# Mode conversion in nondestructive nonlinear acoustic method for defect detection in a layer-structured material

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Nondestructive nonlinear acoustic method in two dimensions is suggested as a useful tool for detecting defects in a composite layer-structured material. Spectrum level changes in fundamental and harmonic frequencies are observed in the presence of a layer type defect compared with in the absence of such a defect. It is proposed in this study that such spectrum changes are due to the mode conversion. The layer type defect makes different normal modes due to different boundary conditions in the thickness direction for the Lamb waves propagating in a layer-structured material. Specifically, the normal mode with the fundamental frequency in the case of the water-layer gap is converted to the normal mode with the second harmonic frequency in the case of the air-layer gap.

## I. INTRODUCTION

Nondestructive nonlinear acoustic diagnosis, as supplementary acoustic evaluation method in addition to nondestructive linear acoustic method, attracts much attention in its applications [1–4]. The crack existence is mainly detected with the nonlinear harmonic observation in nondestructive nonlinear acoustic evaluation method. This technique can be extended to a crack detection method in the two-dimensional plate-type structures and layered plate-type structures. Lamb waves in plate, which are the mixed waves of longitudinal and transverse waves in plane duct, is used to detect the defect. The essential point in nonlinear acoustic evaluation is that the second and higher harmonic modes of incident waves are generated from a defect. The exact reason for the spectrum level increase in the harmonic frequency and the spectrum level decrease in the fundamental frequency is not clarified yet. In this paper, such a reason is suggested by the normal mode conversion process of Lamb waves. The normal mode conversion of Lamb waves on the edge reflection [5] may be extended to the mode conversion in a layer-structured material.

## II. LAMB WAVE THEORY

When an acoustic source is applied to a thin plate, acoustic waves propagating in solid plate material with pressure release boundaries in both sides are normally mixed with the longitudinal and transverse waves and are called Lamb waves.

The equation of motion for a homogeneous, isotropic, linearly elastic body is given by

$$\mu\nabla^2\vec{u}(\vec{r},t) + (\lambda+\mu)\nabla\nabla\cdot\vec{u}(\vec{r},t) = \rho\frac{\partial^2 u(\vec{r},t)}{\partial t^2} - \vec{f}(\vec{r},t)$$

where $\vec{u}$ is the particle displacement vector, $\rho$ is the density, $\mu$ and $\lambda$ are the Lame's elastic constants, and the inhomogeneous term $f(r,t)$ in the right hand side is the external source term. The displacement vector can be expressed in terms of the scalar potential $\Phi$ and the vector potential $\Psi$:

$$\vec{u} = \nabla\Phi + \nabla\times\vec{\Psi}. \tag{1}$$

Using this relation, the equation of motion yields two uncoupled wave equations for the longitudinal wave and transverse wave:

$$\nabla^2\Phi - \frac{1}{c_l}\frac{\partial^2\Phi}{\partial t^2} = 0, \tag{2}$$

$$\nabla^2\vec{\Psi} - \frac{1}{c_s}\frac{\partial^2\vec{\Psi}}{\partial t^2} = 0 \tag{3}$$

where the sound velocity for the longitudinal wave is $c_l = (\frac{\lambda+2\mu}{\rho})^{1/2}$ and the sound velocity for the transverse wave is $c_t = (\frac{\mu}{\rho})^{1/2}$.

In cylindrical coordinates with the origin of $z$ axis at the center of plate, (2) and (3) become

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})\Phi + \frac{\partial^2}{\partial z^2}\Phi - \frac{1}{c_l^2}\frac{\partial^2\Phi}{\partial t^2} = 0, \tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})\Psi_\theta + \frac{\partial^2}{\partial z^2}\Psi_\theta - \frac{1}{c_s^2}\frac{\partial^2\Psi_\theta}{\partial t^2} = 0. \tag{5}$$

The general solutions of (4) and (5) are given by

$$\Phi = (A\cos\alpha z + B\sin\beta z)J_0(kr), \tag{6}$$

$$\Psi_\theta = (C\cos\alpha z + D\sin\beta z)J_0(kr). \tag{7}$$

Using (6), (7), and (1), the components of the displacement vector apart from the time dependent term $e^{-i\omega t}$ are given by

$$u_r = ((A\cos\alpha z + B\sin\alpha z) + \beta(C\sin\beta z - D\cos\beta z))kJ_0'(kr), \tag{8}$$

$$u_z = (-\alpha(A\sin\alpha z - B\cos\alpha z) + k^2(C\cos\beta z + D\sin\beta z))J_0(kr) \tag{9}$$

where $A$, $B$, $C$, and $D$ are constants determined by the initial conditions and $J_0$ is the zeroth Bessel function. The propagation constants $k$, $\alpha$, and $\beta$ have relations

$$\alpha^2 = k_l^2 - k^2,\ \beta^2 = k_s^2 - k^2. \tag{10}$$

If $B = C = 0$, $u_z(z) = -u_z(-z)$. This indicates the symmetric mode of the Lamb wave which has the displacement components

$$u_r = (A\cos\alpha z - \beta D\cos\beta z)kJ_0'(kr), \tag{11}$$
$$u_z = (-\alpha A\sin\alpha z + k^2 + D\sin\beta z)J_0(kr). \tag{12}$$

