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Effect of Specimen Thickness by Simulation of Probabilistic Fatigue Crack Growth

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ABSTRACT: *The evaluation of specimen thickness effect of fatigue crack growth life by the simulation of probabilistic fatigue crack growth is presented. In this paper, the material resistance to fatigue crack growth is treated as a spatial stochastic process, which varies randomly on the crack surface. Using the previous experimental data, the non-Gaussian (eventually Weibull, in this report) random fields simulation method is applied. This method is useful to estimate the probability distribution of fatigue crack growth life and the variability due to specimen thickness by simulating material resistance to fatigue crack growth along a crack path.*

1. INTRODUCTION

An accurate assessment of fatigue crack growth behavior is important to the structural design of fatigue sensitive components. The analysis is accomplished using a fracture mechanics approach that is based on derived fatigue crack growth rate data (Paris and Erdogan, 1963). However, these data, regardless of how carefully they are generated, show considerable scatter which depends on various uncontrolled factors, such as material properties and metallurgical structure, type of loading, environment, and so on. Currently, this scatter of fatigue crack growth data is commonly regarded as an inherent feature of fatigue crack growth process (Sobczyk, 1993).

During the last two decades, experimental and theoretical studies on the randomness of fatigue crack growth have been reported (Virker, Hillbery and Goel, 1979; Tanaka, Ichikawa and Akita, 1981; Kozin and Bogdanoff, 1989; Ortiz and Kiremidjian, 1989; Lapetra Mayo and Dominguez, 1996; Kim, 1999). Such a model assumes the fatigue crack growth as a random process and, therefore, needs to know the random nature of material parameters before any practical application. For this purpose not only mean and variance of crack growth rate but also the spatial distribution of resistance are necessary. The authors studied the spatial correlation of fatigue crack growth resistance of BS 4360 steel (Kim *et al.*, 1993). The results clearly indicate that the effect of autocorrelation function of fatigue crack growth resistance on the specimen thickness has no significant influence, but the variance increases with decreasing specimen thickness. And, the probability distribution functions of the fatigue crack growth resistance obtained from the experimental data are 3-parameter Weibull and show a slight dependence on the specimen thickness of BS 4360 steel.

The effects of specimen thickness on fatigue crack growth have been investigated by many workers (Putatunda and Rigsbee, 1985; Sasaki *et al.*, 1991; Mcmaster *et al.*, 1998; Shim and Kim, 1998) and the results reported in the literature are contradictory in nature. Some workers have reported that specimen thickness had no effect, whereas others have reported either an increase or decrease in the crack growth rate with increasing specimen thickness. But, it was found that there is scatter in their results. And also, the variability of fatigue crack growth life seems to increase with decreasing specimen thickness. However, most of the studies were carried out under constant amplitude loading, and they did not consider the statistical properties of fatigue crack growth resistance and the effect of specimen thickness on the parameters of probability distribution for crack growth life.

The purpose of the present study is, therefore, to evaluate the effect of specimen thickness on the probability distribution of fatigue crack growth life by using the non-Gaussian (especially, Weibull) random fields simulation method (Yamazaki and Shinozuka, 1986; Itragaki *et al.*, 1990; Kim *et al.*, 2000). Applying the previous experimental data (Kim, Itagaki and Ishizuka, 1993), the fatigue crack growth curves were simulated and analyzed for the different specimen thicknesses to determine the probability distribution functions of the fatigue crack growth life. And also, the effect of specimen thickness on the parameter of fatigue crack growth life distribution was investigated.

2. PROBABILISTIC MODEL

Assuming Paris' law (Paris and Erdogan, 1963), the crack growth rate is

$$\frac{da}{dN} = C(\Delta K)^m \quad (1)$$

where, C and m are the material constants, a is the fatigue crack length, and N is the number of cycles to lead. The material constants m and C, hereinafter, called the growth rate exponent and coefficient, respectively, are assumed random. For one dimensional model of fatigue crack growth, they are the random functions of crack length. It is, however, very difficult to determine separately these two random variables from the observed crack growth data even under the test for the condition of constant stress intensity factor range (Kim, 1999). And also, for this purpose many experimental data are necessary. Since the present study is to investigate the effect of specimen thickness on inhomogeneity of fatigue crack growth resistance, it seems unnecessary to use the stochastic model considering the probabilistic properties of the parameters. Therefore, in the present study, for random variables m and C, taking expectation of da/dN gives:

