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Hybrid Filter Design for a Nonlinear System with Glint Noise

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Abstract - In a target tracking problem, the radar glint noise has non-Gaussian heavy-tailed distribution and will seriously affect the target tracking performance. In most nonlinear situations an Extended Robust Kalman Filter(ERKF) can yield acceptable performance as long as the noises are white Gaussian. However, an Extended Robust H_∞ Filter (ERHF) can yield acceptable performance when the noises are Laplacian.
In this paper, we use the Interacting Multiple Model(IMM) estimator for the problem of target tracking with glint noise. In the IMM method, two filters(ERKF and ERHF) are used in parallel to estimate the state. Computer simulations of a real target tracking shows that hybrid filter used the IMM algorithm has superior performance than a single type filter.

1. 서 론

The Kalman filter is widely used in the tracking problem. It can optimally estimate the target motion from noisy data. The optimality of the Kalman filter is based on the assumption of theGaussian noise. If the assumption is violated, the Kalman filter is no longer the optimal filter. There have been a number of researchers who consider the problem of the Kalman filtering in non-Gaussian environments. Unfortunately, few results have been reported regarding this problem and the standard Kalman filter is continuously used in tracking applications.
One of the most effective schemes was proposed by Masreliez[1,2]. He introduced a nonlinear score function as the correction term in the state estimate and the results are often nearly optimal. While this approach seems promising, it encounters the difficulty of implementing the convolution operation involved in the evaluation of the score function. This precludes the practical applications of the method. Wu and Kundu[3] employed an adaptive normal expansion to expand the score function and truncates the higher order terms in the expanded series. Consequently, the score function can be approximated by a few central moments of the observation prediction density. However, the approximate spherical model used to decouple the state components is not a good approximation. We propose a hybrid filtering scheme for systems which allows both Gaussian and non-Gaussian noise called glint noise at a time. In the structure, ERKF and ERHF are used in parallel to estimate the state of a dynamic system with two modes of operation. We deal with a hybrid system with two modes using the IMM algorithm. The main feature of this algorithm is its ability to estimate the state of a dynamic system with several behavior modes which can "switch"from one to another. The ERKF is matched to the dynamic system with Gaussian measurement noise and the ERHF is matched to the same dynamic system but with glint noise modeled as the mixture of a Gaussian noise with high occurrence probability and a Laplacian noise with low occurrence probability.

2. 본 론

2.1 Extended Robust Kalman Filter

Now, an extended robust Kalman filter can be easily derived by using the Krein space extended Kalman filter[4]. Consider the discrete-time uncertain nonlinear system of the form

$$\begin{aligned} x_{j+1} &= f_j(x_j)+g_j(x_j)\,\tilde{u}_j \\ y_j &= h_j(x_j)+\tilde{v}_j \\ s_j &= k_j(x_j) \end{aligned} \quad (1)$$

where the nonlinear functions $f_j(x_j), g_j(x_j), h_j(x_j)$ and $k_j(x_j)$ are repeatedly differentiable and uncertainty inputs $\tilde{u}_j, \tilde{v}_j$ The nonlinear system (1) satisfies the sum of quadratic constraint

$$(x_0-\hat{x}_0)^*\Pi_0^{-1}(x_0-\hat{x}_0)+\sum_{j=0}^{i}\tilde{u}_j^*\,\tilde{Q}_j^{-1}\,\tilde{u}_j + \sum_{j=0}^{i}\tilde{v}_j^*\,\tilde{R}_j^{-1}\,\tilde{v}_j \le \epsilon + \sum_{j=0}^{i}\| s_j\|^2 \quad (2)$$

consider uncertain nonlinear system with contains the norm bounded uncertainty, $\|\Delta_j\|$.

$$(x_0-\hat{x}_0)^*\Pi_0^{-1}(x_0-\hat{x}_0)+\sum_{j=0}^{i}u_j^*Q_j^{-1}u_j+\sum_{j=0}^{i}v_j^*R_j^{-1}v_j\le\epsilon \quad (3)$$

and letting $\xi_j=\Delta_j k_j(x_j)$ then the following inequality hols.

