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Adelphic Integral 을 이용한 비선형 정규모드 진동 해석

A Study on the Nonlinear Normal Mode Vibration Using Adelphic Integral

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ABSTRACT

Nonlinear normal mode (NNM) vibration, in a nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term, is studied in this paper. The existence, bifurcation, and the orbital stability of periodic motions are to be studied in the phase space. In order to find the analytic expression of the invariant curves in the Poincare Map, which is a mapping of a phase trajectory onto 2 dimensional surface in 4 dimensional phase space, Whittaker's Adelphic Integral, instead of the direct integration of the equations of motion or the Birkhoff-Gustavson (B-G) canonical transformation, is derived for small value of energy. It is revealed that the integral of motion by Adelphic Integral is essentially consistent with the one obtained from the B-G transformation method. The resulting expression of the invariant curves can be used for analyzing the behavior of NNM vibration in the Poincare Map.

1. Introduction

The existence, bifurcation, and the orbital stability of periodic motions, which is called Nonlinear Normal Mode, in a nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term as shown in Fig. 1, are under consideration in this paper. In the previous work[1] , the dynamical structure of the same oscillator, was investigated by picturing the Poincare Map, which is a mapping of a phase trajectory onto 2 dimensional surface in 4 dimensional phase space, by direct integration of the equations of motion, and also by generating an approximation for the Poincare Map via Birkhoff-Gustavson[2,3] canonical transformation[4] for small values of energy. In that work, particularly, the existence and the stability of Nonlinear Normal Mode was studied, and it was found that the system has 2 or 4 Similar Nonlinear Normal Modes depending on the values of the nonlinear parameter *k*. The bifurcating modes enter as stable while the mode from which they bifurcated changes from stable to unstable.

In this paper, in order to find the analytic expression of the invariant curves in the Poincare Map, Whittaker's Adelphic Integral[5] , instead of the direct integration of the equations of motion, or the Birkhoff-Gustavson canonical transformation, is derived for small value of energy. We will show that although the calculation process is so much complicated, the resulting integral of motion obtained by Adelphic Integral is consistent with the one obtained from the B-G transformation method.

2. Whittaker's Adephic Integral and Relationship with Birkhoff-Gustavson's Integral

The method using Adelphic Integral, was developed by Whittaker[5], and starts with a canonical transformation which is defined in terms of action –

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angle variables. This method is based on the fact that the Poisson bracket of any integral ϕ and the Hamiltonian H equals zero. The resulting integral ϕ is known as Adelphic integral. Whittaker has divided the method into different cases. The cases arise from problems associated with internal resonance. In this method zero divisors are associated with internal resonances. Because the system R in Fig. 1 has a 1:1 internal resonance, we will consider only internal resonance case in this paper.

Let us begin with a Hamiltonian of the form[5]

$$H(u,v) = H(2)(u,v) + H(3)(u,v) + ... \quad (1)$$

,where $H(2)(u,v) = \sum_{v=1}^{n}(\alpha_v^2 u_v^2 + v_v^2) / 2.$

We can easily transform from (u,v) to (x,y) used in the previous work[1] , with the following canonical transformation ;

$$u_v = x_v / \alpha^{1/2}$$
$$v_v = \alpha^{1/2} y_v . \quad (2)$$

First, we define canonical transformation from (u,v) to (Q,P) ;

$$u = (2Q)^{1/2} \alpha^{-1/2} \cos P \quad (3.1)$$
$$v = (2\alpha Q)^{1/2} \sin P \quad (3.2)$$

The variables Q and P are known as action – angle variables[4,5,6] . From the inverse transformation of Eq. (3), we see that the action variables Q corresponds to amplitude and the angle variable P corresponds to the polar angle locating the trajectory in the (u,v) phase space.

Let us assume the system has two degree of freedom. In terms of the new variables (Q,P) , $H(2)(u,v)$ becomes

$$\widetilde{H}(2)(Q,P) = (\alpha_1 Q_1 + \alpha_2 Q_2) \quad (4)$$

, and $H(s)(u,v)$ becomes a sum of terms proceeding in powers of $Q_1^{1/2}$ and $Q_2^{1/2}$ and in triagonometric functions of multiples of P_1 and P_2 ; that is , terms of the type

$$Q_1^{m/2} Q_2^{n/2} \cos(ip_1 + ip_2) \quad , s = m+n \quad (5)$$

where m,n are nonnegative integers and m-$|i|$, n-$|j|$ are zero or an even integer.