$$\overline{\frac{da}{dN}} = \int_0^\infty \int_0^\infty \left(\frac{da}{dN}\right) \cdot f_C(C) \cdot f_m(m)\, dC dm \quad (2)$$

where, $f_C(\cdot)$ and $f_m(\cdot)$ are the probability density function of random variables C and m, respectively. If ΔK is in constant condition ahead of the crack path, the expectation values are constant. Determining the value of $\overline{da/dN}$ from experimental data for each specimen thickness, we introduce the dimensionless parameter, $S(x)$, which means the inhomogeneity of material properties to fatigue crack growth in front of the crack, then da/dN is written by

$$\frac{da}{dN} = \frac{1}{S(x)} \cdot \overline{\frac{da}{dN}} \quad \text{or} \quad S(x) = \frac{dN}{da} \cdot \overline{\frac{da}{dN}} \quad (3)$$

hereinafter, $S(x)$, is called as the growth resistance coefficient of material to fatigue crack growth, namely the crack growth resistance coefficient.

The spatial stochastic process $S(x)$ is assumed to be a stationary and ergodic process but not necessarily Gaussian, and its autocorrelation function and probability distribution function are determined from experimental data. Using these properties, the random process $S(x)$ is simulated, and then, the fatigue crack growth.

3. PREVIOUS EXPERIMENTAL DATA

In the following the experimental results with a high tensile strength steel, BS 4360 (GR50D), for marine structures are cited from reference (Kim, Itagaki and Ishizuka, 1993) as an example.

The experimental results are listed in Table 1. The test variables are given in the table.

Table 1 Test variables and the results

Thickness (mm)	Specimen No.	FCGR ($\times 10^{-4}$mm/cycle)	COV of $S(x)$ (%)
18	2BS01	1.62	7.9
	2BS02	1.73	9.8
	2BS03	1.71	10.8
	2BS04	1.78	7.9
	2BS05	1.80	8.3
	2BS06	1.74	10.7
	2BS07	1.73	9.5
	2BS09	1.76	10.5
	2BS10	1.64	12.4
	2BS11	1.59	11.3
12	12BS1	1.24	12.9
	12BS4	1.48	16.4
	12BS5	1.38	13.2
	12BS6	1.31	6.4
6	6BS1	1.45	11.4
	6BS2	1.33	15.8
	6BS3	1.16	15.1

One of the examples of the obtained data is shown in Figure 1, the growth rate, da/dN is plotted against crack length, a, together with $\Delta K/\overline{\Delta K}$. As shown in the figure, the range of the stress intensity factor is well controlled and its coefficient of variation is about 0.2 percent. Even under these carefully controlled conditions, the observed crack growth rates have remarkable fluctuations. Throughout the tests all the same figures are obtained.

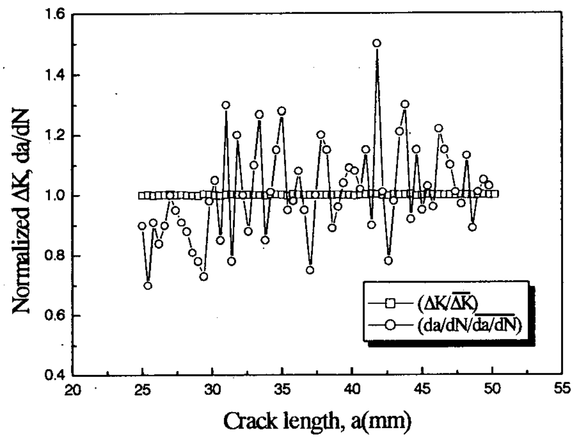

Figure 1 Fatigue crack growth rate versus crack length

The autocorrelation function reflects the correlation between observations of the stochastic process. It was clear from the previous results (Kim, Itagaki and Ishizuka, 1993) that the autocorrelation functions are almost independent of the specimen thickness of the BS 4360 steel, except for the origin, R(0). The variance increases with decreasing specimen thickness. The rate of decay was very rapid. The exponential function seems to be a reasonable shape.

The probability distribution functions of the material resistance coefficients obtained from the experimental data are three parameter Weibull and showed a slight dependence on the specimen thickness (Kim, Itagaki and Ishizuka, 1993). The distribution function is expressed as:

$$F_s(S|\alpha, \beta, \gamma) = 1 - \exp\left[- \left(\frac{S-\gamma}{\beta-\gamma} \right)^{\alpha} \right] \qquad (4)$$

where, α is the shape parameter, β is the scale parameter and γ is the location parameter.

