall formula.

This paper proposes an improved equation to predict pipe hoop stress subjected to both internal and external pressure. Compared to the conventional thin wall formula, the improved formula has additional terms, which improve the agreement with the thick wall formula and account for external pressure. The improved formula is less conservative than the conventional thin wall formula, but slightly more conservative than the thick wall formula. The formula is simpler and easier to use than the thick wall formula and will save pipe material cost as well as installation cost compared to using the conventional thin wall formula. The savings will increase as the water depth increases.

Nomenclatures

D	Pipe outside diameter
D_i	Pipe inside diameter
P_d	Design differential pressure = P_i - P_o
P_o	External pressure
P_i	Internal pressure
S	Pipe specified minimum yield strength (SMYS)
t	Pipe nominal wall thickness
σ_h	Pipe hoop stress

1. Introduction

Subsea pipelines must be designed for installation, hydrotest, and operation. In some cases, the pipe wall thickness may be governed by the internal design pressure or the hydrotest pressure, depending on design factors and code requirements. Pipe wall thickness may also be affected by installation stress limits or free span stress limits due to sea bottom irregularities. If the external pressure exceeds the internal pressure, the pipe must be checked for collapse.

For marine pipelines, the effect of external pressure should be considered in the pipe wall thickness determination. Even though external pressure is considered in the some ASME codes these codes require conservatively heavier wall pipe.

As shown in Figure 1, the hoop stress is not same even though the differential pressure is the same. In the above example, the differential pressure is zero (P_i - P_o = 0 psi), but the hoop stress is -1,000 psi or -3,000 psi. The minus sign indicates that the pipe is in compression. This example illustrates that the hoop stress is a function of not only differential pressure but also external pressure. The external pressure benefits to the pipe wall determination in terms of the internal pressure resistance. In other words, accounting for the external pressure correctly reduces the pipe wall thickness. The role of the external pressure escalates as the water depth increases.

The higher external pressure may require thicker pipe wall due to collapse resistance, and this check should be done before finalizing the wall thickness. This paper focuses only on the minimum pipe wall thickness needed for internal and external pressure. Wall thickness determination to resist collapse buckling is out of the scope of this paper.

The ASME B31.4 and B31.8 codes give the same hoop stress for the same differential pressure, regardless of the magnitude of the external pressure. This paper proposes an improved hoop stress formula to predict pipe wall thicknesses more accurately, for any water depth. The accuracy of the improved formula compared to the thick wall formula is presented for various P_o/P_i ratios.

2. Pipe Hoop Stress Formulas

When a pipe is subjected to internal pressure, a hoop stress will be induced across the pipe wall thickness. As shown in Figure 2, the hoop forces will be in equilibrium with the Y-component forces of the internal pressure, which acts on the pipe ID.

Eq. 1 below is called the "thin wall pipe formula". It is derived assuming a uniform hoop stress across the pipe wall with no external pressure.

$$2\sigma_h t = P_i D_i \Rightarrow \sigma_h = \frac{P_i D_i}{2t} \qquad \text{Eq. 1}$$

ASME B31.4 and B31.8 codes conservatively revise Eq. 1 to use the pipe OD, as follows:

$$\sigma_h = \frac{P_i D}{2t} \qquad \text{Eq. 2}$$

Equation 2 provides reasonable results for D/t ratios greater than 20, with no external pressure.

To account for external pressure, the ASME B31.4 and B31.8 codes simply substitute the pressure differential across the pipe wall for the "P_i" term, as follows:

$$\sigma_h = \frac{(P_i - P_o)D}{2t} \qquad \text{Eq. 3}$$

However, in keeping with thin wall theory, the internal and external pressure forces act respectively on the inner and outer diameter, so the following equation should be used to account for the external pressure properly.

$$2\sigma_h t = P_i D_i - P_o D \Rightarrow \sigma_h = \frac{P_i D_i - P_o D}{2t} \qquad \text{Eq. 4}$$

Eq. 4 contains two unknowns (D_i and t) and it is not simple to solve for the pipe wall thickness t. However, substituting D_i = D − 2t and algebraic manipulation reduces the equation to the following:

$$\sigma_h = \frac{(P_i - P_o)D}{2t} - P_i \qquad \text{Eq. 5}$$

Note that Eq. 5 has an additional "$-P_i$" term compared to the thin wall formula used in the ASME B31.4 and B31.8 codes. This equation is recommended by API RP 2RD [3].