$$\sum_{j=0}^{i} \| \xi_j \|^2 \le \sum_{j=0}^{i} \| s_j \|^2 \quad (4)$$

Adding (3)and (4), one gets

$$(x_0 - \hat{x}_0)^* \Pi_0^{-1} (x_0 - \hat{x}_0) + \sum_{j=0}^{i} \tilde{u}_j^* \begin{bmatrix} Q_j & 0 \\ 0 & 2I \end{bmatrix}^{-1} \tilde{u}_j \quad (5)$$
$$+ \sum_{j=0}^{i} \tilde{v}_j^* (R_j + 2E_{2j}E_{2j}^*)^{-1} \tilde{v}_j \le \varepsilon + \sum_{j=0}^{i} \| s_j \|^2$$

where $\tilde{u}_j = [\, u_j \;\; \xi_j]^*$ and $\tilde{v}_j = v_j + E_{2j}\xi_j$

Therefore, the sum quadratic constraint(2) includes the standard norm bounded uncertainties. To obtain an approximate system model, the nonlinear functions are expanded by using the Taylor series about $\hat{x}_{j|j}$ and $\hat{x}_{j|j-1}$. By neglecting higher order terms, the uncertainty nonlinear system(1) can be linearlized as

$$\begin{aligned} x_{j+1} &= F_j x_j + G_j \tilde{u}_j + p_j \\ y_j &= H_j x_j + \tilde{v}_j + q_j \\ s_j &= K_j x_j + r_j \end{aligned} \quad (6)$$

Note that this linearlized system still satisfies(2). Inequality(2) can be converted to the form related to an indefinite quadratic function, then the extended robust Kalman filtering problem is concerned with the following deterministic minimization problem.

$$\min_{y} J(x_0, \tilde{u}, y) \le \varepsilon \quad (7)$$

By using the Krein space Kalman filter equation in Theorem [4] and the corresponding state-space equation, the Krein space state equation for the extended robust Kalman filtering problem can be expressed by

$$\begin{aligned} x_{j+1} &= F_j x_j + \tilde{G}_j \tilde{u}_j + p_j \\ \begin{bmatrix} y_j \\ 0 \end{bmatrix} &= \begin{bmatrix} H_j \\ K_j \end{bmatrix} x_j + \tilde{v}_j + \begin{bmatrix} q_j \\ r_j \end{bmatrix} \\ s_j &= K_j x_j + r_j \end{aligned} \quad (8)$$

$$\text{with } \left\langle \begin{bmatrix} x_0 \\ \tilde{u}_j \\ \tilde{v}_j \end{bmatrix}, \begin{bmatrix} x_0 \\ \tilde{u}_k \\ \tilde{v}_k \end{bmatrix} \right\rangle = \begin{bmatrix} \Pi_0 & 0 & 0 \\ 0 & \tilde{Q}_j \delta_{jk} & 0 \\ 0 & 0 & \begin{bmatrix} \tilde{R}_j & 0 \\ 0 & -I \end{bmatrix} \delta_{jk} \end{bmatrix} \quad (9)$$

According to the Krein space-state equations of the extended robust Kalman filter and the extended Kalman filter, the extended robust Kalman equation can be expressed as follows:

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + P_{i|i} \begin{bmatrix} H_i \\ K_i \end{bmatrix}^* \begin{bmatrix} \tilde{R}_i & 0 \\ 0 & -I \end{bmatrix}^{-1} \quad (10)$$
$$\times \begin{bmatrix} y_i - h_i(\hat{x}_{i|i-1}) \\ -k_i(\hat{x}_{i|i-1}) \end{bmatrix}$$
$$\hat{x}_{i+1|i} = f_i(\hat{x}_{i|i}) \quad (11)$$

and $P_{i+1|i}$ satisfies the Riccati recursion

$$P_{i+1|i+1}^{-1} = P_{i+1|i}^{-1} + H_{i+1}^* \tilde{R}_{i+1}^{-1} H_{i+1} - K_{i+1}^* K_{i+1} \quad (12)$$
$$P_{i|i}^{-1} = P_{i|i-1}^{-1} + H_i^* \tilde{R}_i^{-1} H_i - K_i^* K_i \quad (13)$$