4. THE RESULTS OF SIMULATION AND DISCUSSION

To simulate the fatigue crack growth in a given specimen thickness, it is necessary to generate a series of $S(x)$ along the crack path. For the simulation of $S(x)$, a non-Gaussian random process simulation method proposed by Yamazaki (Yamazaki and Shinozuka, 1986) are applied for random process $S(x)$ with the obtained autocorrelation function, $R(\tau) = \exp(-1.0|\tau|)$, and probability distribution function, $F_s(S|\alpha,\beta,\gamma)$($\alpha$=4.5, β=1.05, γ=0.61 for 6mm specimens; α=4.2, β=1.05, γ=0.54 for 12mm specimens; α=4.0, β=1.07, γ=0.52 for 18mm specimens), which was obtained by Kim *et al.* (1993). In the simulation, the FFT size is taken 2048. The crack length increment, Δa, is fixed to 0.4mm.

Given a initial crack length a_0, the relation between the crack length, a, and the number of cycles, N, can be determined by the simulated data of $S(x)$ (Kim *et al.*, 2000). 100 simulated $a \sim N$ curves for each specimen thickness are shown in Figure 2, where the crack length a_0 is 25.0mm, and the predetermined crack length a_f is 50.2mm. Comparing the results of the simulations with the previous experimental data (Kim *et al.*, 1993), it can be said that the simulated $a \sim N$ curves describe the experimental data very well. From these curves the probability distribution of fatigue crack growth life can be estimated. The variability of fatigue crack growth life seems to increase with decreasing the specimen thickness.

As one of the examples, the fatigue crack growth lives, $N(a)$, for 6, 12 and 18mm thickness obtained by the simulation are plotted on the Weibull probability paper in Figure 3. It is thought that the location parameter, γ, in the Weibull distribution function must be used. Therefore, 3-parameter Weibull distribution function is used to fit the data. The parameters α, β and γ are estimated by the direct search of optimization method (Buto, Sugie and Okazaki, 1977). The estimated functions are also shown