We call $s = m+n$ the order of the term. The general form of $\widetilde{H}(6)(Q,P)$ is as follows:

$$\begin{aligned}
&Q_1^3 (Y_1 + Y_2 \cos 2P_1 + Y_3 \cos 4P_1 + Y_4 \cos 6P_1) \\
&+ Q_1^{\frac{5}{2}} Q_2^{\frac{1}{2}} \{ Y_5 \cos (P_1 + P_2) + Y_6 \cos (P_1 - P_2) \\
&+ Y_7 \cos (3P_1 + P_2) + Y_8 \cos (3P_1 - P_2) \\
&+ Y_9 \cos (5P_1 + P_2) + Y_{10} \cos (5P_1 - P_2) \} \\
&+ Q_1^2 Q_2 \{ Y_{11} + Y_{12} \cos 2P_1 + Y_{13} \cos 2P_2 \\
&+ Y_{14} \cos(2P_1 + 2P_2) \\
&+ Y_{15} \cos(2P_1 - 2P_2) + Y_{16} \cos 4P_1 \\
&+ Y_{17} \cos(4P_1 + 2P_2) + Y_{18} \cos(4P_1 - 2P_2) \} \\
&+ Q_1^{\frac{3}{2}} Q_2^{\frac{3}{2}} \{ Y_{19} \cos (P_1 + P_2) + Y_{20} \cos (P_1 - P_2) \\
&+ Y_{21} \cos (3P_1 + P_2) \\
&+ Y_{22} \cos (3P_1 - P_2) + Y_{23} \cos (P_1 + 3P_2) \\
&+ Y_{24} \cos (P_1 - 3P_2) \\
&+ Y_{25} \cos (3P_1 + 3P_2) + Y_{26} \cos (3P_1 - 3P_2) \} \\
&+ Q_1 Q_2^2 \{ Y_{27} + Y_{28} \cos 2P_1 + Y_{29} \cos 2P_2 \\
&+ Y_{30} \cos(2P_1 + 2P_2) \\
&+ Y_{31} \cos(2P_1 - 2P_2) + Y_{32} \cos 4P_2 + Y_{33} \cos(2P_1 + 4P_2) \\
&+ Y_{34} \cos(2P_1 - 4P_2) \} \\
&+ Q_1^{\frac{1}{2}} Q_2^{\frac{5}{2}} \{ Y_{35} \cos (P_1 + P_2) + Y_{36} \cos (P_1 - P_2) \\
&+ Y_{37} \cos (P_1 + 3P_2) + Y_{38} \cos (P_1 - 3P_2) \\
&+ Y_{39} \cos (P_1 + 5P_2) + Y_{40} \cos (P_1 - 5P_2) \} \\
&+ Q_2^3 (Y_{41} + Y_{42} \cos 2P_2 + Y_{43} \cos 4P_2 + Y_{44} \cos 6P_2)
\end{aligned} \quad (6)$$

, where Y_i 's are coefficients.

From now on, the tildes above H will be omitted for convenience.

If $\phi(Q,P) = \text{const.}$ is an integral we must have

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial\phi}\dot{Q} + \frac{\partial\phi}{\partial P}\dot{P} = \frac{\partial\phi}{\partial Q}\frac{\partial H}{\partial P} - \frac{\partial\phi}{\partial P}\frac{\partial H}{\partial Q} = (\phi, H) = 0 \tag{7}$$

The notation in Eq. (7) is referred as the Poisson bracket. We expand Eq. (7) and equate terms of equal order. For example, if $H = H(2) + H(6)$ then from the Eq. (3-14) we have

$$(\phi(2) + \phi(4) + \phi(6) + \phi(8) + \phi(10) + \Lambda\ ,\ H(2) + H(6)\,) = 0 \tag{8}$$

Equating terms of equal order, we have

$$(\phi(2), H(2)) = 0 \tag{9.1}$$
$$(\phi(4), H(2)) = 0 \tag{9.2}$$
$$(\phi(6), H(2)) = -(\phi(2), H(6)) \tag{9.3}$$
$$(\phi(8), H(2)) = -(\phi(4), H(6)) \tag{9.4}$$
$$(\phi(10), H(2)) = -(\phi(6), H(6)) \tag{9.5}$$