2.2 Extended Robust H_∞ Filter

Consider the following linearized uncertain nonlinear system

$$\begin{aligned} x_{j+1} &= F_j x_j + G_j \tilde{u}_j + p_j \\ y_j &= H_j x_j + \tilde{v}_j + q_j \\ s_j &= K_j x_j + r_j \\ z_j &= L_j x_j \end{aligned} \quad (14)$$

where uncertainty inputs $\tilde{u}_j$, $\tilde{v}_j$ contain the energy bounded noises and z_j is an arbitrary linear combination of the states should be estimated. If the filtered estimate is defined as $\check{z}_{j|j}$, then the filtered error is given by

$$e_j = \check{z}_{j|j} - z_j \quad (15)$$

Now, for a given scalar γ, the extended robust H_∞ filtering problem is related to find the filtered estimate $\check{z}_{i|i}$ that achieves

$$\| T_i(F) \|_\infty^2 = \sup$$
$$\frac{\sum_{j=0}^{i} e_j^* e_j}{\left(x_0 - \hat{x}_0\right)^* \Pi_0^{-1} \left(x_0 - \hat{x}_0\right) + \sum_{j=0}^{i} \tilde{u}_j^* \tilde{u}_j + \sum_{j=0}^{i} \tilde{v}_j^* \tilde{v}_j} < \gamma^2$$

(16)

where $T_i(F)$ denotes a transfer matrix. Using a virtual energy bound ε, it is assumed that the nonlinear systems (14) satisfy the sum quadratic constraint

$$J_1(x_0, \tilde{u}, y) = \left(x_0 - \hat{x}_0\right)^* \Pi_0^{-1} \left(x_0 - \hat{x}_0\right) + \sum_{j=0}^{N} \tilde{u}_j \tilde{Q}_j^{-1} \tilde{u}_j$$
$$+ \sum_{j=0}^{i} \left(\begin{bmatrix} y_j - q_j \\ 0 - r_j \\ \tilde{z}_{j|j} \end{bmatrix} - \begin{bmatrix} H_j \\ K_j \\ L_j \end{bmatrix} x_j \right)^* \tilde{R}_{\infty,j} \left(\begin{bmatrix} y_j - q_j \\ 0 - r_j \\ \tilde{z}_{j|j} \end{bmatrix} - \begin{bmatrix} H_j \\ K_j \\ L_j \end{bmatrix} x_j \right) \le \varepsilon \quad (17)$$

where $\tilde{R}_{\infty,j} = diag\left(\tilde{R}_j, -I, -\gamma^2 I\right)$. Then, from the above inequality, an auxiliary deterministic minimization problem can be obtained.

$$\min_{y} J_2\left(x_0, \tilde{u}, y\right) \le \varepsilon \quad (18)$$

As in the robust H_∞ filtering problem, the above auxiliary deterministic minimization problem of $J_2(x_0, \tilde{u}, y)$ in Hilbert spaces can be solved by using the recursive Krein space projections with two additional conditions: the positivity condition of $J_1(x_0, \tilde{u}, y)(\min J_1 = \min J_{2|x=0} > 0)$ and the condition for a minimum. The Krein space state-space equation for the extended robust H_∞ filtering problem is

$$\begin{aligned} x_{j+1} &= F_j x_j + \tilde{G}_j \tilde{u}_j + p_j \\ \begin{bmatrix} y_j \\ 0 \\ \check{z}_{i|i} \end{bmatrix} &= \begin{bmatrix} H_j \\ K_j \\ L_j \end{bmatrix} x_j + \tilde{v}_j + \begin{bmatrix} q_i \\ r_j \\ 0 \end{bmatrix} \\ s_j &= K_j x_j + r_j \end{aligned} \quad (19)$$