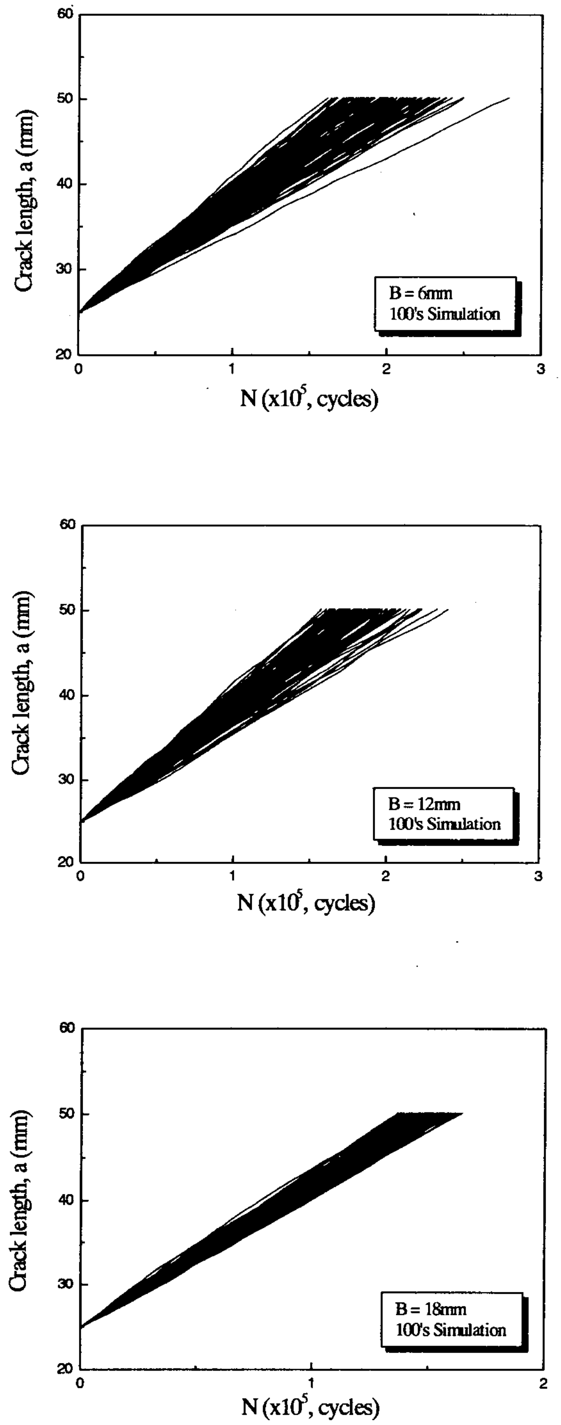

Figure 2 a-N curves under constant ΔK

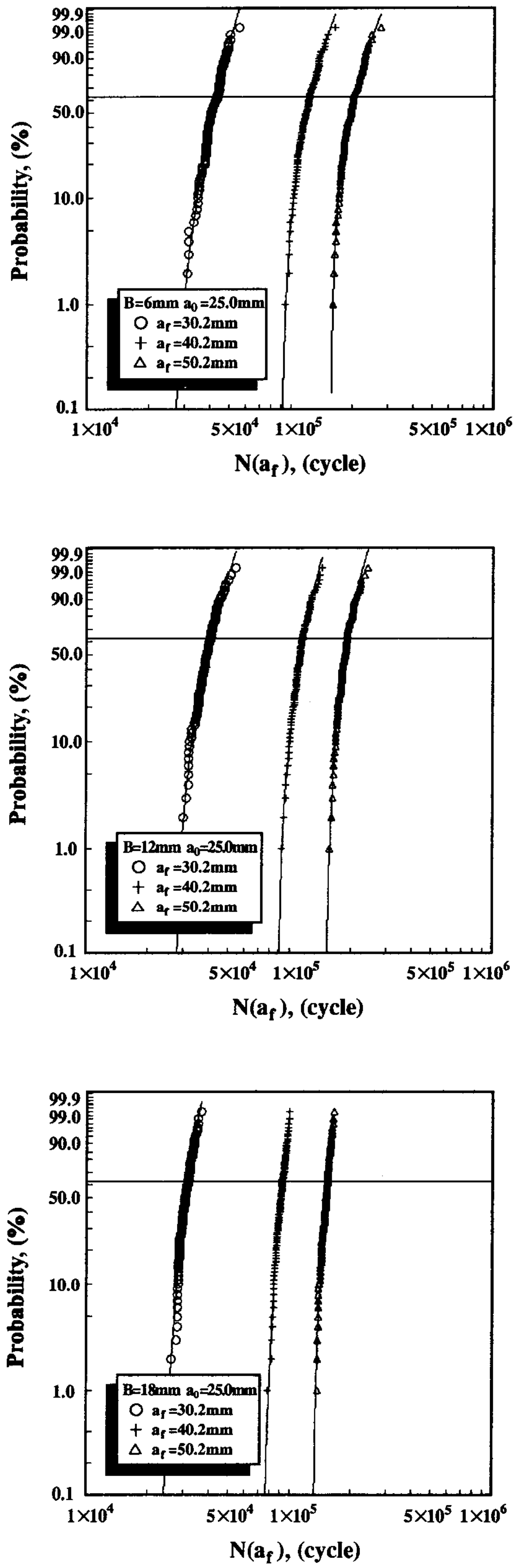

Figure 3 Weibull plots of the simulated life ($\Delta K = 28 MPa\sqrt{m}$)

Table 2 The estimated Weibull parameters (a_0=25.0mm)

Thickness (mm)	final crack (a_f)	Parameter of Weibull Distribution α	β	γ
18	30.2	4.24	31705	22323
	40.2	3.99	91793	73391
	50.2	3.06	151947	130948
12	30.2	2.68	40617	27085
	40.2	2.68	116721	87057
	50.2	2.47	192544	151415
6	30.2	3.62	42897	24704
	40.2	2.32	124026	90060
	50.2	1.92	204448	157157

in the Table 2. It can be said that the fatigue crack growth life, $N(a)$, follows 3-parameter Weibull distribution.

The simulation method is useful for estimation of the probability distribution of fatigue crack growth life and reliability assessment of structures by simulating material resistance to fatigue crack growth.

Figure 4 shows the effect of specimen thickness on Weibull probability of the fatigue crack growth life. It is evident from this figure that there is no negligible influence of specimen thickness on the probability distribution of the fatigue crack growth life. The Weibull shape parameter, α, is increased by increasing the specimen thickness, but the scale parameter, β, and location parameter, γ are decreased.

