Analysis of Complemented CA Derived from a Linear TPMACA

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ABSTRACT

By using basic paths in the 0-tree of a linear TPMACA C we obtain the state-transition diagram of complemented CA C′ derived from C such that the complement vector is a nonzero attractor of C. Also we analyze the behavior of C′.

I. Introduction

An analysis of the state-transition behavior of group cellular automata (abbreviately, CA) was studied by many researchers ([1], [8], [10], [12]). The characteristic matrix of group CA is nonsingular. But the characteristic matrix of nongroup CA is singular. Although the study of nonsingular linear machines has received considerable attention from researchers, the study of the class of machines with singular characteristic matrix has not received due attention. Cho and Kim [6] and others [4],[5] studied nonsingular linear machines. Some properties of nonsingular CA have been employed in several applications ([5], [9] ,[11] ,[12]). In this paper, by using basic paths in the 0-tree of a linear multiple-attractor CA with two predecessor (abbreviately, TPMACA) C we obtain the state-transition diagram of complemented CA C′ derived from C such that the complement vector is a nonzero attractor of C. Also we analyze the behavior of C′. We call C′ the CA corresponding to C. Especially we investigate the behavior of the complemented CA which the complement vector F is taken as a nonzero attractor of C.

II. Preliminaries

Definition 2.1. [2] A state with a self-loop in the state-transition diagram of a nongroup CA are referred to as an attractor.

Remark 2.2. The cycles with length $l(\geq 2)$ in the state-transition diagram of a nongroup CA are not attractors.

Definition 2.3[2]. The nongroup CA for which the state-transition diagram consists of a set of disjoint components forming (inverted) tree-like structures at attractors are referred to as multiple-attractor CA (MACA).

Remark 2.4. (1) In case the number of attractors is one we call single-attractor CA (SACA). (2) A MACA with two predecessor is called a TPMACA. (3) The rank of T is $n-1$ where T is the characteristic matrix of the TPMACA.

The tree tooted at a cyclic state α-tree.

Definition 2.5[2]. The depth of a CA is defined to e the minimum number of clock cycles required to reach the cyclic state from any nonreachable state in the state-transition diagram of the CA.

Since the 0-tree and another tree rooted at a nonzero cyclic state have very interesting relationships, the study of the 0-tree is necessary and very important.

Theorem 2.6[7]. The number of predecessors

of a reachable state and the number of predecessors of the state 0 in a linear nongroup CA are equal.

Definition 2.7[3]. A state X at level l ($l \leq depth$) of the α-tree is a state lying on that tree and it evolves to the state α exactly after l-cycles (l is the smallest possible integer for which $T^l X = \alpha$).

Definition 2.8[3]. A state Y of an n-cell CA is an r-predecessor ($1 \leq r \leq 2^n - 1$) of a state X if $T^r Y = X$, where T is the characteristic matrix of the CA.

Lemma 2.9[11]. Let $\overline{T}^p$ denote p times application of the complemented CA operator $\overline{T}$. Then,

$$\overline{T}^p f(x) = [\, I \oplus T \oplus T^2 \oplus \ldots \oplus T^{p-1}][F(x)] \oplus [\, T_p][f(x)]$$

where T is the characteristic matrix of the corresponding noncomplemented rule vector and [F(x)] is an n-dimensional vector (n=number of cells) responsible for inversion after XNORing. F(x) has '1' entries (i.e., nonzero entries) for CA cell positions where XNOR function is employed.

III. The Behavior of Complemented CA Derived from a Linear TPMACA

By using basic paths in the 0-tree of a linear TPMACA C we obtain the stat-transition diagram of complemented CA C' derived from C such that the complement vector is a nonzero attractor of C. Also we analyze the behavior of C'.

Lemma 3.1. Let C be a linear TPMACA with depth d and F be a nonzero attractor in C as a complement vector. Then the state 0 is a cyclic state in the complemented CA C' corresponding to C. Also the cycle length becomes two.