ASME B31.3 [4] and ASME Boiler and Pressure Vessel (BPV) Code, Section VIII-Division 1 [5] recommend still another hoop stress formula for D/t ratios greater than 6, for internal pressure only.

$$\sigma_h = \frac{P_i D}{2t} - 0.4 P_i \qquad \text{Eq. 6}$$

Substituting the differential pressure, $P_i - P_o$, for the internal pressure yields:

$$\sigma_h = \frac{(P_i - P_o)D}{2t} - 0.4(P_i - P_o) \qquad \text{Eq. 7}$$

This equation appears similar to Eq. 5, except for the last term.

As shown in Figure 3, the thick wall formula for hoop stress (Lame's Equation) is as follows:

$$\sigma_h = \frac{P_i a^2 - P_o b^2 + a^2 b^2 (P_i - P_o)/r^2}{b^2 - a^2} \qquad \text{Eq. 8}$$

The above formula accurately predicts pipe wall hoop stresses at a given radius (positive stresses indicate tension and negative stresses indicate compression). The absolute hoop stress compression or tension, is always maximum at the inner wall surface. The hoop stress varies across the pipe wall and the difference between the inner wall surface and the outer wall surface is the same as the pressure differential (P_i - P_o). Since the pipe is to be designed for maximum stress across the wall, the thick wall pipe formula with r = a, at the inner pipe wall surface, should be used to determine the pipe wall thickness.

By substituting r = a = $D_i/2$, b = D/2, and D = D_i + 2t, followed by considerable algebraic manipulation, Eq. 8 can be rewritten as follows for hoop stress at the pipe ID.

$$\sigma_h = \frac{(P_i - P_o)D}{2t} - 0.5(P_i + P_o) + \frac{(P_i - P_o)t}{2(D-t)} \quad \text{Eq. 9}$$

The Eq. 8 and Eq. 9 give precisely the same hoop stress results at the pipe ID for a given pipe subjected to the same pressures.

3. Improved Pipe Hoop Stress Formula

Neglecting the last term and rewriting the second term with a differential pressure in the thick wall formula (Eq. 9) produce the following equation:

$$\sigma_h = \frac{(P_i - P_o)D}{2t} - 0.5(P_i - P_o) - P_o \qquad \text{Eq. 10}$$

Reconsidering the neglected last term in the thick wall formula by using a smaller coefficient for the second term in the above equation yields:

$$\sigma_h = \frac{(P_i - P_o)D}{2t} - 0.4(P_i - P_o) - P_o \qquad \text{Eq. 11}$$

The above equation has both differential pressure and external pressure terms and will be called the "improved pipe hoop stress formula" in this paper. This equation has additional "$-P_o$" term compared to the modified ASME B31.3 and BPV code formula (Eq. 7).

4. Comparison of Pipe Hoop Stress Formulas

Figure 4 illustrates hoop stresses calculated from various formulas. Figure 5 presents the accuracy of the calculated hoop stresses from various formulas compared to thick wall formula, for a D/t=20 pipe.

The ASME B31.4 and B31.8 thin wall pipe formula gives approximately 10 percent higher stress than the thick wall formula, at P_o/P_i = 0.3. As the external pressure increases, the conservatively high stress from the ASME B31.4 and B31.8 hoop stress formula increases exponentially. This indicates that the ASME B31.4 and B31.8 hoop stress formula should be reviewed for deep water application.

The ASME B31.3 and BPV Code hoop stress formula (Eq. 7) shows almost the same trend with slightly lower hoop stress than the ASME B31.4 and B31.8 formula.

The API RP-2RD (Eq. 5) gives approximately 6 percent lower stress than the thick wall formula, at P_o/P_i = 0.3

The proposed improved formula (Eq. 11) provides very accurate results: maximum 2 percent higher hoop stress than thick wall formula for any P_o/P_i and D/t ratios.

The improved hoop stress formula appears slightly more complicated than the thin wall formula; however, it's use in deepwater should be very beneficial. The example below illustrates the benefit of using the improved hoop stress formula.

Example 1

Given> Determine pipe wall thickness due to internal and external pressures.