Figure 5 shows the effect of the specimen thickness on the variance of fatigue crack growth life. Our results indicate that specimen thickness have significant influence on the variance of fatigue crack growth life for BS 4360 steel. The variance increases with decreasing specimen thickness. This is a good agreement with the experimental results.

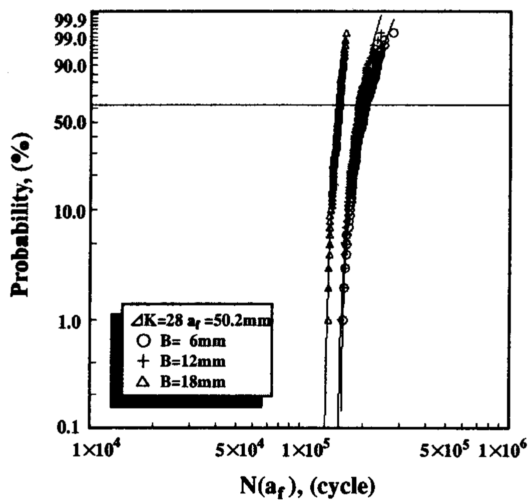

Figure 4 Effect of specimen thickness on Weibull probability distribution of the fatigue crack growth life

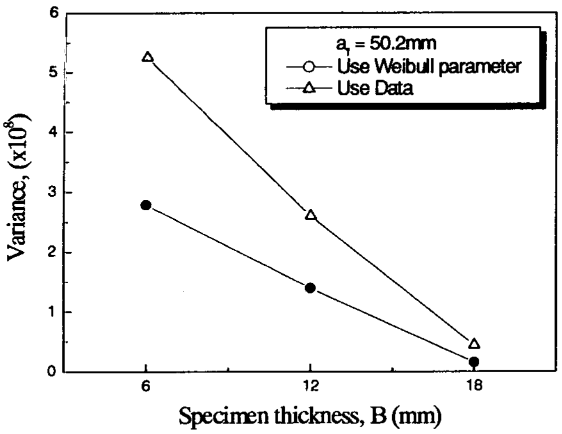

Figure 5 Effect of specimen thickness on the variance of fatigue crack growth life

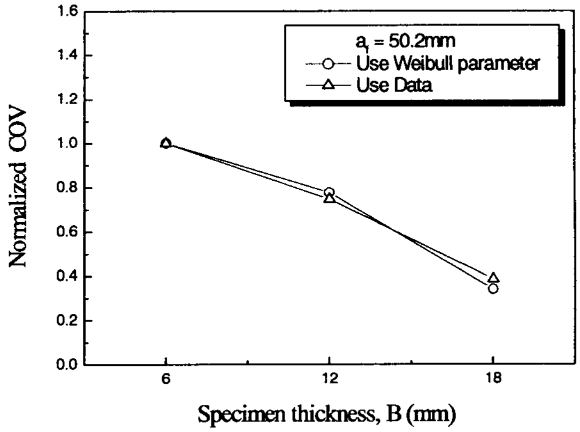

Figure 6 Effect of specimen thickness on the coefficient of variation (COV) of fatigue crack growth life

Figure 6 shows the effect of specimen thickness on the normalized COV for 6mm thickness. In these figures the coefficient of variation is decreased by increasing the specimen thickness. It is evident that specimen thickness has apparent influence on fatigue crack growth life of BS 4360 steel in the thickness range investigated in this study. And also, the coefficient of variation of $N(a)$ is observed to decrease as the crack grows longer. But, for the 6mm specimen thickness, the coefficient of variation of $N(a)$ is almost same although the crack grows. The coefficient of variation for specimen of 6mm thickness is approximately 12 percent for the first 32..6mm growth increment and approximately 11.5 percent at the final crack length.

5. CONCLUSIONS

For the simulation of probabilistic fatigue crack growth, the non-Gaussian random process simulation method is applied for random process $S(x)$ with the obtained autocorrelation function and probability distribution function. The probability distribution of the fatigue crack length after a given number of load cycles or that of the number of load cycles for a crack to reach a given length can be estimated by repeating such simulations. The merit of the presented method is that only a small number of tests are required to estimate the probability distribution function of fatigue crack growth life.

The probability distribution function of the fatigue crack growth life seems to follow a 3-parameter Weibull and show a slight dependence on the specimen thickness. The shape parameter, α, is increased by increasing the specimen thickness, but the scale parameter, β, and location parameter, γ are decreased. The variance of fatigue crack growth life, $N(a)$, and the coefficient of variation is decreased by increasing the specimen thickness.

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