with

$$\left\langle \begin{bmatrix} x_0 \\ \tilde{u}_j \\ \tilde{v}_j \end{bmatrix}, \begin{bmatrix} x_0 \\ \tilde{u}_j \\ \tilde{v}_j \end{bmatrix} \right\rangle = \begin{bmatrix} \Pi_0 & 0 & 0 \\ 0 & \tilde{Q}_j \delta_{jk} & 0 \\ 0 & 0 & \begin{bmatrix} \tilde{R}_j & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} \delta_{jk} \end{bmatrix} \quad (20)$$

Then, the extended robust H_∞ filter and its minimum condition can be easily obtained by using the previous results.

An extended robust H_∞ filter is given by

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + \tilde{R}_f^{-1} P_{i|i} K_i k_i(\hat{x}_{i|i-1}) + \tilde{R}_f^{-1} P_{i|i} H_i^* \tilde{R}_i^{-1} (y_i - h_i(\hat{x}_{i|i-1})) \quad (21)$$

where $\tilde{R}_f = (I + \gamma^{-2} P_{i+1|i+1} L_{i+1}^* L_{i+1})$, $\hat{x}_{i+1|i} = f_i(\text{hat}x_{i|i})$,

and

$P_{i|i-1}$ satisfies the Riccati recursion.

$$P_{i+1|i+1}^{-1} = P_{i+1|i}^{-1} + H_{i+1}^* \tilde{R}_{i+1}^{-1} H_{i+1} - K_{i+1}^* K_{i+1} - \frac{1}{\gamma^2} L_{i+1}^* L_{i+1}$$
$$P_{i+1|i} = F_i P_{i|i} F_i^* + \tilde{G}_i \tilde{Q}_i \tilde{G}_i^* \quad (22)$$

2.3 Hybrid Filter Design

2.3.1 Interacting Multiple Model

The Interaction Multiple Model(IMM) estimator is a suboptimal hybrid filter that has been shown to be one of the most cost-effective hybrid state estimation schemes[5].

A general description for a hybrid system with additive noise is

$$x(k+1)=f[k,x(k),M(k+1)]+g[k,x(k),M(k+1),v[k,M(k+1)]] \quad (23)$$

with mode-dependent noisy measurements

$$z(k)=h[(k,x(k),M(k))+w[k,M(k)]] \quad (24)$$

and a Makovian transition of the system mode

$$P\{M_j(k+1)|M_i(k)\}=\phi[k,x(k),M_i,M_j] \quad (25)$$

where $x(k)$ is the base state, $M(k)$ is the modal state(system mode index) at time k, which denotes the mode in effect during the sampling period ending at k, $P\{\cdot\}$ is probability, $M_j(k)=\{M(k)=j\}$ is the event that mode j is in effect time k, $v[\cdot]$ and $w[\cdot]$ are the mode-dependent process and measurement noise sequences with means $\bar{v}_j$ and $\bar{w}_j$, respectively. In the IMM approach, at time k the state estimate is computed under each possible current model using r filters, with each filter using a different combination of the previous model-conditioned estimates mixed initial condition. The total probability theorem is used as follows to yield r filters running in parallel.

2.3.2 The IMM Algorithm

One cycle of the algorithm consists of the following:

1. Calculation of the mixing probabilities $(i,j=1,\dots,r)$.

The probability that mode M_i was in effect at $k-1$ given that M_j is in effect at k conditioned on Z^{k-1} is

$$\mu_{i|j}(k-1|k-1)=\frac{1}{\bar{c}_j}p_{ij}\mu_i(k-1) \quad i,j=1,\dots,r \quad (26)$$

where the normalizing constants are

$$\bar{c}_j=\sum_{i=1}^{r}p_{ij}\mu_i(k-1) \quad j=1,\dots,r \quad (27)$$

This is what makes it possible to carry out the mixing at the beginning of the cycle, rather than the standard merging at the end of the cycle.