If $A = D = 0$, $u_z(z) = u_z(-z)$. This indicates the asymmetric mode of the Lamb wave which has the displacement components

$$u_r = (B\sin\alpha z + \beta C\sin\beta z)kJ_0'(kr), \tag{13}$$
$$u_z = (\alpha B\cos\alpha z + k^2 C\cos\beta z)J_0(kr). \tag{14}$$

Since the components of the stress tensor are expressed by

$$\sigma_{zz} = \lambda\nabla^2\Phi + 2\mu\frac{\partial}{\partial z}(\frac{\partial\phi}{\partial z} + (\frac{\partial\Psi_\theta}{\partial r} + \frac{1}{r}\Psi_\theta), \tag{15}$$
$$\sigma_{rz} = \mu\frac{\partial}{\partial r}(2\frac{\partial\phi}{\partial z} + (\frac{\partial\Psi_\theta}{\partial r} + \frac{1}{r}\Psi_\theta) + \mu\frac{\partial^2\Psi_\theta}{\partial z^2}, \tag{16}$$

the symmetric mode and the asymmetric mode give dispersion equations when the boundary conditions $\sigma_{zz} = 0$ and $\sigma_{rz} = 0$ at the boundaries $z = \pm h$ are applied:

$$(k^2 - \beta^2)^2\cos\alpha h\sin\beta h + 4\alpha\beta k^2\sin\alpha h\cos\beta h = 0, \tag{17}$$
$$(k^2 - \beta^2)^2\sin\alpha h\cos\beta h + 4\alpha\beta k^2\cos\alpha h\sin\beta h = 0. \tag{18}$$

If $k = 0$, the dispersion relations are simplified by

$$\cos\alpha h\sin\beta h = 0, \tag{19}$$
$$\sin\alpha h\cos\beta h = 0. \tag{20}$$

The symmetric mode thus satisfies the following conditions:

$$\alpha h = \frac{\omega h}{c_l} = \pi/2, 3\pi/2, 5\pi/2 \cdots,$$
$$\beta h = \frac{\omega h}{c_s} = \pi, 2\pi, 4\pi \cdots. \tag{21}$$

The asymmetric mode similarly satisfies the following conditions:

$$\alpha h = \frac{\omega h}{c_l} = \pi, 2\pi, 4\pi \cdots,$$
$$\beta h = \frac{\omega h}{c_s} = \pi/2, 3\pi/2, 5\pi/2 \cdots. \tag{22}$$

The above conditions provide cut-off frequencies for symmetric modes

$$f_c = \frac{c_l}{2d}, \frac{3c_l}{2d}, \frac{5c_l}{2d} \cdots,$$
$$f_c = \frac{c_s}{d}, \frac{2c_s}{d}, \frac{3c_s}{d} \cdots \tag{23}$$

and cut-off frequencies for asymmetric modes

$$f_c = \frac{c_s}{2d}, \frac{3c_s}{2d}, \frac{5c_s}{2d} \cdots,$$
$$f_c = \frac{c_l}{d}, \frac{2c_l}{d}, \frac{3c_l}{d} \cdots \tag{24}$$

with the plate thickness $d = 2h$.

There are thus two types of Lamb waves, symmetric and asymmetric modes, which are the mixed waves of the longitudinal and transverse waves. Symmetric modes are called $S_0$, $S_1$, $S_2 \cdots$ and asymmetric modes are called $A_0$, $A_1$, $A_2 \cdots$.

## III. MODE CONVERSION IN MULTI-LAYER STRUCTURE

There are three types of mode conversion in Lamb waves: longitudinal mode and transverse mode conversion in a certain mode, symmetric mode and asymmetric mode conversion in a Lamb wave type, and lower mode and higher mode conversion in a mode family. The third type mode conversion can be more specified by the same fd mode conversion and the different fd mode conversion. The same fd mode conversion implies the mode conversion with the same fd value and the different mode conversion implies the mode conversion with the different fd value. The mode conversion can be taken place by one of three types and can be done by the mixing of these types. There is small chance in the first and second types of mode conversion as expected normally. In this paper, therefore, the third type of mode conversion is concentrated on since the spectrum chages of the fundamental frequency and the second harmonic frequency seems to be more relevant to it.