6.625-inch OD flowline, API 5L X60
Internal pressure P_i = 6,000 psi at wellhead
Water depth = 4,500 ft

Solution> The allowable hoop stress for flowline is 0.72 x 60,000 = 43,200 psi

Differential pressure $\Delta P = P_i - P_o$
= 6,000 - 4,500(64/144) = 6,000 - 2,000 = 4,000 psi
(P_o/P_i = 2,000/6,000 = 0.33)

(1) Using the ASME B31.4 and B31.8 thin wall pipe formula (Eq. 3),

$$\sigma_h = \frac{(4{,}000)6.625}{2t} \le 43{,}200 \rightarrow t \ge 0.307''$$

(2) Using the API RP 2RD hoop stress formula (Eq. 5),

$$\sigma_h = \frac{(4{,}000)6.625}{2t} - 6{,}000 \le 43{,}200 \rightarrow t \ge 0.269''$$

(3) Using the modified ASME B31.3 and BPV Code formula (Eq. 7),

$$\sigma_h = \frac{(4{,}000)6.625}{2t} - 0.4(4{,}000) \le 43{,}200$$

$$\rightarrow t \ge 0.296"$$

(4) Using the thick wall pipe formula (Eq. 9) with trial and error solution,

$$\sigma_h = \frac{(4{,}000)\,6.625}{2t} - 0.5(8{,}000) + \frac{4{,}000t}{2(6.625-t)}$$

$$\le 43{,}200 \rightarrow t \ge 0.281"$$

(5) Using the improved pipe hoop stress formula (Eq. 11),

$$\sigma_h = \frac{(4{,}000)\,6.625}{2t} - 0.4(4{,}000) - 2{,}000 \le 43{,}200$$

$$\rightarrow t \ge 0.283"$$

As illustrated in the above example, the improved pipe hoop stress formula result (0.283") matches well with the thick wall formula solution (0.281"). It also saves approximately 8 percent of the pipe material cost compared to the thin wall pipe formula (0.307").

5. Conclusions and Recommendation

The ASME B31.4 and B31.8 thin wall pipe formula is not derived specifically to account for external pressure and predicts the most conservative hoop stress compared to any other code. The ASME B31.3 and BPV Code predict less hoop stress than the ASME B31.4 and B31.8 codes, but still predict higher hoop stress than the thick wall formula. However, the API RP 2RD predicts lower hoop stress than the thick wall formula. The inconsistency between codes and conservatism compared to the thick wall formula inspired an improved pipe hoop stress formula presented in this paper.

The improved pipe hoop stress formula (Eq. 11) is presented to properly account for the external pressure. The improved pipe hoop stress formula predicts reasonably accurate hoop stresses with little extra effort. The improved formula will save pipe material and installation costs. For a D/t ratio of 20 and P_o/P_i = 0.3, the improved formula will save approximately 10 percent of the pipe material cost over the ASME B31.4 and B31.8 formula. The savings will increase as the water depth increases.

6. References

[1] ASME B31.4, "Pipeline Transportation Systems for Liquid Hydrocarbons and Other Liquids," 1992, The American Society of Mechanical Engineers, New York.

[2] ASME B31.8, "Gas Transmission and Distribution Piping Systems," 1992, The American Society of Mechanical Engineers, New York.

[2] API RP 2RD, "Design of Risers for Floating Production Systems (FPSs) and Tension-Leg Platforms (TLPs)," 1998, American Petroleum Institute

[4] ASME B31.3, "Chemical Plant and Petroleum Refinery Piping," 1990, The American Society of Mechanical Engineers, New York

[5] ASME Boiler and Pressure Vessel Code, Section VIII – Rules for Construction of Pressure Vessels, Division 1, Appendix 1, 1998, The American Society of Mechanical Engineers, New York.

