LS code pair setting and sequential allocation methods

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Abstract

A new code: LS code was proposed for IMT-2000 CDMA system. The code has special properties during a certain time of interval: 1) perfect autocorrelation 2) perfect crosscorrelation. The perfect autocorrelation means that the autocorrelation has maximum for zero time-offset and zero for other times during a certain time. Moreover the perfect crosscorrelation means that the crosscorrelation has zero during a time of interest.

In the LAS-CDMA system, the LS code is only used in the spreading of data bits in contrast to the conventional CDMA system. Therefore the LS code pair setting and allocation order should be dealt with carefully considering the special properties of LS code.

This paper is intended as an investigation of the setting LS code pair and the sequential allocation method. Firstly, the optimum LS code pair set is proposed in order to minimize PAPR. Secondly, the sequential allocation method is studied to either minimize PAPR or expand IFW.

I. Introduction

LAS(Large Area Synchronization)-CDMA has been proposed as one of candidates for the IMT-2000 standard. LS(Large Synchronization) code is one of key features of LAS-CDMA and has been introduced for the first time in the 3GPP2.[1][2] But little is known about the LS code. Therefore my last study was made on the exact generation method and the several properties of LS code. [3]

What seems to be lacking for LS code studies is the optimum LS code pair setting and the efficient allocation method. As the first step in our analysis, we will devote some space to the discussion of optimum code pair setting in order to minimize the PAPR(peak-to-average power ratio) value. Next let us focus on the efficient sequential allocation of LS code so as to expand temporarily the IFW(interference free window) length.

II. LS Code

Here we revisit shortly LS code. The detailed generation method of LS code was given in [3].

2.1. LS code without IFW(LS code matrix)

LS code with length $LS(=2^m)$ has the number of N codes ($m=2,3,4,\cdots$). LS code matrix LS^N that each row vector represents LS code is defined as follows.

$$LS^N = \begin{bmatrix} LS_0^N \\ \vdots \\ LS_{N-1}^N \end{bmatrix} \quad (1)$$

where LS^N is a $N\times N$ matrix, LS_0^k $(k=0,\cdots,N-1)$ is a row vector expressing the $K-th$ LS code.

2.2. LS code with IFW

In order to make an IFW(Interference Free Window), guard component with the value of zero should be inserted in the LS code. Owing to the guard component with the length L_{GUARD} (L_{GUARD} is a positive integer), the total LS code length increase to the $N(=2^m)+2\times L_{GUARD}$. LS code matrix with guard component is shown below.

$$LS^{N+2\times L_{GUARD}} = \begin{bmatrix} LS_0^{N+2\times L_{GUARD}} \\ \vdots \\ LS_{N-1}^{N+2\times L_{GUARD}} \end{bmatrix} \quad (2)$$

where $LS^{L+2\times L_{GUARD}}$ is a $N\times(N+2\times L_{GUARD})$ matrix, $LS_k^{N+2\times L_{GUARD}}$ ($k=0,\cdots,N-1$) is a $1\times(N+2\times L_{GUARD})$ row vector expressing the $k-th$ LS code.

III. LS code property

LS code has the special properties: perfect autocorrelation and perfect crosscorrelation. The perfect crosscorrelation means the crosscorrelation value is zero during certain time interval. If the crosscorrelation has zero value in some interval, you can mathematically remove all the MAI(Multiple Access

Interference). So, this time interval of interest is called IFW(Interference Free Window). But there is a tradeoff between the IFW length and the number of available LS codes. In order to lengthen the IFW, it is required to decrease the number of available codes or increase the length of guard component.

The detailed properties of LS code were studied in [3].

Ⅳ. LS code pair setting

4.1. LS Code Set Selection

The aperiodic crosscorrelation between $k-th$ LS code $LS_k^{N+2\times L_{GUARD}}$ and $\ell-th$ LS code $LS_\ell^{N+2\times L_{GUARD}}$ with the length of $N(=2^m)+2\times L_{GUARD}$ is defined in (3)

$$C_{k,\ell}(\tau)=\begin{cases}\sum_{j=0}^{N+2\times L_{GUARD}-1-\tau} LS_k^{N+2\times L_{GUARD}}(j)\times LS_\ell^{N+2\times L_{GUARD}}(j+\tau) \\ where\, 0\le\tau\le N+2\times L_{GUARD}-1 \\ \sum_{j=0}^{N+2\times L_{GUARD}-1+\tau} LS_k^{N+2\times L_{GUARD}}(j-\tau)\times LS_\ell^{N+2\times L_{GUARD}}(j) \\ where\, -(N+2\times L_{GUARD}-1)\le\tau<0 \\ 0,\ Otherwise\end{cases} \quad (3)$$

where $LS_k^{N+2\times L_{GUARD}}(j)$ is the $j-th$ code value of $k-th$ LS code, τ is time-offset of crosscorrelation.