It is noted from Eq. (9.1) that

$$\alpha_1\frac{\partial\phi(2)}{\partial P_1} + \alpha_2\frac{\partial\phi(2)}{\partial P_2} = 0 \tag{10}$$

Let us assume

$$\phi(2) = \alpha_1 Q_1 - \alpha_2 Q_2 \tag{11}$$

, which certainly satisfies Eq. (10), and

$$\phi(4) = \phi(8) = 0$$

, which satisfies Eqs. (9.2) and (9.3).
Substituting equation (11) into equation (9.3), we have

$$\alpha_1\frac{\partial\phi(6)}{\partial P_1} + \alpha_2\frac{\partial\phi(6)}{\partial P_2} = \alpha_1\frac{\partial\phi(6)}{\partial P_1} - \alpha_2\frac{\partial\phi(6)}{\partial P_2} \tag{12}$$

This implies that to any term $A\cos(iP_1 + jP_2)$ in $H(6)$ there corresponds a term $\{(i\alpha_1 + j\alpha_2\ /\ i\alpha_1 + j\alpha_2)\}A\cos(iP_1 + jP_2)$ in $\phi(6)$.

Therefore, we can state the general form of ϕ as follows:

$$\phi = \phi(2) + \phi(6) + \Lambda$$

$$= \alpha_1 Q_1 - \alpha_2 Q_2$$

$$+ Q_1^3(Y_2\cos 2P_1 + Y_3\cos 4P_1 + Y_4\cos 6P_1)$$

$$+ Q_1^{\frac{5}{2}} Q_2^{\frac{1}{2}}\{\frac{(\alpha_1 - \alpha_2)}{(\alpha_1 + \alpha_2)}Y_5\cos(P_1 + P_2) + \frac{(\alpha_1 + \alpha_2)}{(\alpha_1 - \alpha_2)}Y_6\cos(P_1 - P_2) + \frac{(3\alpha_1 - \alpha_2)}{(3\alpha_1 + \alpha_2)}Y_7\cos(3P_1 + P_2) + \frac{(3\alpha_1 + \alpha_2)}{(3\alpha_1 - \alpha_2)}Y_8\cos(3P_1 - P_2) + \frac{(5\alpha_1 - \alpha_2)}{(5\alpha_1 + \alpha_2)}Y_9\cos(5P_1 + P_2) + \frac{(5\alpha_1 + \alpha_2)}{(5\alpha_1 - \alpha_2)}Y_{10}\cos(5P_1 - P_2)\}$$

$$+ Q_1^2 Q_2\{Y_{12}\cos 2P_1 + Y_{13}\cos 2P_2 + \frac{(2\alpha_1 - 2\alpha_2)}{(2\alpha_1 + 2\alpha_2)}Y_{14}\cos(2P_1 + 2P_2) + \frac{(2\alpha_1 + 2\alpha_2)}{(2\alpha_1 - 2\alpha_2)}Y_{15}\cos(2P_1 - 2P_2)$$

$$+ Y_{16}\cos 4P_1 + \frac{(4\alpha_1 - 2\alpha_2)}{(4\alpha_1 + 2\alpha_2)}Y_{17}\cos(4P_1 + 2P_2) + \frac{(4\alpha_1 + 2\alpha_2)}{(4\alpha_1 - 2\alpha_2)}Y_{18}\cos(4P_1 - 2P_2)\}$$

$$+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\{\frac{(\alpha_1-\alpha_2)}{(\alpha_1+\alpha_2)}Y_{19}\cos(P_1+P_2)$$
$$+\frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)}Y_{20}\cos(P_1-P_2)$$
$$+\frac{(3\alpha_1-\alpha_2)}{(3\alpha_1+\alpha_2)}Y_{21}\cos(3P_1+P_2)$$
$$+\frac{(3\alpha_1+\alpha_2)}{(3\alpha_1-\alpha_2)}Y_{22}\cos(3P_1-P_2)$$
$$+\frac{(\alpha_1-3\alpha_2)}{(\alpha_1+3\alpha_2)}Y_{23}\cos(P_1+3P_2)$$
$$+\frac{(\alpha_1+3\alpha_2)}{(\alpha_1-3\alpha_2)}Y_{24}\cos(P_1-3P_2)$$
$$+\frac{(3\alpha_1-3\alpha_2)}{(3\alpha_1+3\alpha_2)}Y_{25}\cos(3P_1+3P_2)$$
$$+\frac{(3\alpha_1+3\alpha_2)}{(3\alpha_1-3\alpha_2)}Y_{26}\cos(3P_1-3P_2)\}$$