Figure 1
Hoop Stress Variations for Same Differential Pressure

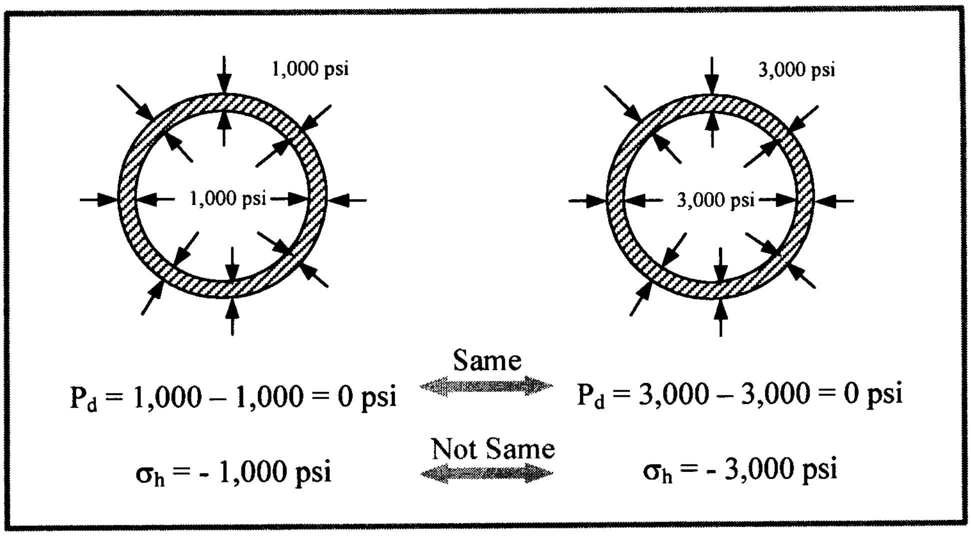
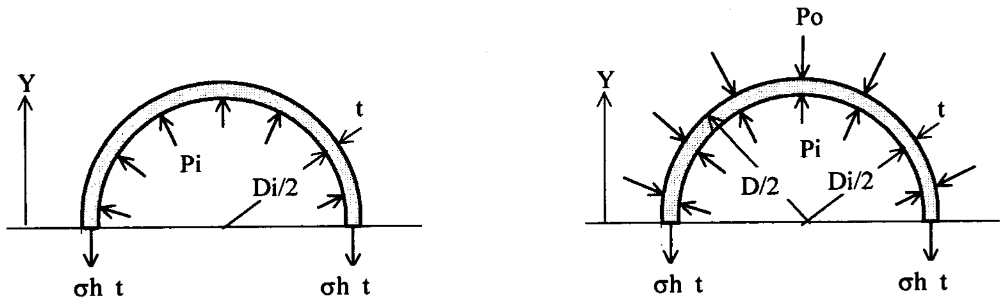