The number of LS codes with the length of $N(=2^m)+2\times L_{GUARD}$ is N. If you assign the whole number of LS codes to one set, then you can define the set as IRS(interference rejection set) or interference free set denoted by I_0^N.

$$I_0^N=\left\{LS_0^{N+2\times L_{GUARD}}, LS_1^{N+2\times L_{GUARD}}, \cdots, LS_{N-1}^{N+2\times L_{GUARD}}\right\} \quad (4)$$

where I_0^N is the first set with N number of elements.

If you divide the IRS I_0^N into two parts, then you can get two IRSs with the 2^{m-1} elements: $I_0^{2^{m-1}}, I_1^{2^{m-1}}$. If you repeat the above method until the number of each set becomes to 2^{m-g}, you can have total 2^g IRSs: $I_0^{2^{m-g}}, I_1^{2^{m-g}}, \cdots, I_{2^g-1}^{2^{m-g}}$. Each IRS is shown in (5).

$$\begin{aligned}
I_0^{2^{m-g}} &= \left\{LS_0^{N+2\times L_{GUARD}}, LS_1^{N+2\times L_{GUARD}}, \cdots, LS_{2^{m-g}-1}^{N+2\times L_{GUARD}}\right\} \\
I_1^{2^{m-g}} &= \left\{LS_{2^{m-g}}^{N+2\times L_{GUARD}}, LS_{2^{m-g}+1}^{N+2\times L_{GUARD}}, \cdots, LS_{2^{m-g}+2^{m-g}-1}^{N+2\times L_{GUARD}}\right\} \\
I_2^{2^{m-g}} &= \left\{LS_{2\times 2^{m-g}}^{N+2\times L_{GUARD}}, LS_{2\times 2^{m-g}+1}^{N+2\times L_{GUARD}}, \cdots, LS_{2\times 2^{m-g}+2^{m-g}-1}^{N+2\times L_{GUARD}}\right\} \\
&\vdots \\
I_k^{2^{m-g}} &= \left\{LS_{k\times 2^{m-g}}^{N+2\times L_{GUARD}}, LS_{k\times 2^{m-g}+1}^{N+2\times L_{GUARD}}, \cdots, LS_{k\times 2^{m-g}+2^{m-g}-1}^{N+2\times L_{GUARD}}\right\} \\
&\vdots \\
I_{2^g-1}^{2^{m-g}} &= \left\{LS_{(2^g-1)\times 2^{m-g}}^{N+2\times L_{GUARD}}, LS_{(2^g-1)\times 2^{m-g}+1}^{N+2\times L_{GUARD}}, \cdots, LS_{(2^g-1)\times 2^{m-g}+2^{m-g}-1}^{N+2\times L_{GUARD}}\right\}
\end{aligned} \quad (5)$$

where $I_k^{2^{m-g}}$ is k-th IRS with the 2^{m-g} elements.

The crosscorrelation between two distinctive LS codes selected from one IRS $I_k^{2^{m-g}}$ $(k=0,1,2,\cdots,2^{m-g}-1)$ is given in (6).

$$C(\tau)=\begin{cases}0 & ,|\tau|\le L_{IFW} \\ & where\, L_{IFW}=\min\left(2^g-1, L_{GUARD}\right) \\ Not fixed, otherwise & \end{cases} \quad (6)$$

where L_{IFW} is a constant for representing the time interval with the zero crosscorrelation value, which is the minimum between the guard length L_{GUARD} and 2^g-1.