2. Mixing $(i,j=1,\dots,r)$

Starting with $\hat{x}^i(k-1|k-1)$ one computes the mixed initial condition for the filter matched to $M_j(k)$ is

$$\hat{x}^{0j}(k-1|k-1)=\sum_{i=1}^{r}\hat{x}^i(k-1|k-1)\mu_{i|j}(k-1|k-1) \quad j=1,\dots,r$$

(28)

The covariance corresponding to the above is

$$P^{0j}(k-1|k-1)=\sum_{i=1}^{r}\mu_{i|j}(k-1|k-1)P^i(k-1|k-1) + \hat{x}^i(k-1|k-1)-\hat{x}^{0j}(k-1|k-1)] \times \hat{x}^i(k-1|k-1)-\hat{x}^{0j}(k-1|k-1)]' \quad (29)$$

3. Mode matched filtering $(j=1,\dots,r)$

The likelihood functions corresponding to the r filters

$$\Lambda_j(k)=p[z(k)|M_j(k),Z^{k-1}]$$

that is,

$$\Lambda_j(k)=N[x(k);\ \hat{z}^j[k|k-1;\ \hat{x}^{0j}(k-1|k-1)],\ S^j[k,P^{0j}(k-1|k-1)]] \quad j=1,\dots,r \quad (30)$$

4. Mode probability update $(j=1,\dots,r)$

This is done as follows:

$$\mu_j(k)=\frac{1}{c}\Lambda_j(k)\bar{c}_j \quad j=1,\dots,r \quad (31)$$

where $\bar{c}_j$ is the expression from (27) and

$$c=\sum_{j=1}^{r}\Lambda_j(k)\bar{c}_j \quad (32)$$

is the normalization constant for (31)

5. Estimate and covariance combination.

Combination of the model-conditioned estimates and covariances is done according to the mixture equations

$$\hat{x}(k|k)=\sum_{j=1}^{r}\hat{x}^j(k|k)\mu_j(k) \quad (33)$$

$$P(k|k)=\sum_{j=1}^{r}\mu_j(k)\{P^j(k|k) + [\hat{x}^j(k|k)-\hat{x}(k|k)][\hat{x}^j(k|k)-\hat{x}(k|k)]'\} \quad (34)$$

2.3 Simulation Results

An incoming ballistic missile model, which contains the standard norm bounded uncertainty $\|\Delta\|\le 1$.

$$x_{k+1}=x_k+\begin{bmatrix} x_{4k} \\ x_{5k} \\ x_{6k} \\ -\frac{1}{2}gx_{4k}x_{7k}\sqrt{x_{1k}^2+x_{2k}^2+x_{3k}^2} \\ -\frac{1}{2}gx_{5k}x_{7k}\sqrt{x_{1k}^2+x_{2k}^2+x_{3k}^2} \\ -\frac{1}{2}gx_{6k}x_{7k}\sqrt{x_{1k}^2+x_{2k}^2+x_{3k}^2}-g \\ -2.9\times10^{-5}x_{6k}x_{7k} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.6\times10^{-5} \end{bmatrix}\Delta_k x_{6k}x_{7k}+u_k$$

$$y_k=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}x_k+v_k$$

where

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}=\begin{bmatrix} x-position \\ y-position \\ z-position \\ x-velocity \\ y-velocity \\ z-velocity \\ atmosphic\ density \end{bmatrix},\ x_0=\begin{bmatrix} 3.2\times10^5 \\ 3.2\times10^5 \\ 2.1\times10^5 \\ -1.5\times10^4 \\ -1.5\times10^4 \\ -8.1\times10^3 \\ 5\times10^{-10} \end{bmatrix}$$

