Reconstruction of ECG Signals using Variable Bandwidth Filter

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Abstract - A variable bandwidth filter (VBF) is proposed for cleaning signals with known instantaneous bandwidth (IB). Also, the behaviors and the characteristics of the VBF are examined in time-frequency domain (TFD). The proposed VBF clips a noisy signal along the boundaries of IB and rejects noise in the outside of the IB of the signal in TFD. It is possible to construct four kinds of VBF that are low-pass VBF, high-pass VBF, band-pass VBF and band-stop VBF.

1. Introduction

Various kinds of signal with time-varying spectral contents are frequently encountered in biological and biomedical measurements, for example, speech, skeletal muscle sounds, electrocardiogram (ECG), electroencephalogram and electromyogram [1]. The concepts of instantaneous frequency (IF) and instantaneous bandwidth (IB) are important in situations where the spectral contents of the signals are not stationary. The IF $\omega(t)$ of an AM-FM signal of the form $x(t)=a(t)\cos\varphi(t)$ is defined by [3] $\omega(t)=D\varphi(t)$ and the IB $b(t)$ of $x(t)$, which is the spread in frequency at a particular time, is defined by [3] $b(t)=|Da(t)/a(t)|$, where $D=d/dt$.

In this letter, a variable bandwidth filter (VBF) is proposed for the more efficient noise rejection of noisy signals with known IB. As an example, a noisy ECG signal corrupted by EMG signals and 60Hz power interference noise, etc, is considered [2]. When looking at the noisy ECG signal in TFD, the spectrum of the ECG signal is within the region of IB, whereas the spectrum of noise is spread out across the wide region of the TFD. Although the bandwidths of the ECG signal and noise in a noisy ECG signal preprocessed by a linear time-invariant (LTI) filter are fully overlapped in Fourier frequency domain, we can find some regions that the spectrums of the ECG signal and noise are not overlapped in TFD since the IB of the ECG signal changes over time. The proposed VBF clips the noisy ECG signal along the boundary of the IB of ECG signals in TFD and rejects noise in the outside of the IB. The IB of ECG signals is estimated in TFD.

2. Variable bandwidth filter

To be specific, let a differential operator G be defined as [5]

$$Gx(t) = g(t)Dx(t) \tag{1}$$

where $D=d/dt$, $0<g(t)<\infty$ and $x(t)$ is a differentiable function variable. Consider the linear quasi-time-invariant(LQTI) system [4,5] described by

$$a_n G^n y + a_{n-1} G^{n-1} y + \ldots + a_0 y = b_m G^m x + \ldots + b_0 x \tag{2}$$

where a_i and b_j are the constant coefficients and $x(t)$ is input, $y(t)$ is the output of the system. The system is stable if the real parts of all the roots of the characteristic equation $a_n \lambda^n + a_{n-1} \lambda$

$^{n-1}+\ldots+a_1\lambda+a_0=0$ with respect to the operator G are negative [5].

Now, let's consider the extended Fourier transform (EFT) $\tilde{F}$ for the frequency analysis and the physical interpretation of the LQTI filters. For a given function $g(t)$, the EFT $\tilde{F}$ and its inverse EFT $\tilde{F}^{-1}$ are defined as follows [4]:

$$\tilde{F}[x(t),g(t)] \equiv \int_{-\infty}^{\infty} (x(t)/g(t))e^{-\int_0^t \frac{i\omega}{g(\xi)}d\xi} dt = X(\omega), \tag{3}$$

$$\tilde{F}^{-1}[X(\omega),g(t)] \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{\int_0^t \frac{i\omega}{g(\xi)}d\xi} d\omega = x(t). \tag{4}$$

With $g(t)=1$, the EFT becomes the traditional Fourier transformation.

Let $x(t)$ be a function whose EFT is denoted as $\tilde{F}[x(t)] = X(\omega)$. Then,

$$\tilde{F}[G^n x(t), g(t)] = (i\omega)^n X(\omega) \tag{5}$$

where n is a non-negative integer [4, 5]. From the definition of $Gx(t)$ and integration by parts, we can get the relation $\tilde{F}[Gx(t),g(t)] = i\omega\tilde{F}[x(t)]$ if $x(t)$ vanishes as $t\to\pm\infty$ [4]. Generally, eqn. 5 is established by iteration of the relation. The system of eqn. 2 can be analyzed by using eqn. 5 in EFT frequency domain. Consider the LQTI system of eqn. 2. Take the EFT for both sides of eqn. 2. Then, by eqn. 5,

$$Y(\omega) = (\sum_{k=0}^{m} b_k (i\omega)^k \Big/ \sum_{k=0}^{n} a_k (i\omega)^k) X(\omega) = H_g(\omega)X(\omega) \tag{6}$$

The output $y(t)$ of eqn. 2 is obtained by using the inverse EFT. That is, $y(t) = \tilde{F}^{-1}[H_g(\omega)X(\omega)]$. The complex function $H_g(\omega)$ is the frequency response of the linear time-varying filter. The physical meaning of frequency variable ω of eqn. 6 depends on the selected function $1/g(t)$. According to eqns. 3 and 4 a time signal $s(t) = \exp(\int_0^t i\omega_0/g(t)\,d\tau)$ corresponds to $\delta(\omega-\omega_0)$ in EFT frequency domain.

In the case that eqn. 2 is the Butterworth polynomial, that is, the constant coefficients a_i and b_j are those of a LTI Butterworth filter, the LQTI system is the Butterworth type VBF. It is possible to construct the Bessel type VBF, the Chebyshev type VBF, etc. There are four kinds of VBF, that is, the low-pass VBF, the band-pass VBF, the high-pass VBF, the band-stop VBF. The cutoff frequencies of VBF in EFT domain are determined by the same method as the traditional LTI filter with the coefficients a_i, b_j of eqn. 2.

