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Modeling of Groundwater Flow Using the Element-Free Galerkin (EFG) Method

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Abstract

The element-free Galerkin (EFG) method is one of meshless methods, which is an efficient method of modeling problems of fluid or solid mechanics with complex boundary shapes and large changes in boundary conditions. This paper discusses the theory of the EFG method and its applications to modeling of groundwater flow. In the EFG method, shape functions are constructed based on the moving least square (MLS) approximation, which requires only a set of nodes. The EFG method can eliminate time-consuming mesh generation procedure with irregular shaped boundaries because it does not require any elements. The coupled EFG-FEM technique was introduced to treat Dirichlet boundary conditions. A computer code EFGGW was developed and tested for the problems of steady-state and transient groundwater flow in homogeneous or heterogeneous aquifers. The accuracy of solutions by the EFG method was similar to that by the FEM. The EFG method has the advantages in convenient node generation and flexible boundary condition implementation.

key word : groundwater flow model, element-free Galerkin (EFG) method, moving least square (MLS) approximation, meshless method, coupled EFG-FEM technique.

1.Introduction

Finite element methods (FEM) have been used for many years for modeling groundwater flow, because it can handle irregular boundaries with reasonable accuracy[1]. The FEM requires elements or a mesh to represent problem domain boundaries. The elements should be of proper sizes and of good shapes in order to obtain accurate solutions of the problems. When elements of larger sizes or accurate angles are used in the FEM, the solutions are less accurate with significant numerical errors[2]. Thus, construction of a proper mesh is essential to acute solutions in the FEM. The more complicated boundaries or discontinuities are considered, the more complicated meshes are required. It is quite burdensome to construct a FEM mesh with elements of proper shapes for very complicated cases such as fractured aquifer systems. Moving boundary problems like seepage in an earthen dam requires a re-meshing procedure in each successive time step, which takes a considerable amount of time. Even though some automatic mesh generators have been developed, manual work is still needed to complete a proper mesh for each problem.

To overcome this difficulty, meshless methods have been developed in astrophysics and solid mechanical engineering approximately two decades ago. These methods only require a set of nodes and description of boundaries to obtain approximate solutions of the problems. Some examples of meshless methods are the smooth particle hydrodynamics method[3], the diffuse element method[4], the element-free Galerkin (EFG) method[5], the reproducing kernel particle method[6], and the partition of unity method[7]. Belytschko et al.[8] reviewed and classified various meshless methods.

In the meshless methods, the EFG method is more stable and consistent than the other methods. The EFG method is also conceptually easier to understand and program computer codes than are other methods. Belytschko et al.[9] demonstrated some examples in modeling of growing cracks and large deformation in solid mechanical engineering by using the EFG method.

The EFG method uses the moving least square (MLS) approximations[9] to construct the discrete equations. The MLS approximations are based upon a weight function, a polynomial basis, and a set of coefficients. A weight function has a compact support that defines the influence domain of each node as shown in Figure 1. The nodal connectivity is defined by overlapping of the influence domains of nodes.

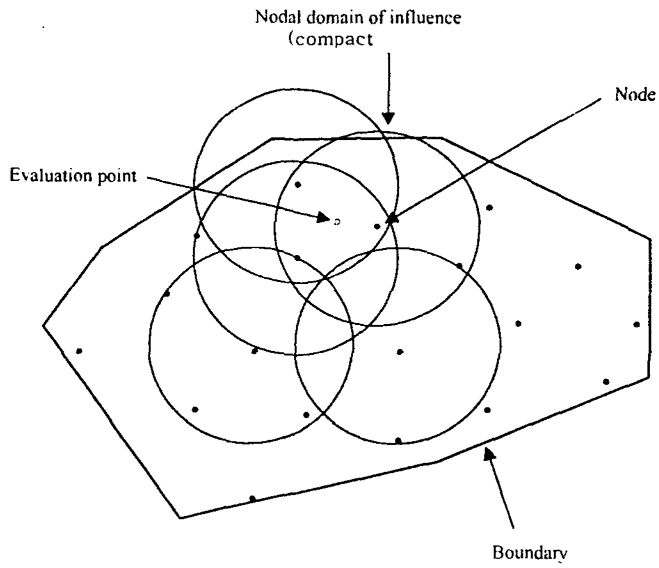

Figure 1. Domain of influence in two-dimensions using a circular compact support.

2. Element-Free Galerkin Method

The hydraulic head $h(x)$ of groundwater in an aquifer system can be approximated with the moving least square (MLS) method. the MLS approximation $\hat{h}(x)$

for $h(x)$ can be defined as

$$\hat{h}(x,t)=\sum_{j=1}^{m}p_j(x)a_j(x,t)\equiv p^T(x)a(x,t) \tag{1}$$

where $p(x)$ represents a polynomial basis of arbitrary order m and a(x) represents a set of coefficients which are functions of the space coordinate x and time t. The unknown coefficients a(x) are determined by the minimizing the difference between the approximation $\hat{h}(x)$ and the hydraulic head $h(x)$ at the point x. The coefficient a(x,t) can be estimated as

$$a(x,t)=A^{-1}(x)B(x)h(t) \tag{2}$$

where

$$A(x)=\sum_{j=1}^{n}w(x-x_j)p(x_j)p^T(x_j), \tag{3}$$

$$B(x)=[\,w(x-x_1)p(x_1)\ \ w(x-x_2)p(x_2)\ \ \ldots\ \ w(x-x_n)p(x_n)], \tag{4}$$

$$h^T(t)=[\,h_1(t)\ \ h_2(t)\ \ldots\ \ h_n(t)]. \tag{5}$$

The weight function in Equation (3) and (4) should be carefully chosen to guarantee the connectivity of nodes[5]. The continuity of the shape function in the EFG method is dependent to the continuity of the weight function. the weight function $w(x-x_j)$ is positive at the compact support of jth node and decrease monotonically as $\|x-x_j\|$ increases. Each node has positive influence only in the compact support. The compact support of each mode should be larger enough to satisfy the connectivity between nodes.