and it is assumed that uncorrelated exogenous noises are u_k, v_k satisfy

$$cov(u_k,u_k)=Q_k=\frac{1}{k+1}diag(0,0,0,100,100,100,2.0 times 10^{-18}),$$

$$cov(v_k,v_k)=R_k=\frac{1}{k+1}diag(150,150,150)$$

The initial error covariance matrix is given by

$$\Pi_0=10^6\times diag(1.5,1.5,1.5,0.015,0.015,0.00015,\ 10^{-6}\exp(-7.38\times10^{-5}))$$

$$L=[0\ 0\ 0\ 1\ 1\ 1\ 0]\ ,\quad \gamma=23$$

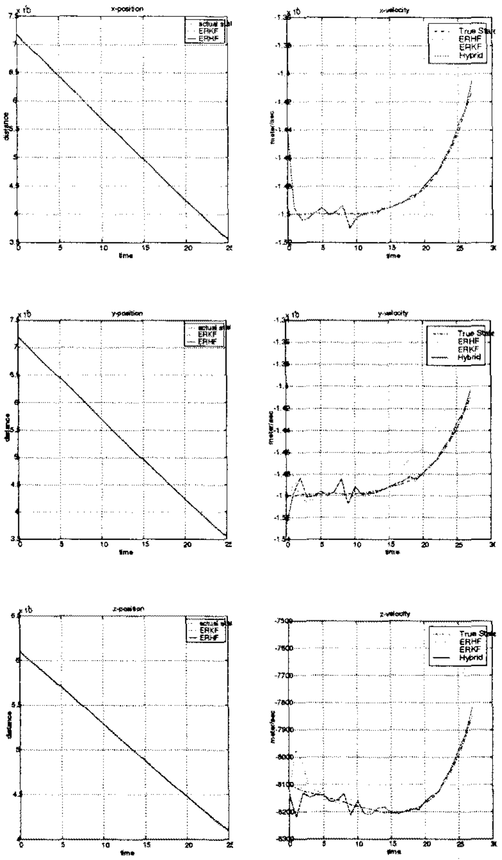
The glint can be modeled as the mixture of a Gaussian noise with moderate variance and a Laplacian noise with large variance.

$$f_t(v)=(1-\varepsilon)f_g(v)+\varepsilon f_l(v)$$

where $f_t(\cdot), f_g(\cdot), f_l(\cdot)$, and ε represent the the glint, the Gaussian, the Laplacian distribution, and glint probability respectively. The ε is given by 0.4.

The two initial mode probabilities were both set to 0.5, and the Markov chain transition matrix for the two-mode system used is

$$p_{ij}=\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$



3. 결 론

In this thesis, the hybrid filter using the IMM algorithm for nonlinear systems which allows both Gaussian and glint non-Gaussian noise at a time. We used the hybrid filter for the problem of target tracking with glint noise mixed Gaussian and Laplacian. It has been shown that the hybrid filter has superior performance compared with a single type filter. We used the hybrid filter and obtained satisfactory results.

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[참 고 문 헌]

[1] Masreliez. C. J. and Martin. R. R., "Robust Bayesian estimation for the linear model and robustifying the Kalman filter", IEEE Trans. Auto. Contr. AC-22, pp. 361-371, 1977.

[2] Masreliez. C. J., "Robust recursive estimation and filtering", Ph. D. dissertation University of Washi ngton, Seattle, 1972.

[3] Wu, W. "Target tracking with glint noise", IEEE Trans. Aero. Electro. Sys., Vol. 29, No. 1, pp. 174-185, 1993.

[4] Ra. W. S., "A unified Approach to Robust using the Krein Space Estimation Theory", M. D. Dissertation, Yonsei Unvi., 2000.

[5] E. Mazor, A. Averbuch, Y. Bar-Shalom, J. Dayan "Interacting Multiple Model Methods in Target Tracking : A Survey", IEEE Trans. Aero. Electro. Sys., Vol. 34, No. 1, pp.103-123, 1998.