$$+Q_1Q_2^2\{Y_{28}\cos 2P_1-Y_{29}\cos 2P_2$$
$$+\frac{(2\alpha_1-2\alpha_2)}{(2\alpha_1+2\alpha_2)}Y_{30}\cos(2P_1+2P_2)$$
$$+\frac{(2\alpha_1+2\alpha_2)}{(2\alpha_1-2\alpha_2)}+Y_{31}\cos(2P_1-2P_2)$$

$$-Y_{32}\cos 4P_2$$
$$+\frac{(2\alpha_1-4\alpha_2)}{(2\alpha_1+4\alpha_2)}+Y_{33}\cos(2P_1+4P_2)$$
$$+\frac{(2\alpha_1+4\alpha_2)}{(2\alpha_1-4\alpha_2)}Y_{34}\cos(2P_1-4P_2)$$

$$+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}\{\frac{(\alpha_1-\alpha_2)}{(\alpha_1+\alpha_2)}+Y_{35}\cos(P_1+P_2)+\frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)}Y_{36}\cos(P_1-P_2)$$
$$+\frac{(\alpha_1-3\alpha_2)}{(\alpha_1+3\alpha_2)}+Y_{37}\cos(P_1+3P_2)+\frac{(\alpha_1+3\alpha_2)}{(\alpha_1-3\alpha_2)}Y_{38}\cos(P_1-3P_2)$$
$$+\frac{(\alpha_1-5\alpha_2)}{(\alpha_1+5\alpha_2)}+Y_{39}\cos(P_1+5P_2)+\frac{(\alpha_1+5\alpha_2)}{(\alpha_1-5\alpha_2)}Y_{40}\cos(P_1-5P_2)$$

$$+Q_2^3(-Y_{42}\cos 2P_2-Y_{43}\cos 4P_2-Y_{44}\cos 6P_2)+\quad\Lambda \tag{13}$$

It should be noted that the Adelphic integral in Eq. (13) is well defined if we have no internal resonance. But the system R has internal resonance $(\alpha_1=\alpha_2=1)$, so we have zero divisors in the expression of ϕ. That is, we obtain $\phi=\phi(2)+\frac{\phi(6)}{D}+\Lambda$, but $\phi(6)$ has terms with vanishing denominators , D =0 . The first integral in this case, becomes $\phi=\phi(6)\quad+\phi(10)\quad+\Lambda$ where $\phi(6)$ consists of those terms that contributed to D =0, neglecting arbitrary constants. Therefore , for the system R

$$\phi(6)=Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}Y_6\cos(P_1-P_2)+Q_1^2Q_2Y_{15}\cos(2P_1-2P_2)$$
$$+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\{Y_{20}\cos(P_1-P_2)+Y_{26}\cos(3P_1-3P_2)\}$$
$$+Q_1Q_2^2Y_{31}\cos(2P_1-2P_2)+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}\;Y_{36}\cos(P_1-P_2) \tag{14}$$

This is equivalent to that if

$$f(Q,P)+\frac{g(Q,P)}{\mu}=const.=r$$

is a first integral, then multiplying by μ and taking the limit

As $\mu\to 0,\gamma\to\infty$, we have

$$g(Q,P)=\lim_{\mu\to 0}(\mu\gamma)=const$$

is the desired form of the integral when $\mu=0$.

We now add

$$C_1Q_1^3+C_2Q_1^2Q_2+C_3Q_1Q_2^2+C_4Q_2^3$$

, which is the complementary solution of Eq. (12), to $\phi(6)$ and determine the arbitrary constants C_1, C_2, C_3 and C_4 by requiring that terms with vanishing denominators disappear from higher order term of ϕ.