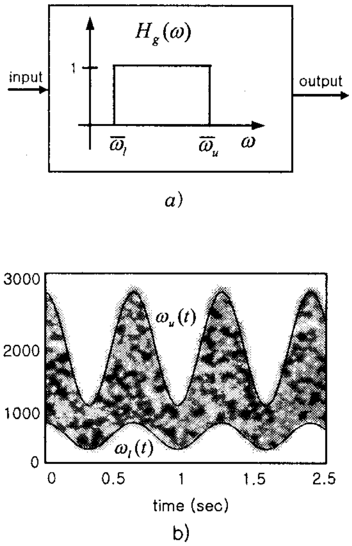
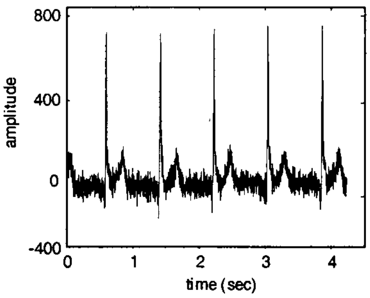


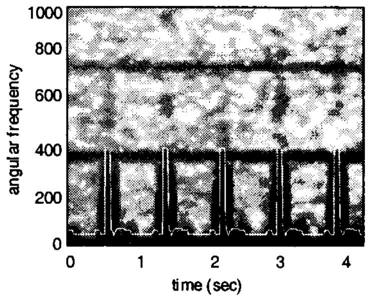
Fig. 1 An ideal band-pass VBF and passing band
a) An ideal band-pass VBF
b) Spectrogram of white noise filtered by BPVBF in Fig. 1*a*

3. Experiment examples

Consider an ideal band-pass variable bandwidth filter (BPVBF) with cutoff frequencies $\bar{\omega}_l$=0.7 rad/sec, $\bar{\omega}_u$=3 rad/sec and a positive function $1/g(t)=2\pi(100+50\cos(10t))$. See Fig. 1*a*, Fig. 1*b* shows the spectrogram of the output of the ideal BPVBF whose input is white noise. The region of pass-band shown in fig. 1b is determined by the cutoff frequencies $\bar{\omega}_l$, $\bar{\omega}_u$ and the positive function $g(t)$. The upper boundary $\omega_u(t)$ and the lower boundary $\omega_l(t)$ of the pass-band in Fig. 1*b* are $\omega_u(t) = \bar{\omega}_u(1/g(t))$ and $\omega_l(t) = \bar{\omega}_l(1/g(t))$ respectively.



a)



b)

Fig. 2 A Noisy ECG signal
a) A noisy ECG signal in time domain
b) Spectrogram of the noisy ECG signal in dB white solid line: the estimated IB of the noisy ECG Signal

From Fig. 1 we know that the signal $x(t)=\cos\varphi(t)$ passes through the ideal BPVBF without any losses if the IF $D\varphi(t)$ is $\overline{\omega}_l/g < D\phi(t) < \overline{\omega}_u/g$ for all t. The ideal BPVBF clips noisy signal along the boundaries of IB in time-frequency domain and rejects noise in the outside of IB. For the diagnosis, not only the QRS complex, the p-wave and the T-wave but also some segments and some intervals of ECG signals are used frequently [6]. From the spectrogram of the noisy ECG signal shown in Fig. 2*b*, we know that the energy of the ECG signal is concentrated on the low frequency region with some peaks in TFD, while the energy of noise is spread throughout the TFD. The white solid line of Fig. 2*b* shows the IB estimated by heuristic method in TFD. Fig. 3 shows the ECG signal reconstructed by the ideal low-pass VBF with the IB shown in Fig. 2*b*. The cutoff frequency is 1 rad/sec. Also, the ECG signal reconstructed by a LTI filter with the cutoff frequency 430 rad/sec is shown in Fig. 3 for comparison. Next, consider the noisy ECG signal of Fig. 2*a* measured by a portable 24-hours health monitoring system [2].

We know that the QRS complex, P-wave and T-wave are reconstructed accurately. According to experiments, the improvement of SNR in energy is >11 dB as compared with the LTI filter. The LTI filter cannot reject noise among the peaks of the IB. The proposed method based on VBF and time-frequency analysis can reject noise among the peaks of IB if the spectrums of signal and noise are not overlapped in TFD.

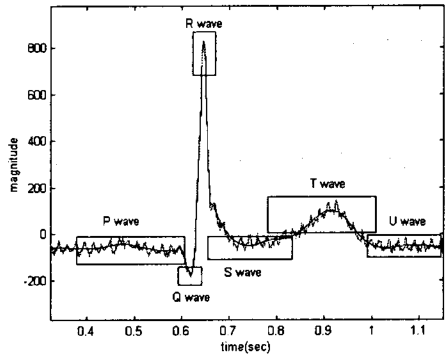


Fig. 3 Comparison of the reconstructded ECG signals
—— reconstruction by LTI filter
--- reconstruction by VBF

4. Conclusions and further studies

In this paper, a VBF is proposed for the reconstruction of signals with time-varying IB. The proposed VBF rejects the noise in the outside of the region of interest along the boundaries of time-varying IB of a signal in time-frequency domain (TFD). If the spectrums of signal and noise are not overlapped in TFD, the VBF rejects the noise whose spectrum is not overlapped with the spectrum of signal. The VBF is represented by rational form in the EFT frequency domain. The results show that the VBF-based noise rejection method produces reliable reconstruction of underlying signals. It is necessary to study the systematic estimation method of the IB and the elimination of the window effects for the more accurate reconstruction of signals.

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