### A. Normal Modes

Normal modes in a multi-layer structured material are studied. Normal modes in the $z$ direction are classified with the fundamental, second harmonic, third harmonic frequency, etc. in frequency domain regardless of longitudinal and transverse waves. As shown in the above, there are two types of normal modes in Lamb waves, one type of which is for the longitudinal mode and the other type of which is for the transverse mode.

If the pressure release conditions at the boundary, $\sigma_{zz} = 0$ and $\sigma_{rz} = 0$, is satisfied, the normal mode waves are produced. The air-layer defect also satisfies the pressure release boundary conditions if the defect is not negligible. Symmetric normal modes are given by (21) and asymmetric normal modes are given by (22). When symmetric modes are considered, the lowest wave length is $d/2$ and the corresponding fundamental frequency is $f_1 = 2v/d$ with the sound velocity $v$. When asymmetric modes are considered, the lowest wave length is $d$ and the corresponding fundamental frequency is $f_1 = v/d$. The second harmonic frequency is $f_2 = 2f_1$ and the third harmonic frequency is $f_3 = 3f_1$. The symmetric $S_0$ mode is faster in velocity and lower in amplitude than the asymmetric $A_0$ mode is in the low fd region, with the frequency f and the thickness d. The symmetric $S_0$ mode is the transverse wave and the asymmetric $A_0$ mode is the longitudinal wave in the low fd value.

### B. Spectrum Levels in Multi-layer Structure

In nondestructive nonlinear acoustic evaluation, the spectrum level changes of the fundamental and second harmonic frequency are observed due to the defect. A possible reason for the spectrum change in multi-layer structure is the mode conversion among normal modes.

Frequency response for two glass plate specimen with the water-layer gap is shown in Figure 1 a). Frequency response for two glass plate specimen with the air-layer gap is shown in Figure 1 b). In the case of the two glasses with the water-layer gap, the spectrum level of the fundamental frequency is high and the spectrum level of the second harmonic frequency is relatively low. However, in the case of the two glasses with the air-layer gap, the spectrum level of the fundamental frequency decreases and the spectrum level of the second harmonic frequency increases. Similarly, three glass plate with air gaps in interfaces increases the second and third harmonic spectra and decreases the fundamental spectra compared with three glass plate with no air gaps in interfaces.

These phenomena are explained by the mode conversion process. Some part of the fundamental spectrum level is converted to the second harmonic spectrum level in the case with the air-layer gap. The fundamental frequency in two glass plate does not exist in one glass plate and the first harmonic frequency in two glass plate becomes the fundamental frequency in the one glass plate. The reason for the spectrum level increase in the second harmonic frequency and the spectrum level decrease in the fundamental frequency is thus that normal modes of the fundamental frequency in two plate with the water-layer gap are converted to normal modes of the second harmonic frequency in two plate with the air-layer gap. In the presence of the air-layer gap, the fundamental frequency in the half thickness plate becomes the second harmonic frequency in the twice thickness plate.

Similarly, the spectrum level increase in the third harmonic frequency and the spectrum level decrease in the fundamental frequency is that normal modes of the fundamental frequency in the three plate specimen with the water-layer gap are converted to normal modes of the third harmonic frequency in the three plate specimen with the air-layer gap.

In the mode conversion, the boundary condition imposed by the defect, which is represented by the air-layer gap, plays the essential role. Two types of pressure normal mode conversion for multiple frequencies are shown in Figure 2. The first group is the easy type in the normal mode conversion, where normal modes satisfy the boundary conditions imposed by the air-layer gap in the interface. The family mode conversion between modes with the fundamental and second harmonic frequencies is possible. The second group is the difficult type in the normal mode conversion where normal modes do not satisfy the boundary conditions imposed by the air-layer gap to maintain the same family mode. Since the condition of the family mode conversion is not maintained, for example, the symmetric mode with the fundamental frequency changes to the asymmetric mode with the second harmonic frequency.

### C. Clues for Mode Conversion

There are several clues for the normal mode conversion in multi-layer structure. A few examples, which can be easily confirmed by measurement, are described.

Mode conversion can take place more in asymmetric mode family than in symmetric mode family. The $A_0$ mode easily produces the second harmonic in the presence of the air-layer gap than the $S_0$ mode does in the low fd region: the $A_0$ mode, which satisfies the boundary condition at the half thickness, is the transverse mode but the $S_0$ mode, which does not satisfy the boundary condition at the half thickness, is the longitudinal mode.