Thus

$$\phi(6)$$
$$=Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}Y_6\cos(P_1-P_2)+Q_1^2Q_2Y_{15}\cos(2P_1-2P_2)$$
$$+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\{Y_{20}\cos(P_1-P_2)+Y_{26}\cos(3P_1-3P_2)\}$$
$$+Q_1Q_2^2Y_{31}\cos(2P_1-2P_2)+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}Y_{36}\cos(P_1-P_2)$$
$$+C_1Q_1^3+C_2Q_1^2Q_2+C_3Q_1Q_2^2+C_4Q_2^3 \tag{15}$$

To solve for the constants $C_{1,}$ $C_{2,}$ $C_{3,}$ and $C_{4,}$, it is required that

$$(\phi(10), H(2)) \quad = \quad -(Q(6), H(6)) \qquad (9.5)$$

, where $\phi(6)$ is given by Eq. (15).

First, we expand the right hand side of Eq. (9.5)

$$\frac{\partial\phi(6)}{\partial Q_1}\frac{\partial H(6)}{\partial P_1} - \frac{\partial\phi(6)}{\partial P_1}\frac{\partial H(6)}{\partial Q_1} + \frac{\partial\phi(6)}{\partial Q_2}\frac{\partial H(6)}{\partial P_2} - \frac{\partial\phi(6)}{\partial P_2}\frac{\partial H(6)}{\partial Q_2} \qquad (16)$$

and find the coefficients of the terms that contribute to $\sin(P_1 - P_2), \sin(2P_1 - 2P_2), \sin(3P_1 - 3P_2), \Lambda$ and require them vanish.

Finally, we find

$$C_1 = C_4 = C, C_2 = C_3 = 3C - (5/4)(k+1) + (15/4) \qquad (17)$$

Thus, Whittaker's Adelphic integral for the system R is

$$\begin{aligned}\phi &= \phi(6) \\ &= (-5/2)Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}\cos(P_1 - P_2) + (5/2)Q_1^2 Q_2 \cos(2P_1 - 2P_2) \\ &+ (-15/2)Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\cos(P_1 - P_2) - (5/6)Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\cos(3P_1 - 3P_2)\} \\ &+ (5/2)Q_1 Q_2^2 \cos(2P_1 - 2P_2) - (5/2)Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}\cos(P_1 - P_2) \\ &+ C_1 Q_1^3 + C_2 Q_1^2 Q_2 C_3 Q_1 Q_2^2 + C_4 Q_2^3\end{aligned} \qquad (18)$$

,where $C_1, C_2\ C_3$ and C_4 are related by Eq. (17) and the constants $Y_6, Y_{15}, Y_{20}, Y_{31}$, and Y_{36} are obtained by the Hamiltonian

$$\begin{aligned}H &= (1/2)(u_1^2 + v_1^2 + u_2^2 + v_2^2) \\ &+ (k/6)(u_1^6 + u_2^6) + (1/6)(u_1 - u_2)^6.\end{aligned} \qquad (19)$$

In order to write ϕ in terms of the original coordinates (u, v), we use the inverse transformation

$$\tan P = v/u$$
$$Q = (u^2 + v^2)/2.$$

Thus we obtain

$$\begin{aligned}\phi &= (-5/16)\{(u_1^2 + v_1^2)^2 + 3(u_1^2 + v_1^2)(u_2^2 + v_2^2) \\ &+ (u_2^2 + v_2^2)^2\} \times (u_1 u_2 + v_1 v_2) \\ &+ (5/16)(u_1^2 + v_1^2 + u_2^2 + v_2^2) + (u_1^2 - v_1^2)(u_2^2 - v_2^2) \\ &+ (5/4)(u_1^2 + v_1^2 + u_2^2 + v_2^2)u_1 u_2 v_1 v_2 \\ &+ (-5/48)(u_1^3 - 3u_1 v_1^2)(u_2^3 - 3u_2 v_2^2) \\ &+ (3u_1^2 v_1 - v_1^3)(3u_2^2 v_2 - v_2^3)\} \\ &+ (C/8)\{(u_1^2 + v_1^2)^3 + (u_2^2 + v_2^2)^3\} \\ &+ (1/32)(12C - 5k + 10)(u_1^2 + v_1^2) \\ &\times (u_2^2 + v_2^2)(u_1^2 + v_1^2 + u_2^2 + v_2^2)\end{aligned} \qquad (20)$$