If you select two codes in one IRS, you can get a desired crosscorrelation property in (6). But if you choose 1st code from one IRS and 2nd code from another IRS, then you can not have an above crosscorrelation characteristic. So you should choose only one IRS to keep the desired crosscorrelation property. The selected IRS $I_k^{2^{m-g}}$ among $I_0^{2^{m-g}}, I_1^{2^{m-g}}, \cdots, I_{2^g-1}^{2^{m-g}}$ can be defined as the representative interference rejection set: $R^{2^{m-g}}$. The representative IRS $R^{2^{m-g}}$ is given in (7).

$$\begin{aligned} R^{2^{m-g}} &= \left\{r_0^{2^{m-g}}, r_1^{2^{m-g}}, \cdots, r_{2^{m-g}-1}^{2^{m-g}}\right\} \\ where\ R^{2^{m-g}} &= I_0^{2^{m-g}}, or\ I_1^{2^{m-g}}, \cdots, or\ I_{2^g-1}^{2^{m-g}} \end{aligned} \quad (7)$$

The crosscorrelation between $r_k^{2^{m-g}}$ and $r_\ell^{2^{m-g}}$ in the set $R^{2^{m-g}}$ is shown in (8).

$$C_{k,\ell}(\tau)=\begin{cases}0 & ,|\tau|\le L_{IFW} \\ & where\, L_{IFW}=\min\left(2^g-1, L_{GUARD}\right) \\ Not fixed, otherwise & \end{cases} \quad (8)$$

where L_{IFW} is a constant for representing the time interval with the zero crosscorrelation value, which is the minimum between the guard length L_{GUARD} and 2^g-1.

From above equations, we can observe that the crosscorrelation property of LS code within the time-offset interval of $\left[-L_{IFW}, L_{IFW}\right]$ is perfect.

4.2. LS Code Pair Selection

When the QPSK spreading is used in the CDMA system, one code is applied to In-phase branch and the other is used in Quadrature-phase branch. In order to decrease the PAPR value, it is necessary to reduce the number of 180° phase difference between two spreading codes. [4] Contrary to the conventional CDMA system, only LS code is used for spreading the data in the LAS-CDMA. Therefore it is important to set the code pairs considering the 180° phase difference between two spreading codes. The optimum code pair sets are proposed so as to minimize the PAPR value.

If you divide the initial LS code matrix($LS^{N+2\times L_{GUARD}}$)

g times, then you can get the total 2^g IRSs and select representative IRS denoted by $R^{2^{m-g}}$.

$$R^{2^{m-g}} = \{r_0^{2^{m-g}}, r_1^{2^{m-g}}, \cdots, r_{2^{m-g}-1}^{2^{m-g}}\} \quad (9)$$

Without loss of generality, we can assume that the representative IRS should be selected from the first IRS. The representative IRS $R^{2^{m-g}}$ is given as follows.

$$\begin{aligned} R^{2^{m-g}} &= \{r_0^{2^{m-g}}, r_1^{2^{m-g}}, \cdots, r_{2^{m-g}-1}^{2^{m-g}}\} \\ &= \{LS_0^{N+2\times L_{GUARD}}, LS_1^{N+2\times L_{GUARD}}, \cdots, LS_{2^{m-g}-1}^{N+2\times L_{GUARD}}\} \end{aligned} \quad (10)$$

When the codes from the set $R^{2^{m-g}}$ are applied to I branch and Q branch, the optimum code pairs for minimum 180° phase difference are given in (11).

$$\begin{aligned} &(I_{branch}Code, Q_{branch}Code) \text{ or } (Q_{branch}Code, I_{branch}Code) \\ &= (LS_0^{N+2\times L_{GUARD}}, LS_{2^{m-g-1}}^{N+2\times L_{GUARD}}), (LS_1^{N+2\times L_{GUARD}}, LS_{2^{m-g-1}+1}^{N+2\times L_{GUARD}}) \\ &\quad , \cdots, (LS_{2^{m-g-1}-1}^{N+2\times L_{GUARD}}, LS_{2^{m-g}-1}^{N+2\times L_{GUARD}}) \end{aligned} \quad (11)$$

The pair sets in (11) are optimum with respect to PAPR. The number of 180° phase difference between two codes is minimum in the above pair set and is verified by computer simulation thoroughly.

V. LS code sequential allocation

When it comes to allocating the LS codes to users, two criteria can be considered. One is to minimize the PAPR, the other is to expand temporarily the IFW length. Two criteria are mutually exclusive. They cannot be satisfied at the same time.