Substituting Equation (2) into Equation (1) leads the MLS approximation to

$$\hat{h}(x,t)=\sum_{j=1}^{n}p^T(x)A^{-1}(x)B_j(x)h_j(t)=\sum_{j=1}^{n}\Phi_j(x)h_j(t) \tag{6}$$

From Equation (6), the shape function (or the basis function) $\Phi_j(x)$ is defined as

$$\Phi_j(x)=\sum_{k=1}^{m}p_k(x)\,(A^{-1}(x)B(x))_{kj}=P^T(x)A(x)^{-1}B_j(x) \tag{7}$$

where the MLS approximation of shape function h(x,t) can be expressed as

$$\hat{h}(x,t)=\sum_{j=1}^{n}\Phi_j(x)h_j(t)=\Phi(x)h(t). \tag{8}$$

The governing partial differential equation (PDE) and boundary conditions for transient groundwater flow in a two-dimensional confined aquifer is given by

$$\nabla\cdot(T\nabla h)+R=S\frac{\partial h}{\partial t}\ \text{ in }\ \Omega \tag{9}$$

$$(T\nabla h)\cdot n=\bar{q}\ \text{ on }\ \Gamma_q \tag{10}$$

$$h=\bar{h}\ \text{ on }\ \Gamma_h \tag{11}$$

where ∇ is a vector differential operator, T is transmissivity, h is hydraulic head, R is a source or sink strength, S is storativity, and t is time.

By Galerkin method with the MLS approximation $\hat{h}(x)$, the governing PDE and boundary conditions can be expressed as

$$\int\int_{\Omega}\left[T_{ij}\frac{\partial\hat{h}}{\partial x_j}\frac{\partial\Phi}{\partial x_i}\right]+\int\int_{\Omega}\left[S\frac{\partial\hat{h}}{\partial t}\right]\Phi d\Omega=\int_{\Gamma}\left[T_{ij}\frac{\partial\hat{h}}{\partial x_j}n_i\right]\Phi d\Gamma-\int\int_{\Omega}R\Phi d\Omega \tag{12}$$

where i,j=1,2 for two-dimensional Cartesian coordinates.

The MLS approximations do not satisfy Kronecker delta property at the Dirichlet boundary, which causes difficulties in dealing with the Dirichlet boundary conditions. Belytschko et al.[11] developed a coupling technique with the finite element method to deal with Dirichlet boundary conditions. The Coupled EFG-FEM technique employs interface elements, where the EFG and FEM shape functions are combined.

3. Conclusion

A FORTRAN code EFGGW was developed to simulate groundwater flow problems and tested with confined and unconfined aquifer systems. The code was revised to simulate groundwater flow in fractured aquifer systems. Solutions by the EFG method was similar in accuracy to that by the FEM. The versatility of the EFG method was better than that of the EFM in the problems with arbitrary shaped boundaries.

The most attactive features of the EFG method are flexible node generation and elimination of time-consuming meshing procedure. Especially, the EFG method can handle complicated boundaries in the problems of groundwater flow in fractured aquifer systems.

4. References

1) Istok, J. D., Groundwater modeling by the finite element method, *American Geophysical Union*, DC., 1989

2) Wang, H. F. and Anderson, M. P., Introduction to groundwater modeling, Finite difference and finite element methods, Acdemic Press, Inc., NY., 1995

3) Lucy, L. B., A numerical approach to the testing of the fission hypothesis, *The Astronomical Journal* 82(12), 1013-1024., 1977

4) Nayroles, B., Touzot, G. and Villon, P., Generalizing the finite element method: diffuse approximation and diffuse elements, *Computational Mechanics* 10, 307-318., 1992

5) Belytschko, T., Lu, Y. Y. and Gu, L., Element-Free Galerkin Methods, *International Journal for Numerical Methods in Engineering* 37, 229-256.5), 1994

6) Liu, W. K., Jun, S. and Zhang, Y. F., Reproducing Kernel particle methods, *Internatinal Journal for Numerical Methods in Engineering* 20, 1081-1106.6), 1995

7) Duarte, C. A. and Oden, T. J., H-p cloudsan h-p meshless method. *Numerical Methods for Partial Differential Equations*, 1-34., 1996

8) Belytschko, T., Krongauz, Y., Organ, D., Fleming, M., and Krysl, P., Meshless methods: An overview and recent developments, *Computer Methods in Applied Mechanics and Engineering* 139, Mar-47., 1996

9) Belytschko, T., Lu, Y. Y., and Gu, L., Crack Propagation by Element-Free Galerkin Methods, *Engineering Fracture Mechanics* 51(2), 295-315., 1995

10) Lancaster, P. and Salkauskas K., Surface generated by moving least squares methods, *Mathematics of Computation* 37, 141-158., 1981