Lemma 3.2. Let C be a linear TPMACA and α be a nonzero attractor in C as a complement vector. Then α lies on the two-length cycle including the state 0 of C' corresponding to C.

Theorem 3.3. Let C be a linear TPMACA and α be a nonzero attractor in C as a complement vector. Then the following hold:

(1) If β is an attractor of C, then $\beta \oplus \alpha$ is also an attractor of C.

(2) If β is an attractor of C, then β and $\beta \oplus \alpha$ are coalesced to form a two-length cycle, and thus β and $\beta \oplus \alpha$ lie on the same two-length cycle in the complemented CA C' corresponding to C.

Theorem 3.4. Let C be a linear TPMACA. Let α be a nonzero attractor in C as a complement vector. If x is a state at the level 2m in the β-tree of C, then x is a state at the level 2m in the β-tree of C' corresponding to C.

Theorem 3.5. Let C be a linear TPMACA. Let α be a nonzero attractor in C as a complement vector. If y is a state at the level 2m-1 in the β-tree of C, then y is rearranged at the level 2m-1 in the ($\beta \oplus \alpha$)-tree of C' corresponding to C.

Now we construct the state-transition diagram of the complemented CA corresponding to a linear TPMACA.

Definition 3.6. Let C be a linear TPMACA and the depth of C be d. Let β be a nonreachable state of the α-tree of C. Then we call the path $\beta \rightarrow T\beta \rightarrow \cdots \rightarrow \alpha$ a α-basic path of the α-tree of C.

Remark 3.7. Let C be a linear TPMACA with depth d.

Then $S_{d,0} \rightarrow S_{d-1,0} \rightarrow \ldots \rightarrow S_{1,0} \rightarrow 0$ is a 0-basic path of the 0-tree of C, where $T S_{i+1,0} = S_{i,0}$ ($1 \leq i \leq d-1$) and $S_{i,0}$ is the leftmost state of level I of the 0-tree of C.

Theorem 3.8[7]. Let C be a linear TPMACA. Given a 0-basic path of the 0-tree of C' corresponding to C we can construct the state-transition diagram of the 0-tree of C' as the following : If the states of the state-transition diagram of C(resp. C') are labeled such that $S_{l,k}$ ($rep.$ $\overline{S}_{l,k}$) be the (k+1)-th state in the l-th level, then

$$\overline{S}_{l,k} = S_{l,0} \oplus \sum_{i=1}^{l-1} b_i S_{i,0}$$

where $b_{l-1}\ b_{l-2} \cdots b_1$ is the binary representation of k and the maximum value of k is $2^{l-1}-1$.

The next theorems deal with rearrangements of the tree-structures between a linear TPMACA C and its complemented CA C′.

Theorem 3.9. Let C be a linear TPMACA and the depth of C be d. Let α be a nonzero attractor in C as a complement vector. Given a 0-basic path $S_{d,0} \rightarrow S_{d-1} \rightarrow \cdots \rightarrow 0$ of the 0-tree of C, we can construct a 0-basic path $\overline{S}_{d,0} \rightarrow \overline{S}_{d-1} \rightarrow \cdots \rightarrow 0$ of the 0-tree of the complemented CA C′ corresponding to C as the following:

$$\overline{S}_{l,0} = \begin{cases} S_{l,0} & \text{if } l \text{ is even} \\ S_{l,0} \oplus \beta & \text{if } l \text{ is odd} \end{cases}$$

Lemma 3.10. Let C be a linear TPMACA. The states lying at the i-th level of the β-tree of C′ corresponding to C satisfy the following:

$\overline{B}_{i,k} = \overline{S}_{i,k} \oplus \beta$ ($k = 0, \cdots, 2^{i-1}-1$)

where $\overline{B}_{i,k}$ is the (k+1)-th state in the i-th level of the β-tree of C′ and $\overline{S}_{i,k}$ is in Theorem 3.8.