5.1. LS code allocation to minimize PAPR

Firstly, the codes can be allocated to the users using the optimum code pair set discussed earlier. This criterion is based on minimization PAPR value.

If a user requests spreading codes, then one of pair set in (11) is allocated. Since the pair set of (11) is used, the PAPR value keeps minimized.

5.2. LS code allocation to lengthen IFW temporarily

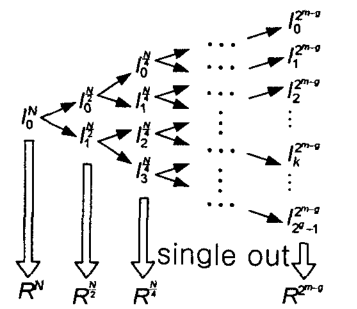
The illustration of this second criterion is like as follows: If the codes are not fully used in the system, the IFW can be expanded temporarily. The temporary expanded IFW length makes the MAI value reduced and brings about the better system performance.

As a first step of our analysis we will examine the relationship of IRS to representative IRS shown in the <Fig.1>.

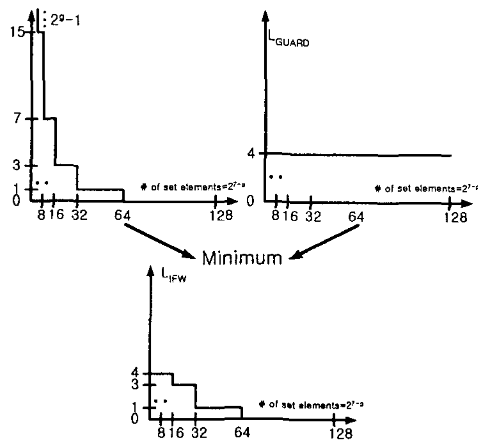
Next, let us take an example to show the determination of IFW of LS code. The LS code with the length $128+2\times4$ has the crosscorrelation property shown in the <Fig.2>. As shown in the <Fig. 2>, you can determine the time interval with zero crosscorrelation value by selection of minimum between 2^g-1 and L_{GUARD}. In the <Fig.2>, x-axis represents the number of elements in the selected set which has $128/2^g(=2^{7-g})$. The y-axis of upper-left graph in the <Fig.2> is 2^g-1, that of upper-right graph means L_{GUARD}, and that of lower graph stands for L_{IFW} which is the minimum value between 2^g-1 and L_{GUARD}.

Here is another example to show the crosscorrelation property of LS code. If you select 32 LS codes from the above example that is the representative IRS is $R^{2^{7-2}} = I_0^{2^{7-2}}$ given in (12), you can get the crosscorrelation between $LS_0^{128+2\times4}$ and $LS_1^{128+2\times4}, \cdots, LS_{31}^{128+2\times4}$ shown in <Fig.3>.

$$R^{2^{7-2}} = I_0^{2^{7-2}} = \{LS_0^{128+2\times4}, LS_1^{128+2\times4}, \cdots, LS_{31}^{128+2\times4}\} \quad (12)$$



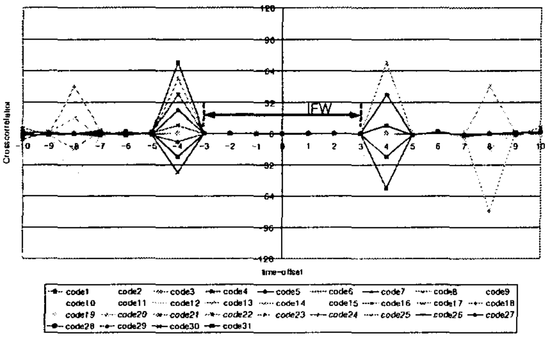
<Fig. 1> Expansion of interference rejection set tree and selection of representative interference rejection set