In two glass system, the increase of the second harmonic level is manifest and in three glass system, the increase of the third harmonic level is distinct due to the same fd mode conversion, in which the same fd value is maintained during the mode conversion. In three glass system, the increase of the second harmonic level takes place due to the different fd mode conversion, in which the fd value is changed.

The spectrum level of the fundamental frequency decreases and the spectrum level of the second harmonic increases for the $A_0$ mode, which is dominated by the transverse wave, in the presence of the air-layer gap. The spectrum level ratio of the second harmonic frequency to the fundamental frequency decreases as the fd value increases in the low fd region.

Phase and group velocity changes are expected in the family mode conversion such as the mode conversion from the $A_0$ mode to the $A_1$ mode since each mode has different sound velocity.

## VI. CONCLUSIONS

Nondestructive nonlinear acoustic method as supplementary acoustic evaluation method can be used to detect and locate defects in addition to nondestructive linear acoustic method. The second and higher harmonic modes of incident waves are easily generated from a defect.

In this paper, nondestructive nonlinear acoustic method in two dimensions is used to detect defects in a composite layer-structured material. Spectrum level changes in fundamental and harmonic frequencies are observed in the presence of a layer type defect compared with in the absence of such a defect. It is proposed in this study that such spectrum changes in a layer-structured material are due to mode conversion.

The reason for the spectrum level increase in the harmonic frequency and the spectrum level decrease in the fundamental frequency is that normal modes of the fundamental frequency in the plate with the water-layer gap are converted to normal modes of the harmonic frequency in the plate with the air-layer gap. In the presence of the air gap, the fundamental frequency in a thinner plate becomes the harmonic frequency in a thicker plate whose thickness is the integer multiple of the thinner plate thickness. The mode conversion can take place more in asymmetric mode family than in symmetric mode family due to the boundary condition imposed by the layer-type defect.

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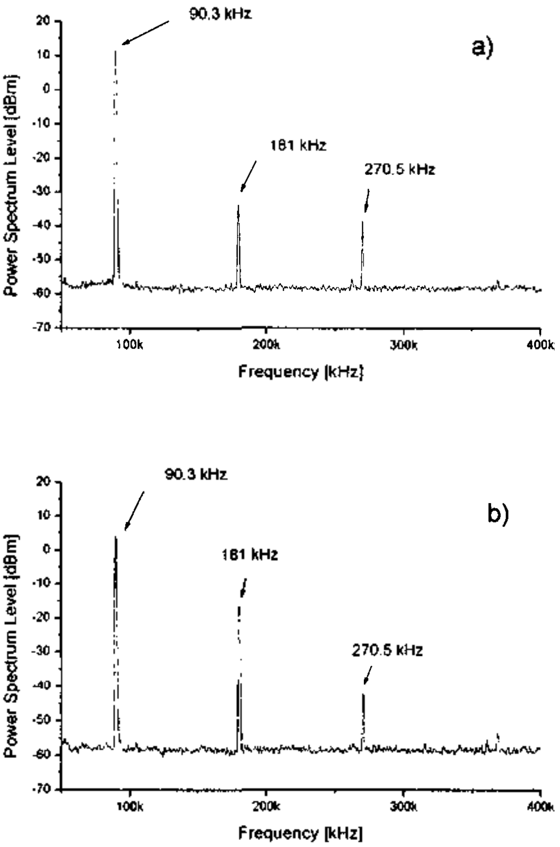


Fig. 1. Frequency response for two glass plates: a) with the water-layer interface b) with the air-layer interface.

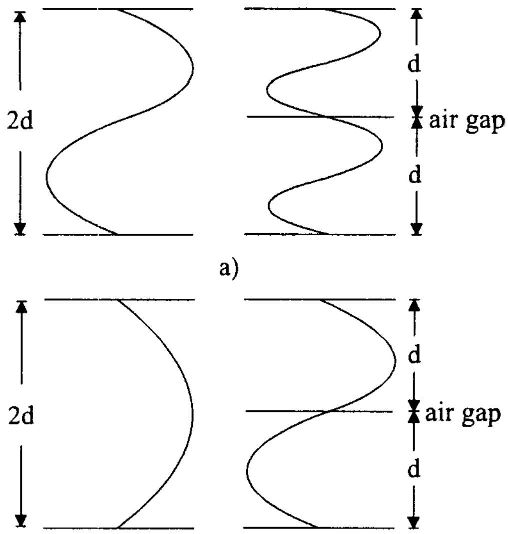


Fig. 2. Pressure mode conversion with multiple frequencies of Lamb waves: a) easy mode conversion b) difficult mode conversion.