Theorem 3.11. Let C be a linear TPMACA with depth d. A β-basic path of the β-tree of C′ corresponding to C is $\overline{B}_{d,0} \rightarrow \overline{S}_{d-1,0} \rightarrow \cdots \rightarrow \beta$ where $\overline{B}_{l,0}$ is the state in Lemma 3.10 and $\overline{S}_{l,0}$ is in Theorem 3.8 ($1 \le i \le d$).

Theorem 3.12. Let C be a linear TPMACA. The (k+1)-th state lying at the l-th level of the β-tree of C′ corresponding to C satisfies the following:

If $\overline{B}_{l,k}$ is the state in Lemma 3.10, then

$$\overline{B}_{l,k} = \overline{S}_{l,0} \oplus \sum_{i=1}^{l-1} b_i S_{i,0}$$

where $b_{l-1}\ b_{l-2} \cdots b_1$ is the binary representation of k and the maximum value of k is $2^{l-1}-1$.

Example 3.13. Let C be a five-cell linear TPMACA with the rule <102, 102, 60, 240, 204>.

Then

$$T = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Now the characteristic polynomial of T is $c(x) = x^3 (1+x)^4$ and the minimal polynomial of T is $m(x) = x^3(1+x)$. The state-transition diagram of C is as the following:

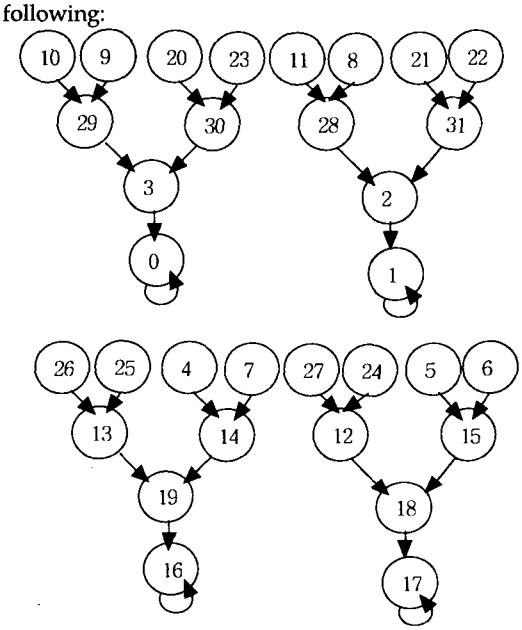
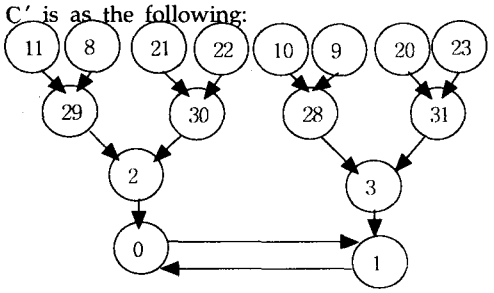


Figure 1: The state-transition diagram of C

For the case $F = (00001)^T$ is the complement vector, the state-transition diagram of C′ is as the following:



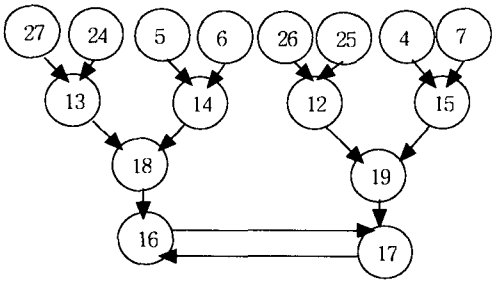


Figure 2: The state-transition diagram of C′

The 0-tree and the 1-tree of C are closed in C′. Also the state at the even levels of Analysis of complemented CA.

The 0-tree and the 1-tree of C remain unaltered in C′ whereas the states at the odd levels of C get interchanged between the trees in C′.

V. Conclusion

By using basic paths in the 0-tree of TPMACA, we obtain the state-transition diagram of complemented CA derived from CA such that the complement vector is a nonzero attractor of given TPMACA. Also we analyze the behavior of complemented CA. Especially we investigate the behavior of the complemented CA which the complement vector is taken as a nonzero attractor of given TPMACA.

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