Figure 2
Thin Wall Pipe Hoop Stress Diagram



(a) Internal Pressure only

(b) Internal + External Pressures

Figure 3
Thick Wall Pipe Hoop Stress Diagram

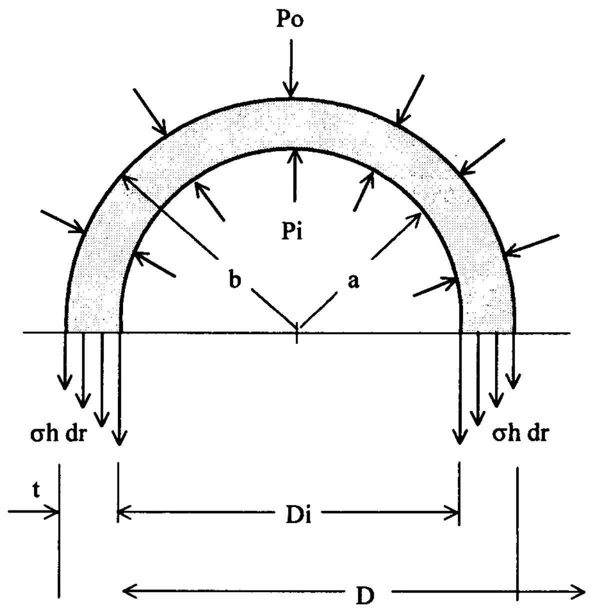


Figure 4
Hoop Stresses from Various Formulas

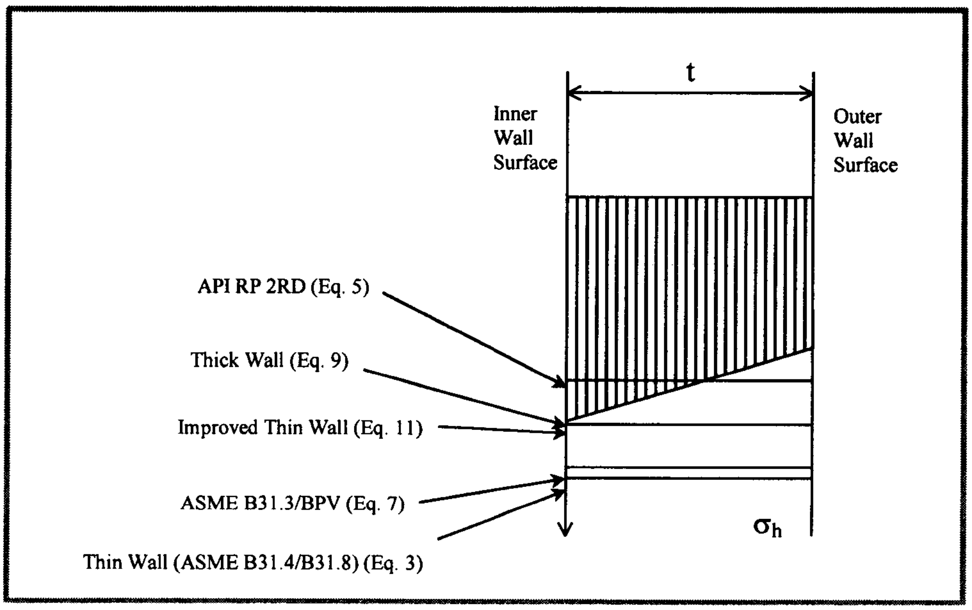
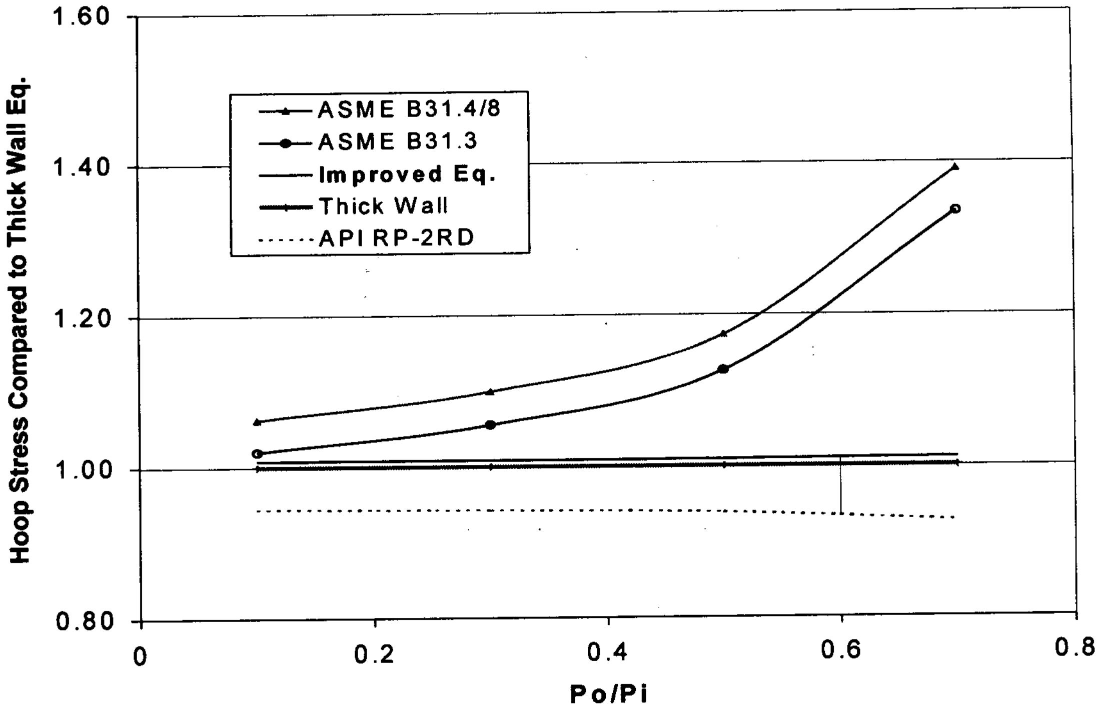


Figure 5



Hoop Stress Comparison (D/t = 20 Pipe)