It is very interesting to note that if we compare Eq. (20) to Eq. (29) in the reference (1), we see that if $C = (5/12)(k+1)$ then Whittaker's Adelphic integral (ϕ_w) and Birkhoff – Gustavson's (ϕ_{B-G}) integrals identical. Therefore, if we take $C = \lambda + (5/12)(k+1)$ so that $\lambda = 0$ corresponds to $\phi_w = \phi_{B-G}$. Then, it can be seen that

$$\phi_w = \phi_{B-G} + \frac{\lambda}{8} + [(u_1{}^2 + v_1{}^2) + (u_2{}^2 + v_2{}^2)]^3$$

, which means that

$$\phi_w = \phi_{B-G} + \lambda[H^{(2)}(u, v)]^3. \qquad (21)$$

Thus ϕ_w and ϕ_{B-G} differ to $O(6)$ by a cubic function of the Hamiltonian.

We can construct the analytic expression of the invariant curves in the Poincare Map by combining the integral of motion in Eq. (20) with the Hamiltonian in Eq. (19) as discussed in the reference (1). The detailed procedure to calculate the expression will not be shown in this paper. Using the resulting analytic expression we can easily construct the Poincare Map. As can be seen in the figures in the reference (1), the analytic expression represents essentially identical invariant curves compared to the Poincare Map obtained by the direct integration of the equations of motion for small value of energy. Fig. 2, taken here from the reference (1), shows an example of the level lines calculated by Eqs. (19) and (20). As a result we can see that the nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term considered in this paper has 2 or 4 Similar Nonlinear Normal Modes depending on the values of the nonlinear parameter k. The bifurcating modes enter as stable while the mode from which they bifurcated changes from stable to unstable.

3. Conclusions

Nonlinear normal mode vibration, in a nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term, is studied in this paper. In order to find the analytic expression of the invariant curves in the Poincare Map, Whittaker's Adelphic Integral, instead of the direct integration of the equations of motion or the Birkhoff-Gustavson canonical transformation, is derived for small value of energy. It is revealed that the integral of motion by Adelphic Integral is essentially consistent with the one obtained from the B-G transformation method. They differ to the order of 6 by a cubic function of the Hamiltonian. The resulting expression of the invariant curves can be used for analyzing the behavior of NNM vibration in the Poincare Map. It can be clearly seen that the system considered in this paper has 2 or 4 Similar Nonlinear Normal Modes depending on the values of the nonlinear parameter. The bifurcating modes enter as stable while the mode from which they bifurcated changes from stable to unstable.

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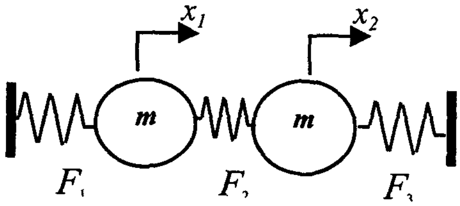

$$F_1 = F_3 = d + k\,d^5$$
$$F_2 = d^5$$

Fig. 1 The nonlinear oscillator R

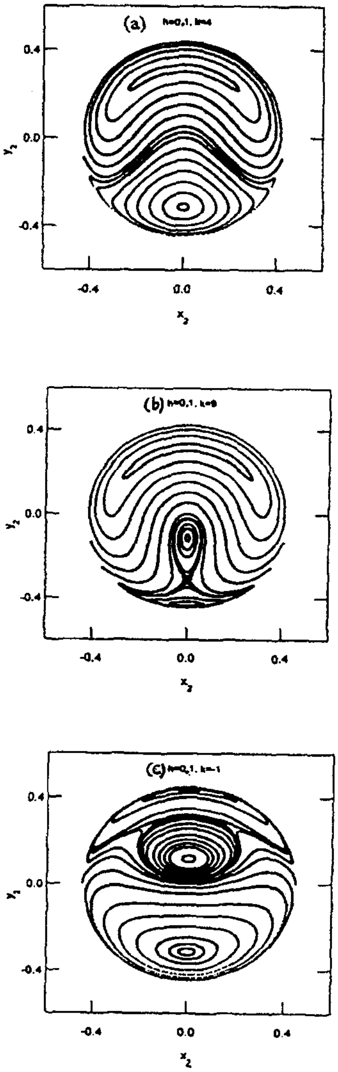

Fig. 2 Invariant Curves in the Poincare Map calculated using the Integral of Motion