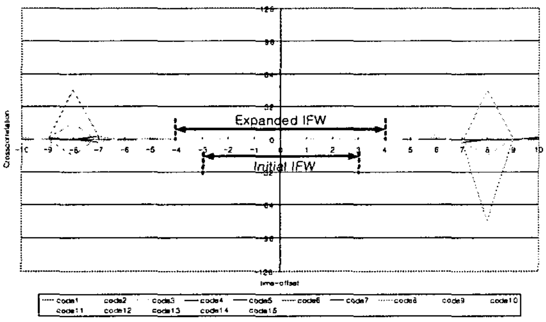
<Fig. 2> Determination of L_{IFW}

From the <Fig. 1> and <Fig. 2>, we can observe the two facts. One is that the higher(i.e. close to the root) representative IRS in the IRS tree of <Fig.1> includes lower representative IRSs; that is to say, the lower representative IRS is the subset of higher representative IRS. The other observation is that the lower(i.e. far from the root) representative IRS the larger IFW length. From the above two observations, we can expand the IFW temporarily. If the smaller number of codes are needed, you can allocate the codes from the lower representative IRS which is the subset of the original(i.e. initially selected) representative IRS. It makes the IFW be lengthened as long as the LS codes are allocated from the lower representative IRS.

An illustration of that point can be seen in <Fig.3> and <Fig.4>. While <Fig.3> depicts the crosscorrelation between two codes in the higher IRS $r_0^{2^{7-2}}$, <Fig.4> illustrates the crosscorrelation between two codes in the lower IRS $r_0^{2^{7-3}}$. The smaller codes are used, the larger IFW length can be obtained, as can be seen in the following <Fig.4>.



<Fig.3> Crosscorrelation between $LS_0^{128+2\times4}$ and $LS_1^{128+2\times4},\cdots,LS_{31}^{128+2\times4}$



<Fig.4> Crosscorrelation between $LS_0^{128+2\times4}$ and $LS_1^{128+2\times4},\cdots,LS_{15}^{128+2\times4}$

In summary, we formulate this allocation rule in (14). In order to simplify the explanation, we assume that the representative IRS should be selected from the first IRS. The representative IRS $R^{2^{m-g}}$ is given as follows.

$$\begin{aligned} R^{2^{m-g}} &= \left\{ r_0^{2^{m-g}}, r_1^{2^{m-g}}, \cdots, r_{2^{m-g}-1}^{2^{m-g}} \right\} \\ &= \left\{ LS_0^{N+2\times L_{GUARD}}, LS_1^{N+2\times L_{GUARD}}, \cdots, LS_{2^{m-g}-1}^{N+2\times L_{GUARD}} \right\} \end{aligned} \tag{13}$$

When the codes from the set $R^{2^{m-g}}$ are applied to both of I branch and Q branch, you should keep the following code pairs allocation order in (14) to expand the IFW temporarily. If you don't conform to the order in (14), the expanded IFW doesn't take effect.

$$\begin{aligned} &(I_{branch}Code, Q_{branch}Code) \text{ or } (Q_{branch}Code, I_{branch}Code) \\ &= (LS_0^{N+2\times L_{GUARD}}, LS_{2^{m-g-2}}^{N+2\times L_{GUARD}}), (LS_1^{N+2\times L_{GUARD}}, LS_{2^{m-g-2}+1}^{N+2\times L_{GUARD}}) \\ &\quad ,\cdots, (LS_{2^{m-g-2}-1}^{N+2\times L_{GUARD}}, LS_{2^{m-g-1}-1}^{N+2\times L_{GUARD}}), (LS_{2^{m-g-2}+1}^{N+2\times L_{GUARD}}, LS_{2^{m-g-1}+1}^{N+2\times L_{GUARD}}) \\ &\quad ,(LS_{2^{m-g-2}+2}^{N+2\times L_{GUARD}}, LS_{2^{m-g-1}+2}^{N+2\times L_{GUARD}}),, (LS_{2^{m-g-1}+2^{m-g-2}-1}^{N+2\times L_{GUARD}}, LS_{2^{m-g}-1}^{N+2\times L_{GUARD}}) \end{aligned} \tag{14}$$

As long as the number of codes used in (13) is smaller than half of the maximum code number, the IFW interval is larger than the guaranteed minimum IFW length.

VI. Conclusion

The optimum LS code pair setting and the sequential allocation method are investigated in this paper. While both PN and Walsh code are used in the conventional CDMA system, the LS code is only used in the spreading of data bits in the LAS-CDMA system. Therefore the LS code pair setting and allocation order should be dealt with carefully considering the special properties of LS code. At first the optimum LS code pair set is proposed in order to minimize PAPR. Furthermore the sequential allocation method is studied to either minimize PAPR or expand IFW.

Many aspects of LS code remain as a matter to discussed further. Therefore further research on LS code will be needed to clarify the LAS-CDMA.

VII. References

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